Market power, randomization and regulation*

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Abstract

This paper provides an introduction to and overview of the mechanism design approach to textbook monopoly and monopsony pricing problems. Specifically, assuming that agents are privately informed about their values and costs, it shows that the optimal selling and procurement mechanisms quite generally involve rationing, provided the underlying mechanism design problem does not satisfy the regularity assumption of Myerson (1981). Rationing takes the form of underpricing in the case of a monopoly seller and of involuntary unemployment and efficiency wages in the case of a monopsony employer. The paper illustrates these phenomena, as well as the effects of price ceilings and minimum wages, with a leading example that permits closed-form solutions. It also explains why resale tends to undermine the firm's benefits from rationing without eliminating them and discusses emerging issues for the theory of regulation.

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JEL-Classification: C72, D47, D82

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1 Introduction

Monopoly and monopsony pricing problems are of long-standing interest in economics and have received renewed attention in the digital age with the emergence of large online platforms. Traditional analysis restricts the firm's contracting space by assuming that the firm sets a uniform price or wage. This assumption serves two purposes. First, it introduces a tradeoff between social surplus and profit. This tradeoff is central to much of economics and would not arise if firms could perfectly price discriminate. Second, it makes the analysis tractable. At times, restrictions of this form are justified on empirical grounds; for example, by citing the observation that uniform pricing is frequently seen in practice. In contrast, perfect price discrimination is generally not observed—a fact that is sometimes explained by invoking frictions that make it costly for the firm to set too many prices or by referring to resale that prevents the firm from engaging in price discrimination.

In this paper, we provide an introduction to and overview of an alternative approach to otherwise textbook monopoly and monopsony problems that leverage mechanism design techniques. This approach assumes that the agents—consumers or workers—are privately informed about their values and costs without imposing any constraints on the contracts the firm can offer. It has a number of appealing properties. First, perfect price discrimination is simply not possible: If the allocation of a consumer who is privately informed about its value is the same for two or more of its possible values, then its payment must be the same. (The argument is simple and rudimentary — the consumer would optimally behave as the type whose payment is the smallest if the payments differed.) Second, it predicts that the firm optimally uses a uniform price or wage under some conditions and two prices or wages otherwise. Almost ironically, the mechanism design approach thus shows that having a single price or wage is not without loss of generality but that allowing for two prices or wages is. Third, this approach is not only predictive about when the firm price discriminates but also about the form this discrimination takes. When a monopoly seller optimally uses two prices, it does so to deliberately induce excess demand and random rationing at the low price, and when a monopsony employer sets two wages for workers of the same productivity, it does to to induce excess labor supply and involuntary unemployment at the high wage, which can thus be interpreted as an *efficiency wage*. While phenomena like underpricing are often thought to be puzzling[1] and to require explanations that involve some sort of frictions, they arise in the incomplete information framework merely as the result of optimal behaviour by a firm that exerts market power.

Random rationing and involuntary unemployment that arise from the exertion of market

[1]See, for example, Becker (1991).

power also offer new scope for price and wage regulation because—even keeping the quantity sold or the level of employment fixed—social surplus and consumer or worker surplus can be increased by imposing appropriately chosen price ceilings or minimum wages. Perhaps most fundamentally, inherent in the incomplete information approach is a tradeoff between social surplus and profit or rent extraction that derives from the primitives of the model.

Of course, the allocative inefficiency associated with randomly rationing consumers gives rise to the possibility of efficiency-restoring resale markets. While in models with price discrimination resale is often ruled out, the incomplete information approach offers a more nuanced analysis of the effects of resale. Specifically, if resale is modelled as a perfectly competitive market that operates with some known probability, then one can still deploy the mechanism design machinery to derive the optimal selling mechanism in the presence of resale. Among other things, this analysis shows that even though the benefits from random rationing decrease as the probability that the resale market operates increases, they only vanish in the limit as the resale market becomes perfectly competitive.[2] Perhaps surprisingly, resale can harm consumers because it can induce the firm to sell a substantively smaller quantity than it would sell without resale.[3]

The mechanism design approach also reveals novel aspects of price ceilings and minimum wages. For example, because involuntary unemployment can arise under the optimal mechanism under laissez-faire, appropriately chosen minimum wages can reduce and potentially eliminate involuntary unemployment. Similarly, price ceilings can reduce the extent of rationing, even to the point of eliminating it. That said, these regulatory instruments are not a panacea. As a case in point, increasing a minimum wage may induce involuntary unemployment, even as it increases total employment. If this happens, a marginal increase in the minimum wage reduces social surplus because the benefit derived from increased employment is second order to the harm resulting from the random allocation associated with involuntary unemployment. By the same token, even if it increases output, a binding price ceiling can make rationing optimal and thereby reduce social and consumer surplus. Moreover, it is possible that in the presence of a minimum wage or a price ceiling, some workers are paid more than the marginal product of labor and that some consumers pay less than the marginal cost of production. Interestingly, the feature of pricing below marginal costs—which is usually associated with predatory or entry-deterring pricing—arises here solely as part of the optimal selling mechanism in the presence of a price ceiling.

[2] Unless workers can subcontract, resale—or its labor-market equivalent, subcontracting—is not a concern in labor applications. However, as we will show, laws requiring pay transparency (Cullen and Pakzad-Hurson, 2023) can be interpreted as having the same effects as resale to the extent that they diminish a firm's ability to use discriminatory wages.

[3] See Loertscher and Muir (2022).

In many applications of interest, such as issues pertaining to consumers' privacy and the value of consumer data, incomplete information is an indispensable assumption. More fundamentally, the incomplete information approach has, as mentioned, the benefit of having a tradeoff between social surplus and profits that derives from the primitives of the setup rather than from restrictions of the contracting space.[4] In light of the *Lucas critique*, this property is valuable because it disciplines the analysis and ensures optimal behavior before and after a change in the environment of interest.[5] This is particularly so in monopoly and monopsony models. For example, consider a situation in which a firm serving one market sets a uniform price and suppose that this market is then integrated into another market (perhaps due to the internet). Analysis based on contractual restrictions—having observed uniform pricing, the firm is assumed to use a single price—would maintain the assumption that the firm uses a single price after market integration. In contrast, the mechanism design approach merely stipulates that the firm uses the optimal mechanism before and after integration, and whether that involves one or two prices after integration then depends on the specifics of the setting. In contrast to oligopoly models, where the notion of equilibrium provides a disciplining device and at least partial immunity to the Lucas critique, it is difficult to see where that discipline can could come from in models involving a single firm other than from incomplete information and the assumption that the firm behaves optimally, given the constraints incomplete information entails.

The purpose of this paper is to provide a summary of and an accessible introduction to the mechanism design approach to monopoly and monopsony pricing problems. The settings we study are based on standard, textbook models with only two points of departure. First, the firm is assumed to use the optimal mechanism, which is subject to the agents' incentive compatibility and individual rationality constraints and well-defined because the agents are privately informed about their values or costs. In particular, and in contrast to much of the literature, the firm is not restricted to setting a uniform price or wage. Second, no restrictions on the curvature of the firm's revenue or procurement cost function are imposed.[6] As we will see, these assumptions are related insofar as a firm optimally

[4]This point is also made by Loertscher and Marx (2022).

[5]The Lucas critique, named after Lucas (1976), has been most influential in macroeconomics. In a nutshell, it stipulates that models that are fitted to match historical data are not robust to changes in the policy environment. Evidently, the same applies to models in industrial organization.

[6]Like much of the theoretical literature, the empirical literature in IO, as well as the literature concerning the empirics of auctions, typically assumes that revenue is concave. However, when the assumption of concavity of revenue—or, equivalently, monotonicity of the virtual value function—is tested, it is often rejected. See, for example, Celis et al. (2014), Appendix D in Larsen and Zhang (2018) and Section 5 in Larsen (2021)). Henderson et al. (2012) development a method that imposes monotonicity of virtual value functions under nonparametric estimation. If this method is required, then this suggests that the underlying data-generating process may not generate a monotone virtual value function.

departs from a uniform price or wage only if the revenue or procurement cost function is not concave or convex, respectively. For illustrative purposes, much of the analysis here focuses on fully parameterized specifications in which the demand or labor supply function is piecewise linear. However, the analysis generalizes completely. Interested readers are referred to Loertscher and Muir (2022, 2024a).

The literature the present paper relates to includes analyses of monopoly pricing problems without restricting the revenue function to be concave which, following Myerson (1981), is often referred to as *regularity*. The monopoly and monopsony pricing literature that works with the textbook model in which the demand and revenue function are given includes Hotelling (1931), Mussa and Rosen (1978), Wilson (1987), Bulow and Roberts (1989) and our aforementioned papers (Loertscher and Muir, 2022, 2024a). Relative to Loertscher and Muir (2024a), which studies a monopsony problem, the present paper provides a closed-form solution for the optimal mechanism in the presence of a minimum wage for the piecewise linear specification and translates this to the monopoly problem. Compared to Loertscher and Muir (2022), it provides an alternative proof of the optimality of a two-price mechanism under laissez-faire, and it extends the analysis concerning the implications of resale for consumer surplus by studying settings with continuous and increasing marginal costs of production. Incomplete information industrial organization ("Triple-IO") models are also analyzed by Loertscher and Marx (2019, 2022) but these differ from the textbook settings studied here by having aggregate uncertainty in the form of independent private values and in the latter case by treating the market as the mechanism designer. Analysis of market power in labor markets and the employment-increasing effects of minimum wages dates back to Robinson (1933) and Stigler (1946) and has received renewed interest recently; see, for example, Card (2022a,b), and, of course, Card and Krueger (1994). The labor market model we analyze makes the same assumptions regarding payoff functions as Lee and Saez (2012). Yellen (1984) surveys earlier work on efficiency-wage theories of involuntary unemployment, noting that "[a]ll these models suffer from a similar theoretical difficulty—that employment contracts more ingenious than the simple wage schemes considered, can reduce or eliminate involuntary unemployment." In contrast, in the incomplete information model studied here, involuntary unemployment, when it occurs, is optimal for the firm. Armstrong and Vickers (1991) analyze the welfare effects of price discrimination across markets by a monopolist who faces an "average price" cap. In contrast, the analysis of price regulation and price discrimination in the present paper as well as in Loertscher and Muir (2024a, 2022) focuses on homogeneous goods markets in which two-price (or two-wage) mechanisms can become optimal if the underlying mechanism design problem the firm faces is not regular.

The remainder of this paper is organized as follows. Section 2 introduces the setups. In

Section 3, we derive the conditions under which rationing that takes the form of underpricing or involuntary employment is optimal, and we analyze the effects of resale that arises from random rationing. Price ceilings and minimum wages are analyzed in Section 4, and Section 5 concludes the paper.

2 Setup

In this section we introduce two settings: a monopoly pricing problem and a monopsony pricing problem.

2.1 Monopoly pricing problem

We consider a monopoly firm that sells units of a homogeneous good to a unit mass of risk-neutral consumers with single-unit demand, quasilinear utility and private values $v \in [0,1]$. The expected payoff of a consumer with value $v \in [0,1]$ that receives a unit of the good with probability $x \in [0,1]$ and makes a payment of $p \geq 0$ to the monopsony upon receiving the good is given by $x(v-p)$. The firm faces a commonly known inverse demand function $P(Q)$ satisfying $P' < 0$ and a cost function $K(Q) \geq 0$ that is convex and continuously differentiable in $Q \in [0,1]$. Both P and K are such that $P(0) > K'(0)$ and $P(1) < K'(1)$. Much of our analysis assumes

$$P(Q) = \begin{cases} 1 - \frac{8}{3}Q, & \text{if} \quad Q \in [0, 1/4) \\ \frac{4}{9}(1-Q), & \text{if} \quad Q \in [1/4, 1] \end{cases}. \tag{1}$$

This inverse demand function is illustrated in Figure 1 and implies the demand function $D(p) = \frac{1}{4}(4-9p)$ for $p \in [0, 1/3]$ and $D(p) = \frac{3}{8}(1-p)$ for $p \in (1/3, 1]$. For concreteness, we will often use the specification $K(Q) = \frac{1}{10}Q^2$ for the cost function.

Under market-clearing (or uniform) pricing, the firm's revenue $R(Q)$ from selling Q units is given by

$$R(Q) = QP(Q) = \begin{cases} Q\left(1 - \frac{8}{3}Q\right), & Q \in [0, 1/4) \\ \frac{4}{9}Q(1-Q), & \text{if} \quad Q \in [1/4, 1] \end{cases}.$$

As we will see, an object that will play an important role in the analysis is the concavification $\overline{R}$ of the function revenue function R, which is the smallest concave function $\overline{R}$ such that

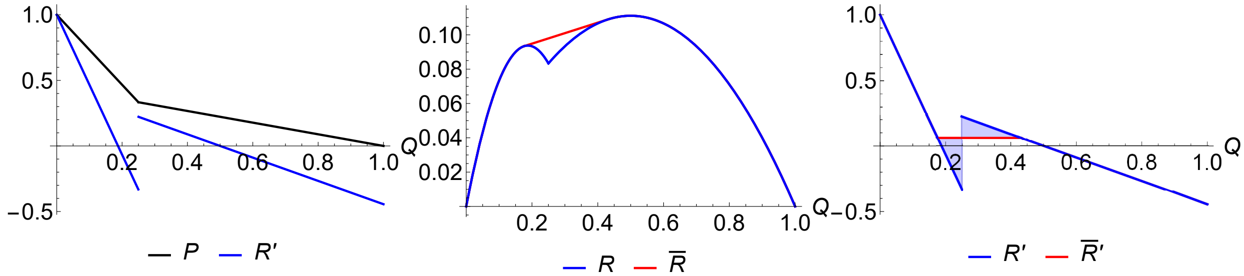

Figure 1: The left-hand panel displays the inverse demand function P (black) and the marginal revenue function R' (blue). The centre panel displays the revenue function R (blue) and its concavification $\overline{R}$ (red). The right-hand panel displays the marginal revenue function R' (blue) and the ironed marginal revenue function $\overline{R}'$ (red).

$\overline{R}(Q) \geq R(Q)$ holds for all $Q \in [0,1]$. For P in (1), this function is given by

$$\overline{R}(Q) = \begin{cases} Q\left(1 - \frac{8}{3}Q\right), & Q \in [0, Q_1^*) \\ \left(1 - \alpha\left(Q, Q_1^*, Q_2^*\right)\right) R(Q_1^*) + \alpha(Q, Q_1^*, Q_2^*) R(Q_2^*), & Q \in [Q_1^*, Q_2^*] \\ \frac{4}{9}Q(1-Q), & Q \in (Q_2^*, 1] \end{cases},$$

where $Q_1^* = \frac{6+\sqrt{6}}{48}$, $Q_2^* = \frac{1+\sqrt{6}}{8}$ and, for $Q, Q_1, Q_2 \in [0,1]$ with $Q_1 < Q < Q_2$, $\alpha(Q, Q_1, Q_2) := \frac{Q-Q_1}{Q_2-Q_1}$. Note that $\overline{R}$ is a linear function of Q on the interval $[Q_1^*, Q_2^*]$ and that Q_1^* and Q_2^* are pinned down by the first-order condition

$$R'(Q_1^*) = R'(Q_2^*) = \frac{R(Q_2^*) - R(Q_1^*)}{Q_2^* - Q_1^*}.$$

Figure 1 also illustrates the revenue function R and its concavification $\overline{R}$ as well as the derivatives of these functions.

2.2 Monopsony pricing problem

We consider a monopoly firm that faces a unit mass of equally productive risk-neutral workers with quasilinear utility. Each worker inelastically supplies a single unit of labor at a private opportunity cost $c \in [0,1]$. The expected payoff of a worker with cost $c \in [0,1]$ that supplies $x \in [0,1]$ units of labor at a wage of $w \geq 0$ is therefore given by $x(w-c)$. The firm faces a commonly known inverse labor supply function $W(Q)$ satisfying $W' > 0$, which for much of

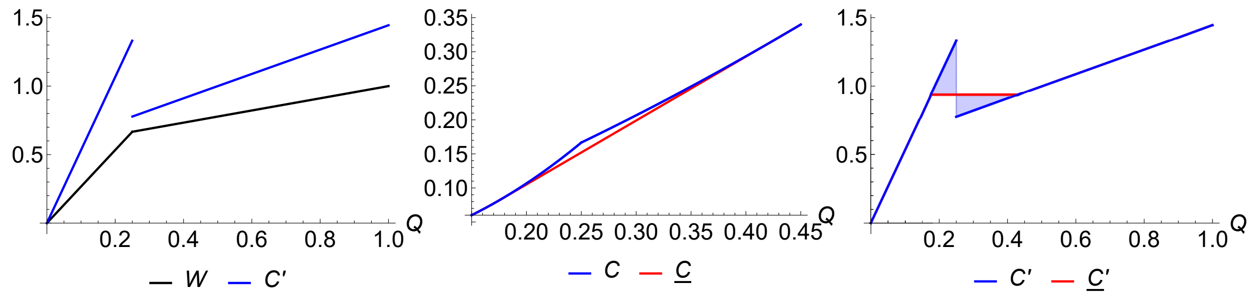

Figure 2: The left-hand panel displays the inverse labor supply function W (black) and the marginal cost function C' (blue). The center panel displays the procurement cost function C (blue) and its convexification $\underline{C}$ (red). The right-hand panel displays the marginal cost function C' (blue) and the ironed marginal cost function $\underline{C}'$ (red).

the analysis is assumed to be given by

$$W(Q) = \begin{cases} \frac{8}{3}Q, & \text{if} \quad Q \in [0, 1/4) \\ \frac{1}{9}(5+4Q), & \text{if} \quad Q \in [1/4, 1] \end{cases}. \tag{2}$$

A brief inspection show that W in (2) is such that $W(Q) = 1 - P(Q)$, where $P(Q)$ given by (1). The firm has a marginal revenue product of labor function $V(Q)$ that is concave and continuously differentiable in $Q \in [0, 1]$, satisfying $V(0) > W(0)$ and $V(1) < W(1)$. If the firm sets a market-clearing wage, then the firm's cost of procuring Q units of labor is given by

$$C(Q) = QW(Q) = \begin{cases} \frac{8}{3}Q^2, & \text{if} \quad Q \in [0, 1/4) \\ \frac{1}{9}Q(5+4Q), & \text{if} \quad Q \in [1/4, 1] \end{cases}.$$

For this problem, an object that will play an important role in the analysis is the convexification $\underline{C}$ of the cost function C, which is the largest convex function $\underline{C}$ such that $\underline{C}(Q) \le C(Q)$ for all $Q \in [0, 1]$. To compute the convexification $\underline{C}$, we can exploit the fact that $C(Q) = Q - R(Q)$, as this implies that[7]

$$\underline{C}(Q) = Q - \overline{R}(Q) = \begin{cases} \frac{8}{3}Q^2, & Q \in [0, Q_1^*) \\ (1 - \alpha(Q, Q_1^*, Q_2^*))\, C(Q_1^*) + \alpha(Q, Q_1^*, Q_2^*) C(Q_2^*), & Q \in [Q_1^*, Q_2^*] \\ \frac{1}{9}Q(5 + 4Q), & Q \in (Q_2^*, 1] \end{cases}.$$

An illustration of the inverse labor supply function W, the cost function C and its convexification $\underline{C}$, as well as the derivatives C' and $\underline{C}'$ is provided in Figure 2.

2.3 Motivation of assumptions

A brief discussion of a number of assumptions is warranted. The assumptions that P (as in (1)) is such that that R is non-concave and similarly that W (as in (2)) is such that C is not convex are, admittedly, not standard. Readers might therefore, quite rightly, ask why one should be interested in problems of this form and, specifically, why R should be non-concave or C be non-convex. There are, at least, three mutually non-exclusive reasons for why these problems are of interest.

First, even though the concavity and convexity assumptions are standard, there is no intrinsic underlying economic rationale for why R should be concave or C should be convex. As we will see, under market-clearing prices and wages, little is gained by dropping these standard assumptions, but of course, there is nothing in the real world per se that prevents firms from departing from such uniform prices and wages and, more fundamentally, that makes these convenient curvature assumptions the empirically relevant ones. Second, while there are no reasons per se for R to be concave or C to be convex, there are compelling reasons for the opposite. Consider, for example, the integration of two markets (i.e. demand sides) into a single one. This may occur due to globalization or through digitalization. If the maximum willingness to pay in the standalone markets differs prior to integration, then the resulting revenue function R after integration is not concave even if it is concave in each standalone market prior to integration.[8] Finally, there is, as mentioned in footnote 6 in

[7] This is a general feature in the following sense. For any real-valued function H, let $\overline{H}$ denote its concavification and $\underline{H}$ denote its convexification. Letting $J(x) = x - H(x)$, we have

$$\underline{J}(x) = x - \overline{H}(x). \tag{3}$$

To establish (3), suppose to the contrary that it is violated at some $\hat{x}$. Assume first that $\underline{J}(\hat{x}) > \hat{x} - \overline{H}(\hat{x})$ holds. This is equivalent to $\hat{x} - \underline{J}(\hat{x}) < \overline{H}(x)$. Since $\underline{J}(x)$ is convex, both $-\underline{J}(x)$ and $x - \underline{J}(x)$ are concave, contradicting that $\overline{H}(x)$ is the smallest concave function not smaller than H. Conversely, if $\underline{J}(\hat{x}) < \hat{x} - \overline{H}(\hat{x})$ holds for some $\hat{x}$, this contradicts that $\underline{J}(x)$ is the largest convex function not bigger than $x - H(x)$ since $x - \overline{H}(x)$ is convex by the concavity of $\overline{H}(x)$.

[8] See Loertscher and Muir (2022) for a formal argument.

the Introduction, empirical evidence in IO and in the literature on the empirics of auction showing that R is not concave—or, equivalently, that R' is not monotone—in many data sets. While we are not aware of any parallel studies in labor markets, it would be surprising if C were consistently found to be convex—or, equivalently, C' were consistently found to be monotone—as this would mean that a property that is frequently violated for output markets would systematically hold for input markets.

3 Optimality of rationing

In this section we determine the optimal selling mechanism for the monopoly problem and the optimal procurement mechanism for the monopsony problem introduced in Section 2, as well as the associated optimal level of production and employment. We then anaylyze resale that arises from random rationing.

3.1 Underpricing in product markets

We now return to the monopoly pricing problem introduced in Section 2. The significance of the concavification $\overline{R}$ of the monopoly's revenue function R is that $\overline{R}(Q)$ is the monopoly's revenue when it sells Q units using an optimal selling mechanism. This result, which follows from Mussa and Rosen (1978) as well as the ironing procedure of Myerson (1981), is formally stated in the following proposition.

Proposition 1. *Revenue under any optimal mechanism for selling the quantity Q is given by $\overline{R}(Q)$. The optimal quantity sold Q^* then satisfies the first-order condition*

$$\overline{R}'(Q^*) = K'(Q^*).$$

Whenever Q^* is such that $\overline{R}(Q^*) > R(Q^*)$, the optimal selling mechanism involves rationing and underpricing *(i.e. a positive mass of consumers that pay a price strictly below the market-clearing price $P(Q^*)$).*

Figure 3 illustrates the first-order condition in Proposition 1. The left-hand panel plots marginal revenue under market-clearing (uniform) pricing and the marginal cost associated with the cost function $K(Q) = \frac{1}{10}Q^2$. There are two points of intersection between the marginal revenue function R' and the marginal cost function K', each of which characterizes a local maximum since $R'' - K'' < 0$ and which of the two is the global maximum under market-clearing pricing depends on the specifics of the model. This cumbersome multiplicity of local maxima under market-clearing pricing possibly explains why the assumption of

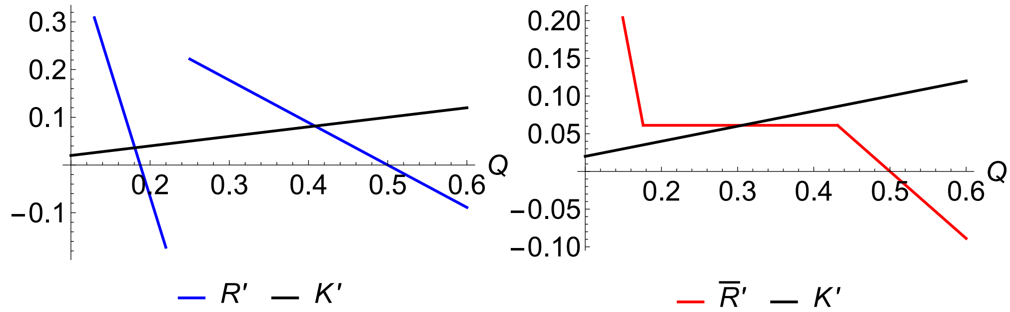

Figure 3: The right-hand panel provides an illustration of Proposition 1 for $K(Q) = \frac{1}{10}Q^2$. The quantity Q^* that is produced by the monopoly under the optimal selling mechanism is given by the intersection of $\overline{R}'$ (red) and K' (black). The left-hand panel shows the multiplicity of local maxima that arise when the firm is restricted to uniform price-setting.

monotone marginal revenue (or, equivalently, concave revenue) is so widespread: Few insights seem to be available from relaxing the assumption under market-clearing pricing, but the analysis becomes considerably more complicated. The right-hand panel depicts $\overline{R}'$, which is weakly monotone, and the marginal cost function K'. Provided the marginal cost is strictly increasing, there is a unique solution to the first-order condition. If this intersection occurs at a Q^* such that $\overline{R}(Q^*) > R(Q^*)$ (which is the case for the specification depicted) then the monopoly optimally uses a selling mechanism that involves rationing and underpricing.

In contrast, if the marginal cost schedule is constant, then there is either a single point of intersection with $\overline{R}'$ (which occurs outside the interval $[Q_1^*, Q_2^*]$) or there is a continuum of quantities such that $\overline{R}'$ is equal to the marginal cost schedule. The latter case arises if and only if the constant marginal cost happens to be equal to $\overline{R}'(Q)$ for $Q \in [Q_1^*, Q_2^*]$. All $Q \in (Q_1^*, Q_2^*)$ are associated with optimal selling mechanisms that involve rationing and underpricing. However, as noted by Bulow and Roberts (1989), a selling mechanism with these features is never uniquely optimal if the marginal cost schedule is constant since the monopoly could always make the same profit without inducing rationing by choosing $Q = Q_1^*$ or $Q = Q_2^*$. Yet, as we will see below, a binding price ceiling may make rationing uniquely optimal even when the production technology exhibits a constant marginal cost function.

To prove Proposition 1, we start by explicitly constructing a mechanism for selling the quantity Q that achieves revenue $\overline{R}(Q)$. In particular, we introduce a class of mechanisms that will play an important role throughout this paper: *two-price mechanisms*.

Consider the problem of selling Q units using a mechanism involving two prices (p_1, p_2) with $p_1 > p_2 > 0$ and construct an equilibrium such that consumers pay the high price p_1 in order to purchase the good with certainty, while there is rationing at the lower price of p_2.

Such a mechanism can be parameterized by three quantities (Q, Q_1, Q_2) with $Q_1 < Q \le Q_2$.[9] We construct an equilibrium such that Q_1 is the mass of consumers that purchase a unit of the good at the high price p_1 and Q_2 is the total mass of consumers that participate in the mechanism. Notice that this implies that the probability of service for consumers that attempt to purchase a unit in the lottery at the low price p_2 are allocated a good with probability $\alpha(Q, Q_1, Q_2) := \frac{Q-Q_1}{Q_2-Q_1}$. The two prices $p_1(Q, Q_1, Q_2)$ and $p_2(Q_2)$ that implement this equilibrium are then pinned down by the consumers' incentive compatibility and individual rationality constraints. In particular, the individual rationality constraint for consumers with a valuation of $P(Q_2)$—and are indifferent between not participating in the selling mechanism and participating in the lottery—implies that $p_2(Q_2) = P(Q_2)$. Moreover, the incentive compatibility constraint for consumers with a valuation of $P(Q_1)$, who must be indifferent between paying the high price $p_1(Q, Q_1, Q_2)$ to receive the good with certainty and participating in the lottery where they obtain the good at the low price $p_2(Q_2)$ with probability $\alpha(Q, Q_1, Q_2)$, implies that $P(Q_1) - p_1(Q, Q_1, Q_2) = \alpha(Q, Q_1, Q_2)(P(Q_1) - p_2(Q_2))$.[10] Substituting $p_2(Q_2) = P(Q_2)$ into this expression and rearranging yields

$$p_1(Q, Q_1, Q_2) = (1 - \alpha(Q, Q_1, Q_2))P(Q_1) + \alpha(Q, Q_1, Q_2)P(Q_2). \tag{4}$$

Notice that the high price $p_1(Q, Q_1, Q_2)$ is given by a convex combination of the market-clearing price $P(Q_1)$ for selling Q_1 units and the market-clearing price $P(Q_2)$ for selling Q_2 units, where the weight on the lower price $P(Q_2)$ is given by the probability $\alpha(Q, Q_1, Q_2)$ of success in the lottery. Consequently, $p_1(Q, Q_1, Q_2)$ is a decreasing function of Q.

The seller's revenue $R(Q, Q_1, Q_2)$ under the two-price mechanism parameterized by (Q, Q_1, Q_2) is then given by

$$\begin{aligned} R(Q, Q_1, Q_2) &= Q_1 p_1(Q, Q_1, Q_2) + (Q - Q_1)p_2(Q_2) \\ &= (1 - \alpha(Q, Q_1, Q_2))Q_1 P(Q_1) + P(Q_2)[\alpha(Q, Q_1, Q_2)Q_1 + (Q - Q_1)]. \end{aligned}$$

[9] Note that by allowing for the possibility that $Q_2 = Q$ we capture single-price mechanisms that involve setting a market-clearing price ($Q_1 = 0$ and $Q_2 = Q$) or rationing at a single price ($Q_1 = 0$ and $Q_2 > Q$) as a special case of two-price mechanisms.

[10] A single-crossing property that holds for this setting implies that all consumers with values above $P(Q_1)$ then strictly prefer paying the high price $p_1(Q, Q_1, Q_2)$ in order to secure a unit with certainty, all consumers with values between $P(Q_2)$ and $P(Q_1)$) strictly prefer participating in the lottery at the low price $p_2(Q_2)$ and all consumers with values below $P(Q_2)$ strictly prefer to not participate in the mechanism.

Using $Q - Q_1 = \alpha(Q, Q_1, Q_2)(Q_2 - Q_1)$ this simplifies to

$$\begin{aligned} R(Q, Q_1, Q_2) &= (1 - \alpha(Q, Q_1, Q_2))Q_1 P(Q_1) + \alpha(Q, Q_1, Q_2)Q_2 P(Q_2) \\ &= (1 - \alpha(Q, Q_1, Q_2))R(Q_1) + \alpha(Q, Q_1, Q_2)R(Q_2). \end{aligned}$$

We can therefore see that the seller's revenue is also given by a convex combination of the revenue from selling Q_1 units at the market-clearing price $P(Q_1)$ and the revenue from selling Q_2 units at the market-clearing price $P(Q_2)$, where the weight on the revenue associated with selling the quantity Q_2 is given by the probability $\alpha(Q, Q_1, Q_2)$ of success in the lottery. By construction, the seller's maximum revenue from selling the quantity Q under a two-price mechanism is then given by

$$\max_{Q_1 < Q \leq Q_2} R(Q, Q_1, Q_2) = \max_{Q_1 < Q \leq Q_2} \{(1 - \alpha(Q, Q_1, Q_2))R(Q_1) + \alpha(Q, Q_1, Q_2)R(Q_2)\} = \overline{R}(Q)$$

and the quantities Q_1^* and Q_2^* that parameterize the optimal two-price selling mechanism correspond to those computed in Section 2.

We have now explicitly constructed a mechanism that achieves revenue $\overline{R}(Q)$ from selling the quantity Q. It remains to show that the seller cannot do any better than this. We refer the reader to the appendix for this argument.

There are numerous ways in which the monopoly seller could implement the optimal selling mechanism whenever it takes the form of a non-degenerate two-price mechanism that involves rationing and underpricing. However, one natural implementation is a dynamic implementation whereby the seller first sets a high price, before then rationing any remaining goods a lower price. Such a format is very common in the events industry (where high-priced tickets are frequently sold first) and in the fashion industry in the form of seasonal sales.

3.1.1 Resale

As we just noted in the previous section, whenever the optimal quantity sold by the seller is such that $Q^* \in (Q_1, Q_2)$ (in which case $\overline{R}(Q^*) > R(Q)$), the optimal selling mechanism involves rationing for all consumers with $v \in [P(Q_2^*), P(Q_1^*)]$ that participate in the lottery offered at the low price $p_2(Q_2^*)$. This creates scope for gains from trade in a resale market. Loertscher and Muir (2022) analyze a variety of specifications of such resale markets that differ with respect to their generality and tractability. Using a simple revealed preference argument Loertscher and Muir show that the primary market seller is always harmed by resale markets. The implications of resale for the welfare of consumers is more subtle. In particular, holding the primary market selling mechanism fixed, resale is always in the interest of consumers who benefit from realizing any opportunities for trade created by rationing in the primary market. If the primary market seller responds to the possibility of resale by also increasing the equilibrium quantity sold, then consumers clearly benefit from resale. However, the primary market seller may respond to the possibility of resale by reducing the equilibrium quantity sold in the primary market. If this *quantity effect*—which harms consumers—is sufficiently large, then consumers may be worse off as a result of resale. In such cases, both the primary market seller and consumers are better off when resale is banned.

A particularly tractable model that captures the first-order effects of resale is one that Loertscher and Muir (2022) refer to as *ρ-competitive resale*. Under ρ-competitive resale, a perfectly competitive resale market operates following the primary market with probability $\rho \in [0, 1]$ and no resale market operates otherwise. Clearly, the primary market seller can always foreclose the resale market by simply setting a market-clearing price in the primary market. So suppose instead that the primary market seller uses a two-price mechanism parameterized by the quantities (Q, Q_1, Q_2). If the resale market operates then the equilibrium price in the resale market is $P(Q)$. Consumers with values $v \in [P(Q_2), P(Q))$ that secured a unit in the primary market become sellers in the resale market and consumers with values $v \in (P(Q), P(Q_1)]$ that were rationed in the primary market become buyers in the resale market. The quantity traded in the resale market is $\frac{(Q_2-Q)(Q-Q_1)}{Q_2-Q_1}$.

By focusing on ρ-competitive resale, we can provide a simple illustration of the main results of Loertscher and Muir (2022) because this specification permits a closed-form characterization of optimal primary market mechanism in the face of resale. In particular, if the resale market operates then any consumer with $v < P(Q)$ that is allocated a good in the primary market will sell that good in the resale market at the market-clearing price of $P(Q)$. Similarly, any consumer with $v > P(Q)$ that does not receive a good in the primary market will buy that good in the resale market at the market-clearing price of $P(Q)$. Consequently, the willingness to pay in the primary market for any consumer with value v is $(1-\rho)v + \rho P(Q)$. Let $\overline{R}_\rho(Q)$ denote revenue under the optimal mechanism for selling the quantity Q in the primary market with ρ-competitive resale. Repeating the proof of Proposition 1 but with the inverse demand curve replaced by $(1-\rho)v+\rho P(Q)$, we can prove that $\overline{R}_\rho(Q) = (1-\rho)\overline{R}(Q) + \rho R(Q)$. Consequently, for any quantity $Q \notin (Q_1^*, Q_2^*)$ such that $\overline{R}(Q) = R(Q)$, the optimal selling mechanism still involves setting the market-clearing price $P(Q)$ and no resale occurs in equilibrium. Moreover, for any quantity $Q \in (Q_1^*, Q_2^*)$ such that $\overline{R}(Q) > R(Q)$, the optimal selling mechanism is still a two-price mechanism parameterized by the quantities (Q, Q_1^*, Q_2^*). However, the prices that implement these mechanisms need to be adjusted under ρ-competitive resale. Letting $p_2^\rho(Q)$ and $p_1^\rho(Q, Q_1^*, Q_2^*)$

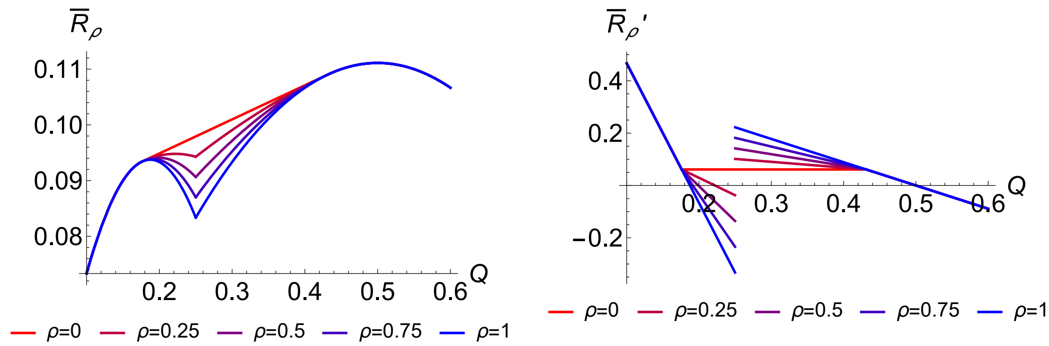

Figure 4: An illustration of the primary market seller's revenue and marginal revenue envelopes under the optimal selling mechanism for a variety of values of $\rho \in [0,1]$.

respectively denote the low and the high price under the optimal two-price mechanism for selling Q units under ρ-competitive resale, we have $p_2^\rho(Q) = (1-\rho)p_2(Q) + \rho P(Q)$ and $p_1^\rho(Q, Q_1^*, Q_2^*) = (1-\rho)p_1(Q, Q_1^*, Q_2^*) + \rho P(Q)$. Finally, let Q_ρ^* denote the optimal quantity sold by the seller under ρ-competitive resale. Note that although Q_ρ^* satisfies $\overline{R}_\rho'(Q_\rho^*) = K'(Q_\rho^*)$, this first-order condition does not necessarily uniquely characterize Q_ρ^* since $\overline{R}_\rho$ is not a concave function for all $\rho \in (0,1]$.

From the preceding analysis, we can immediately see that the seller is always worse off under ρ-competitive resale since we have $\overline{R}(Q) \geq \overline{R}_\rho(Q)$ for all $Q \in [0,1]$ and $\rho \in [0,1]$. An illustration of this is provided in Figure 4 for a variety of values of ρ. As the figure illustrates, as ρ increases from 0 to 1 the revenue envelope $\overline{R}_\rho$ is continuously deformed from $\overline{R}$ to R. Intuitively, as the resale market becomes increasingly effective at implementing an ex post efficient allocation, this undermines the seller's ability to benefit from randomization. For the extreme case where $\rho = 1$ and the resale market is perfectly competitive, the seller cannot benefit from rationing in the primary market and cannot do better than setting a market-clearing price in the primary market.

Whether or not consumers benefit from resale depends on both how effective the resale market is and how the resale market affects the equilibrium quantity sold by the primary market seller. To illustrate this, we consider four specifications of the cost function: $K_1(Q) = 2Q^2 - 0.8Q + 0.1$ (Example 1), $K_2(Q) = 5Q^2 - 3Q + 0.45$ (Example 2), $K_3(Q) = 10Q^2 - 5Q + 0.65$ (Example 3), and $K_4(Q) = 2Q^2 - Q + 0.15$ (Example 4). Each of these examples is illustrated in figures 5 and 6. In each example, for sufficiently small values of ρ, the equilibrium quantity sold under ρ-competitive resale (Q_ρ^*) is close to the equilibrium quantity sold absent resale (Q^*). Consequently, if $Q^* < 1/4$ (as in Example 1), then Q_ρ^* is decreasing in ρ for sufficiently small values of ρ. Moreover, if $Q^* > 1/4$ (as in examples 2, 3 and 4),

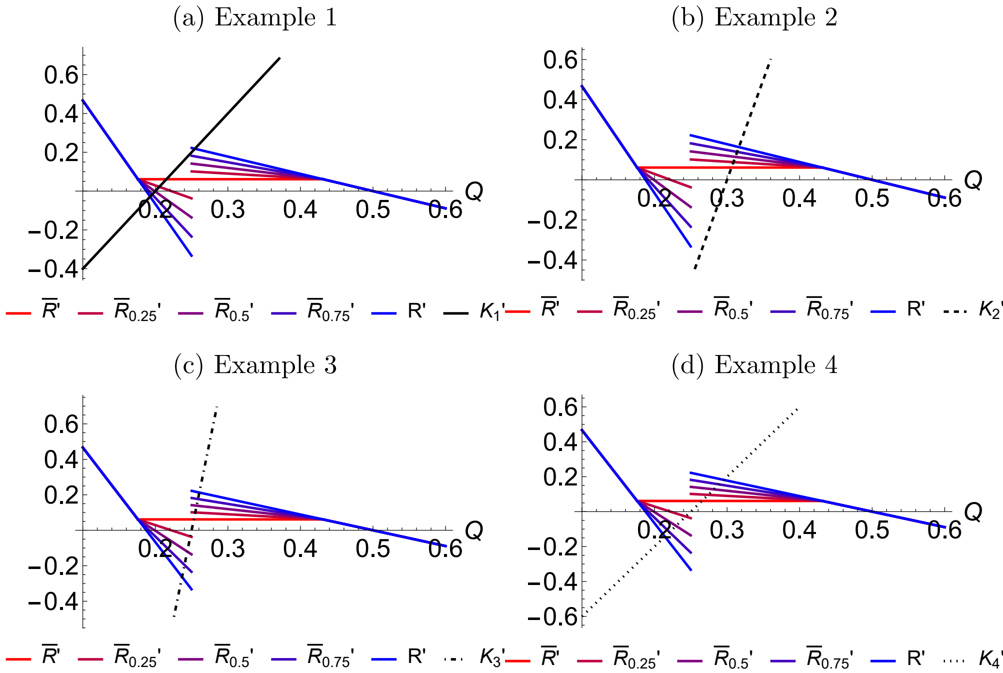

Figure 5: The four example cost functions used to illustrate the implications of ρ-competitive resale for the welfare of consumers.

then Q^*_ρ is increasing in ρ for sufficiently small values of ρ. More generally, for $\rho \in (0, 1]$, the primary market seller's revenue function fails to be concave and its profit function may exhibit up to two local maxima: a low-quantity one with $Q < 1/4$ and a high-quantity one with $Q < 1/4$. For an interval of ρ values such that $Q^*_\rho < 1/4$, Q^*_ρ is decreasing in ρ. Similarly, for an interval of ρ values such that $Q^*_\rho > 1/4$, Q^*_ρ is increasing in ρ. In examples 1 and 2 Q^*_ρ is monotone in ρ. However, in examples 3 and 4 a discontinuous decrease in the equilibrium quantity sold occurs at the value of ρ where the global maximum in the seller's profit function switches from a high-quantity local maximum to a low-quantity local maximum. At such points there is a discontinuous decrease in consumer surplus.

For intervals such that Q^*_ρ is increasing in ρ, consumer surplus is, of course, increasing in ρ. Consumers benefit from both the increase in the quantity sold in the primary market and the increased gains from trade generated by the resale market.[11] Such intervals appear in examples 2, 3 and 4. In contrast, for intervals such that Q^*_ρ is decreasing in ρ, the implications for consumer surplus are more subtle as there are countervailing effects. As ρ increases consumers benefit from the increased gains from trade in the resale market but are

[11] Note that for any $\rho \in [0, 1)$ interval such that $Q^*_\rho \in (Q^*_1, Q^*_2)$, the optimal selling mechanism is a two-price mechanism parameterized by (Q^*_ρ, Q^*_1, Q^*_2). The only parameter of the optimal selling mechanism that then varies with ρ is the quantity sold Q^*_ρ.

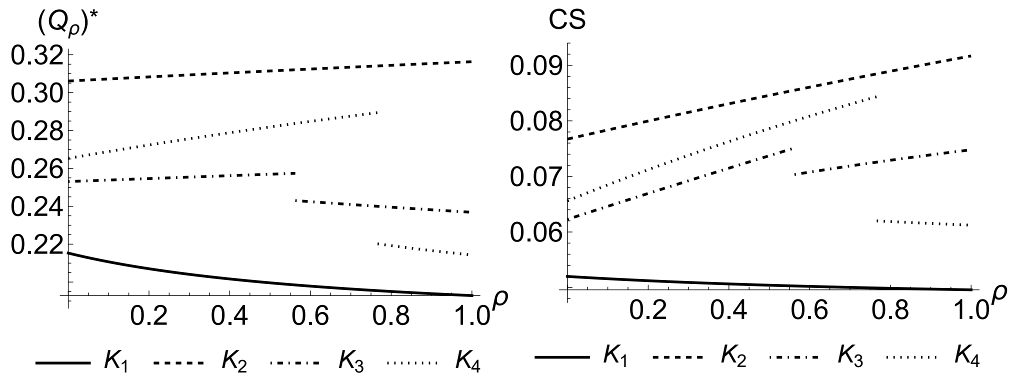

Figure 6: The left-hand panel illustrates the equilibrium quantity sold by the primary market seller as a function of ρ for each of the four cost functions and the right-hand panel provides a corresponding illustration of consumer surplus.

worse off as a result of the decrease in the quantity sold in the primary market. Examples 1 and 4 exhibit intervals where both the equilibrium quantity sold and consumer surplus decrease in ρ, as the negative quantity effect dominates. Example 3 illustrates an interval where the equilibrium quantity sold decreases in ρ and consumer surplus increases in ρ, as the positive effect associated with increased gains from trade in the resale market dominates.

Example 1 is such that consumers are always harmed by resale, and both consumers and the primary market seller benefit from banning resale. Examples 2 and 3 are such that consumers always benefit from resale, and banning resale harms consumers and benefits the primary market seller. Finally, countervailing effects play a larger role in Example 4, with consumers benefiting from resale for sufficiently small values of ρ and being harmed by resale for larger values of ρ.

3.2 Efficiency wages and involuntary unemployment

We now consider the monopsony pricing problem introduced in Section 2. Similarly to what we saw for the monopsony pricing problem, the significance of the convexification $\underline{C}$ of the monopsony's cost function C is that $\underline{C}(Q)$ represents the monopsony's cost of procurement when it hires Q workers using an optimal procurement mechanism. Analogously to what we had for monopoly pricing, we have the following proposition.

Proposition 2. *The procurement cost under any optimal mechanism for hiring Q workers is given by $\underline{C}(Q)$. The optimal level of employment Q^* then satisfies the first-order condition*

$$\underline{C}'(Q^*) = V(Q^*).$$

Whenever Q^* is such that $\underline{C}(Q^*) < C(Q^*)$, the optimal selling mechanism involves inducing involuntary unemployment *(i.e. rationing of workers) and an* efficiency wage *(i.e. a wage strictly above the market-clearing wage $W(Q^*)$ that is paid positive mass of workers).*

The proof of Proposition 2 is completely analogous to the proof of Proposition 1. In particular, we can restrict attention to *two-wage* selling mechanisms that are parameterized by three quantities (Q, Q_1, Q_2) with $Q_1 < Q \leq Q_2$. Here, Q is the level of employment; Q_1 is the mass of workers that are hired with certainty at a low wage $w_1(Q, Q_1, Q_2)$ and $Q_2 - Q_1$ is the total mass of workers that attempt to secure employment at a high wage $w_2(Q_2)$ and are hired with probability $\alpha(Q, Q_1, Q_2) = \frac{Q-Q_1}{Q_2-Q_1}$. The individual rationality constraint for worker with a cost of $c = W(Q_2)$, who must be indifferent between not participating in the mechanism and participating in the lottery at the high wage, implies that $w_2(Q_2) = W(Q_2)$. The incentive compatibility constraint for workers with a cost of $c = W(Q_1)$, who must be indifferent between securing employment at the low wage with certainty and securing employment with probability $\alpha(Q, Q_1, Q_2)$ at the high wage, implies that

$$w_1(Q, Q_1, Q_2) = (1 - \alpha(Q, Q_1, Q_2))W(Q_1) + \alpha(Q, Q_1, Q_2)P(Q_2).$$

Straightforward calculations then show that the monopsony's cost of procurement $C(Q, Q_1, Q_2)$ under the two-wage procurement mechanism parameterized by (Q, Q_1, Q_2) is given by

$$C(Q, Q_1, Q_2) = (1 - \alpha(Q, Q_1, Q_2))C(Q_1) + \alpha(Q, Q_1, Q_2)C(Q_2)$$

By construction, the seller's minimum cost from hiring Q workers under a two-wage procurement mechanism is then given by

$$\min_{Q_1 < Q \leq Q_2} C(Q, Q_1, Q_2) = \min_{Q_1 < Q \leq Q_2} \{(1 - \alpha(Q, Q_1, Q_2))C(Q_1) + \alpha(Q, Q_1, Q_2)C(Q_2)\} = \underline{C}(Q)$$

and the quantities Q_1^* and Q_2^* that parameterize the optimal two-price procurement mechanism correspond to those computed in Section 2. Repeating the mechanism design analysis from the proof of Proposition 1 then shows that the restriction to two-wage procurement mechanisms is without loss of generality and the two-wage mechanism parameterized by (Q, Q_1^*, Q_2^*) is in fact the optimal mechanism for procuring Q workers. Notice that for $Q \in (Q_1^*, Q_2^*)$, $w_1(Q, Q_1^*, Q_2^*)$ is a strictly increasing function of Q.

As noted in the statement of Proposition 2, the optimal level of employment Q^* then satisfies $\underline{C}'(Q^*) = V(Q^*)$. Moreover, if $\underline{C}(Q^*) < C(Q^*)$ then the optimal procurement mechanism involves an efficiency wage $w_2(Q_2^*)$ and generates involuntary unemployment. In particular, there is a mass of workers $(1 - \alpha(Q^*, Q_1^*, Q_2^*))(Q_2^* - Q_1^*)$ that are rationed at the efficiency wage $w_2(Q_2^*) = W(Q_2^*) > W(Q^*)$. As discussed in Loertscher and Muir (2024a), there are numerous ways in which the firm may ration workers at the high wage. However, one natural possibility is for the firm to hire full-time workers at a low hourly wage proportional to $w_1(Q^*, Q_1^*, Q_2^*)$ and part-time workers (who have a proportion $\alpha(Q^*, Q_1^*, Q_2^*)$ of a full time job) at a high hourly wage proportional to $w_2(Q_2^*)$. Alternatively, the firm can make each worker a random take-it-or-leave-it offer. With probability $\alpha(Q^*, Q_1^*, Q_2^*)$, the wage offer is $W(Q_2^*)$ and with probability $1 - \alpha(Q^*, Q_1^*, Q_2^*)$ it is $W(Q_1^*)$. Workers with opportunity costs below $W(Q_1^*)$ accept either offer whereas workers with opportunity costs of working between $W(Q_1^*)$ and $W(Q_2^*)$ only accept the high wage offer.

3.2.1 Pay transparency

When the optimal level of employment Q^* is such that $\underline{C}(Q^*) < C(Q^*)$, then the optimal procurement mechanism involves wage discrimination. Recent political pressure and legislative changes have aimed at making pay "transparent" (see, for example, Cullen and Pakzad-Hurson (2023)), but wage discrimination and lack of transparency have long been a concern in policy debates.[12] One natural situation where *pay transparency* arises is when firms hire workers using an efficient procurement mechanism, rather than a procurement-cost minimizing mechanism that may involve setting two wages and rationing some workers. To the extent that the imposition of full pay transparency prevents a monopsony firm from engaging in wage discrimination, increasing pay transparency has the same effects as increasing the probability ρ that a competitive resale market operates in the product market setting analyzed above. As we saw there, the effects of increasing ρ on the equilibrium quantity produced and on consumer surplus are rich and non-monotone because of the multiplicity of local maxima under market-clearing pricing. The same will hold in a labor market context.[13]

[12]For example, the influential investigation by the journalist Malcolm Johnson in the late 1940s into criminal behavior and abuse of power on the waterfront in New York City lamented, among other things, that the widely used 'shape-up" (the hiring scheme depicted in movies such as "Cinderella Man" and "On the Waterfront" whereby workers waiting in a line are randomly offered a day job or are sent home) lacked transparency about wages and vacancies. Subsequently, it was replaced by a centralized hiring scheme, essentially an efficient auction, that resulted in wage transparency by clearing the market at a uniform wage. See Johnson and Schulberg (2005), which contains the original newspaper articles and an account of the political and institutional reforms in their wake.

[13]Cullen and Pakzad-Hurson (2023) document that pay transparency tends to decreases workers' pay (or, more precisely, slow their pay increases), without significantly affecting the equilibrium level of employment.

4 Price ceilings and minimum wages

We now turn to the analysis of the effects of price ceilings and minimum wages, which are widely used instruments to curb the market power of firms. As we will see, these instruments can—on top of the traditional, socially desirable effects of increasing equilibrium output and employment—have the effects of reducing or even eliminating rationing and involuntary unemployment that occur under laissez-faire. That said, price ceilings and minimum wages can also induce the firm to find rationing and involuntary unemployment optimal when—without the pricing constraint—it would use market-clearing prices. If this occurs, the positive social surplus effect of marginally increasing output or employment via a marginal tightening of the pricing constraint is dwarfed by the socially undesirable effect of random allocation. Finally, price ceilings and minimum can make it optimal for the monopoly to sell some units below marginal costs and for the monopsony to pay some workers more than their marginal product.

4.1 Price ceilings

We now reconsider the monopoly pricing problem introduced in Section 2 and suppose that a regulator imposes a legally binding price ceiling (or cap) $\overline{p}$, which prevents the firm from selling any units at a price above $\overline{p}$. However, the firm can still use the optimal selling mechanism, subject to the additional constraints imposed by $\overline{p}$. We denote by $\overline{R}_C(Q, \overline{p})$ the maximum revenue the firm can obtain when selling the quantity Q, subject to the consumers' incentive compatibility and individual rationality constraints, as well as the constraint imposed by the price cap $\overline{p}$. Denoting by $D(p) = P^{-1}(p)$ the demand function, it is easy to see that $\overline{R}_C(Q, \overline{p}) = \overline{p}Q$ for all $Q \leq D(\overline{p})$: Although the firm is allowed to set multiple prices, none of them can be higher than $\overline{p}$ and so if the firm were to engage in price discrimination in this case it would earn strictly less than $\overline{p}Q$. Similarly, recalling that $p_1(Q, Q_1^*, Q_2^*)$ as defined in (4) is a decreasing function in the quantity Q and denoting its inverse by $p_1^{-1}(p)$, it is straightforward to see that for $Q > p_1^{-1}(\overline{p})$, we have $\overline{R}_C(Q, \overline{p}) = \overline{R}(Q)$ because the price cap is not binding. Note that we can extend the function of p_1 to quantities $Q \notin (Q_1^*, Q_2^*)$ by setting $p_1(Q, Q_1^*, Q_2^*) = P(Q)$ for such quantities, so this result encompasses the case where under laissez-faire the firm sets a uniform price. The interesting aspect of the analysis therefore pertains to the derivation of the optimal selling mechanism for $Q \in (D(\overline{p}), p_1^{-1}(\overline{p}))$, which is a non-empty set only if $\overline{p} \in (P(Q_2^*), P(Q_1^*))$.

This problem is non-trivial. Even though the firm cannot charge any consumer more than $\overline{p}$ when selling $Q \in (D(\overline{p}), p_1^{-1}(\overline{p}))$, this does not imply that it cannot screen over consumers by using two-price mechanisms and find it optimal to do so by serving fewer than

$D(\overline{p})$ of them at the high price. Intuitively, if $Q \in (Q_1^*, Q_2^*)$ and there is no price ceiling, then a two-price mechanism generates strictly more revenue than setting a uniform price. Consequently, continuity implies that a two-price mechanism must still outperform uniform pricing whenever a binding price ceiling sufficiently close to $p_1(Q, Q_1^*, Q_2^*)$ is imposed. The following analysis formally shows that this is indeed the case.

The most challenging aspect of the analysis involving price ceilings consists of showing that focusing on two-price mechanisms is also without loss of generality in the presence of a price ceiling. This is formally shown in Loertscher and Muir (2024b).[14] As in the case without price ceilings, any two-price mechanism with prices p_1 and p_2 with $p_1 \geq p_2$ can conveniently be parameterized by quantities Q_1 and Q_2 satisfying $p_2(Q_2) = P(Q_2)$ and $p_1(Q, Q_1, Q_2) = (1-\alpha(Q, Q_1, Q_2))P(Q_1) + \alpha(Q, Q_1, Q_2)P(Q_2)$, where $\alpha(Q, Q_1, Q_2) = \frac{Q-Q_1}{Q_2-Q_1}$. Letting Q_1^C and Q_2^C denote the optimal values of Q_1 and Q_2 given $\overline{p} \in (P(Q_2^*), P(Q_1^*))$ and $Q \in (D(\overline{p}), p_1^{-1}(\overline{p}))$, a result in the online appendix of Loertscher and Muir (2024b) further implies that Q_1^C and Q_2^C satisfy

$$P'(Q_1^C)P'(Q_2^C) = \left(\frac{P(Q_2^C) - P(Q_1^C)}{Q_2^C - Q_1^C}\right)^2. \tag{5}$$

For the piecewise linear specification of the inverse demand function P in (1), P' is constant on $Q \in [0, 1/4)$ and $Q \in (1/4, 1]$ and we always have $Q_1^C \in [0, 1/4)$ and $Q_2^C \in (1/4, 1]$. Combining this with (5) shows that Q_1^C and Q_2^C satisfy $\frac{P(Q_2^C)-P(Q_1^C)}{Q_2^C-Q_1^C} = \frac{P(Q_2^*)-P(Q_1^*)}{Q_2^*-Q_1^*}$, where Q_1^* and Q_2^* are the optimal mechanism parameters under laissez-faire. This in turn implies the following useful property: For $\overline{p} \in (P(Q_2^*), P(Q_1^*))$ and $Q \in (D(\overline{p}), p_1^{-1}(\overline{p}))$, the quantities $Q_1^C(Q, \overline{p})$ and $Q_2^C(Q, \overline{p})$ that parameterize the optimal selling mechanism can be computed by taking an appropriate parallel shift of the function $p_1(Q, Q_1^*, Q_2^*)$ so that $Q_1^C(Q, \overline{p})$ satisfies

$$\overline{p} = P(Q_1^C(Q, \overline{p})) + (Q - Q_1^C(Q, \overline{p}))\frac{P(Q_2^*) - P(Q_1^*)}{Q_2^* - Q_1^*}, \tag{6}$$

where the dependence of $Q_i^C(Q, \overline{p})$ on Q and $\overline{p}$ is now made explicit.[15]

This parallel-shift property is useful because it permits closed-form solutions for $Q_1^C(Q, \overline{p})$ and $Q_2^C(Q, \overline{p})$ and implies that—when the price ceiling is binding—the optimized revenue

[14] The gist of the reasonably involved argument is to first show that without loss of generality the price ceiling can be imposed as a binding constraint for the buyer with the highest willingness to pay only. Second, the resulting Lagrangian takes the expectation over a virtual type function that involves the value of the Lagrange multiplier associated with this constraint. Because this virtual type is not monotone, it needs to be ironed with respect to the appropriate probability measure. This ironing in turn implies that the focus on at most two prices is without loss of generality.

[15] Note that this last equation exploits the property that under a binding price ceiling $\overline{p}$, the high price must satisfy $p_1(Q, Q_1^C, Q_2^C) = \overline{p}$.

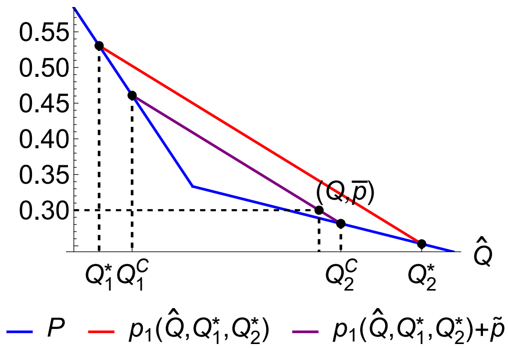

Figure 7: This figure illustrates the parallel shift property that provides a tractable characterization of the parameters of the optimal mechanism under a price ceiling for piecewise linear specifications of demand. The figure is plotted for $\overline{p} = 0.3$ and $Q = 0.35$. The figure illustrates the inverse demand function P (blue) and the function $p_1(\cdot, Q_1^*, Q_2^*)$ (red). The parameters that characterize the optimal procurement mechanism for the quantity $Q = 0.35$ and the price ceiling $\overline{p} = 0.3$ can be found by taking the parallel shift of the function $p_1(\cdot, Q_1^*, Q_2^*)$ that passes through the point $(0.35, \overline{p})$ (purple). The two points of intersection of this new schedule with the inverse demand curve P pin $Q_1^C(Q, \overline{p})$ and $Q_2^C(Q, \overline{p})$.

function $\overline{R}_C(Q, \overline{p})$ is quadratic in Q. Figure 7 plots $P(Q_1^C(Q, \overline{p})) + (\hat{Q} - Q_1^C(Q, \overline{p}))\frac{P(Q_2^*)-P(Q_1^*)}{Q_2^*-Q_1^*}$ as a function of $\hat{Q}$ for $\overline{p} = 0.3$ and $Q = 0.325$. By construction, this function—which is a parallel shift of the function $p_1(\hat{Q}, Q_1^*, Q_2^*)$—is equal to $\overline{p}$ at $\hat{Q} = Q$. Moreover, the right-hand-side of (6) is linear in Q_1^C and in Q and independent of Q_2^C. It therefore pins $Q_1^C(Q, \overline{p})$ down as a linear of function of $\overline{p}$ and Q and straightforward calculations then reveal that

$$Q_1^C(Q, \overline{p}) = \frac{9 - 9\overline{p} - 4\sqrt{6}Q}{4(6 - \sqrt{6})}.$$

The value of $Q_2^C(Q, \overline{p})$ is finally pinned down by solving $\frac{P(Q_2^C(Q,\overline{p}))-P(Q_1^C(Q,\overline{p}))}{Q_2^C(Q,\overline{p})-Q_1^C(Q,\overline{p})} = \frac{P(Q_2^*)-P(Q_1^*)}{Q_2^*-Q_1^*}$ for $Q_2^C(Q, \overline{p})$. That is, by equalizing the slope of the high-price function $p_1(\hat{Q}, Q_1^C(Q, \overline{p}), Q_2^C(Q, \overline{p}))$ under the two-price mechanism parameterized by Q_1^C and Q_2^C and the slope of $p_1(\hat{Q}, Q_1^*, Q_2^*)$ (which is given by $\frac{P(Q_2^*)-P(Q_1^*)}{Q_2^*-Q_1^*}$). This solution is also linear in Q and $\overline{p}$ and is given by

$$Q_2^C(Q, \overline{p}) = \frac{4\left(6\left(\sqrt{6}-1\right)Q + \sqrt{6} - 6\right) - 9\left(\sqrt{6}-6\right)\overline{p}}{28\sqrt{6} - 48}.$$

Putting all of this together, for $Q \in (D(\overline{p}), p_1^{-1}(\overline{p}))$, the maximal revenue as a function of Q

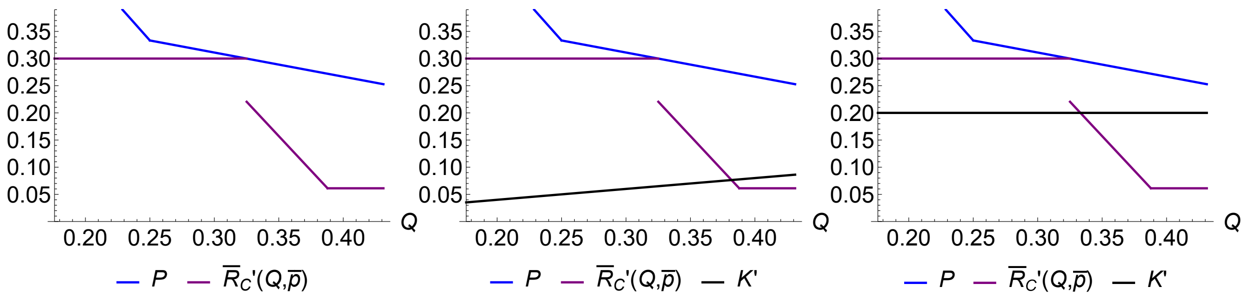

Figure 8: The left-hand panel illustrates marginal revenue $\overline{R}_C'(Q,\overline{p})$ under the optimal selling mechanism (purple) as a function of Q for $\overline{p}=0.3$. The decreasing section corresponds to the region where the price ceiling is binding and a two-price mechanism is optimal. The first flat section is given by $\overline{p}$ and the second corresponds to $\overline{R}'$. The center panel adds the increasing marginal cost function $K'(Q)=Q/5$ (black). The right-hand panel adds a constant marginal cost function $K'(Q)=0.2$ (black) such that the price ceiling makes rationing uniquely optimal (and the firm sets a market-clearing price under laissez-faire).

given $\overline{p}\in(P(Q_2^*),P(Q_1^*))$ is

$$\begin{aligned}\overline{R}_C(Q,\overline{p}) &= \overline{p}Q_1^C(Q,\overline{p})+(Q-Q_1^C(Q,\overline{p}))P(Q_2^C(Q,\overline{p})\\ &= \frac{-27\sqrt{6}\overline{p}^2+3\overline{p}\left(13\sqrt{6}-48Q\right)-4\sqrt{6}\left(8Q^2-11Q+3\right)}{84\sqrt{6}-144},\end{aligned}$$

which is quadratic in Q as claimed. Consequently, the marginal revenue in this range is linear in Q and given by

$$\overline{R}_C'(Q,\overline{p})=\frac{\sqrt{6}(11-16Q)-36\overline{p}}{21\sqrt{6}-36}, \tag{7}$$

where $\overline{R}_C'$ is the derivative of $\overline{R}_C$ with respect to Q. Observe that for $Q\in(D(\overline{p}),p_1^{-1}(\overline{p}))$, decreasing $\overline{p}$ increases marginal revenue $\overline{R}_C'$.[16] This means that if the point of intersection of the marginal cost schedule with $\overline{R}_C'$ occurs at some $Q\in(D(\overline{p}),p_1^{-1}(\overline{p}))$, then marginally decreasing the price ceiling (that is, making the price cap tighter) increases the quantity produced.

Figure 8 displays the optimal marginal revenue schedule in the presence of a price ceiling. It consists of two flat parts: for $Q\le D(\overline{p})$, $\overline{R}_C'(Q,\overline{p})=\overline{p}$ and for $Q\ge p_1^{-1}(\overline{p})$, $\overline{R}_C'(Q,\overline{p})=\overline{R}'(Q)$. The decreasing part occurs for $Q\in(D(\overline{p}),p_1^{-1}(\overline{p}))$ and corresponds to the expression in (7). If there is a point of intersection between $\overline{R}_C'(Q,\overline{p})$ and $K'(Q)$—which

[16]This is a general property, that is, it holds for any inverse demand function that is such that R is not concave; see Loertscher and Muir (2024b).

is unique if K' is strictly increasing—then this point pins down the optimal quantity produced. Selling this quantity, denoted $Q_C^*(\overline{p})$, involves a non-degenerate two-price mechanism if and only if $Q_C^*(\overline{p}) \in (D(\overline{p}), Q_2^*)$. If there is no such point of intersection, then the monopoly optimally sells the quantity $D(\overline{p})$. As illustrated in panel (c) Figure 8, a unique point of intersection can occur within the interval $(D(\overline{p}), p_1^{-1}(\overline{p}))$ with a constant marginal cost function K'. This means that using a two-price mechanism involving rationing can become uniquely optimal with a constant marginal cost function in the presence of a price ceiling whereas—as discussed—without a price ceiling this would never be the case.

A cautionary tale regarding the effects of price ceilings emerges more generally because tighter price ceilings (that is, decreases in $\overline{p}$) can make rationing optimal even while increasing total output. As the equilibrium transitions from a market-clearing price to a two-price mechanism, the loss in social surplus (and consumer surplus) from the random allocation dominates the gain from the increased output. Thus, tightening price ceilings can reduce social surplus even as they increase total output.[17]

The constant marginal cost case has another noteworthy property in the presence of a price ceiling. Denoting by k the constant marginal cost, Lemma 2 in Loertscher and Muir (2024a) implies that if k is such that $k = \overline{R}_C'(Q, \overline{p})$ for some $Q \in (D(\overline{p}), p_1^{-1}(\overline{p}))$, then $Q_C^*(\overline{p})$ varies with $\overline{p}$ in such a way that $Q_1^C(Q_C^*(\overline{p}), \overline{p}) = Q_1^*$ and $Q_2^C(Q_C^*(\overline{p}), \overline{p}) = Q_2^*$. This implies in particular that the consumers who are served at the low price pay $p_2 = P(Q_2^*)$. This is *less* than the marginal cost of production. That is, in the presence of a price ceiling a firm may find it optimal to sell some units below marginal costs. Importantly, this has nothing to do with predatory or entry-deterring pricing. It is purely a feature of the optimal selling mechanism in the presence of a price ceiling.

One notable feature of the optimal marginal revenue envelope $\overline{R}_C'$ is the discontinuity at $Q = D(\overline{p})$. The analysis in Loertscher and Muir (2024a) shows that this is a generic property of the optimal mechanism in the presence of a price constraint. However, a moment's reflection reveals that this discontinuity is not specific to two-price mechanisms. In particular, the marginal revenue schedule under market-clearing pricing for a firm that faces a price ceiling of $\overline{p}$ is simply $\overline{p}$ for $Q \leq D(\overline{p})$ and $R'(Q) = P(Q) + QP'(Q)$ otherwise. This schedule is discontinuous at $Q = D(\overline{p})$ since $P' < 0$. Moreover, as illustrated in Figure 9, the quantity-increasing effects of price ceilings under market-clearing pricing derive precisely from instances where the firm's marginal cost at $Q = D(\overline{p})$ lies within the discontinuity (that is, is less than $\overline{p}$ but larger than $R'(Q)|_{Q=D(\overline{p})}$). As $\overline{p}$ marginally decreases, the firm will continue to find it optimal to produce $D(\overline{p})$ because it is a price-taker on these units

[17]See Loertscher and Muir (2024a) for a formal statement in the context of a procurement model and minimum wage effects.

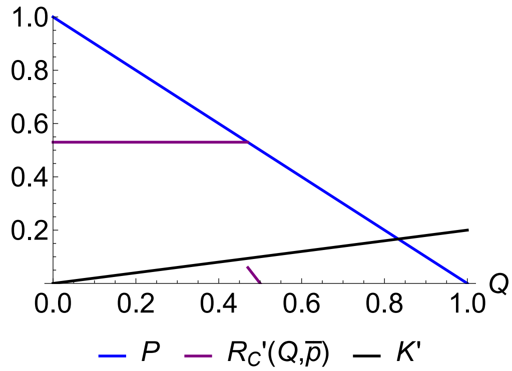

Figure 9: This figure illustrates the discontinuous marginal revenue envelope (in purple) that arises under optimal market-clearing pricing in the presence of a price ceiling and with concave revenue. The figure is plotted for the linear inverse demand function $P(Q) = 1 - Q$ (blue), $K'(Q) = Q/5$ (black) and $\overline{p} = 0.53$.

and marginal revenue $\overline{p}$ exceeds the marginal costs. It will not produce more because the marginal cost is larger than marginal revenue on any additional units.

4.2 Minimum wages

The effects and mechanics of minimum wages in labor markets mirror those of price ceilings in product markets discussed above. In particular, denoting by $S(w) := W^{-1}(w)$ the labor supply function given the wage w, the minimal cost of hiring Q workers given a minimum wage $\underline{w}$, denoted $\underline{C}_M(Q, \underline{w})$, is simply $\underline{w}Q$ for $Q \leq S(\underline{w})$ and $\underline{C}(Q)$ for $Q \geq w_1^{-1}(Q)$, where $w_1(Q) := w_1(Q, Q_1^*, Q_2^*)$ for $Q \in (Q_1^*, Q_2^*)$ and $w_1(Q) := W(Q)$ otherwise. For $Q \in (S(\underline{w}), w_1^{-1}(\underline{w})$, the derivation of the optimal procurement mechanism and of $\underline{C}_M(Q, \underline{w})$ is non-trivial. (Notice that $S(\underline{w}) < w_1^{-1}(\underline{w})$ only if $\underline{w} \in (W(Q_1^*), W(Q_2^*)$.) But, just like in the product market setting, the optimal mechanism is a two-wage mechanism and $\underline{C}_M(Q, \underline{w})$ is quadratic in Q, implying that the marginal cost of procurement, $\underline{C}'_M(Q, \underline{w})$ is linear and increasing in Q for $Q \in (S(\underline{w}), w_1^{-1}(\underline{w})$.

Rather than replicating this analysis in the procurement context, which at any rate is also available in complete generality in Loertscher and Muir (2024a), we highlight here an aspect of minimum wage regulation that is somewhat surprising in light of the perceived wisdom: Appropriately chosen minimum wages can *eliminate* involuntary unemployment. To see this, assume that the equilibrium quantity under laissez-faire, Q^*, is such that $Q_1^* < Q^* < Q_2^*$. This implies that there is involuntary unemployment in equilibrium under laissez-faire. Let Q^p denote the equilibrium quantity under price-taking behavior, which is the unique quantity satisfying $V(Q^p) = W(Q^p)$. Since $\underline{C}'(Q) > W(Q)$, we have $Q^p > Q^*$. By imposing the minimum wage $\underline{w} = W(Q^p)$, the firm can be induced to hire the mass of workers Q^p at the minimum wage: For any $Q \leq Q^p$ its marginal cost of procurement is $\underline{w}$ and—since $\underline{C}_M(Q, \underline{w})$ is convex in Q—for any $Q > Q^p$ its procurement cost is larger than $\underline{w}$ and hence larger than $V(Q^p)$, which implies that the firm will hire precisely Q^p workers. Thus, in the presence of monopsony power an appropriately chosen minimum wage can not only increase employment, workers' pay and social surplus, as noticed by Robinson (1933) and Stigler (1946), but also eliminate involuntary unemployment. Since $W(Q^p)$ is the equilibrium wage that prevails under price-taking behavior, this may raise the question of what information a regulator would require in order to know precisely what that wage is. However, such precise knowledge is not actually required because, by a continuity property, all the aforementioned properties—increased employment, social surplus, workers' pay and no involuntary unemployment—continue to hold for minimum wages below but sufficiently close to $W(Q^p)$.[18]

5 Discussion

We conclude this paper with a formalization of the sense in which incomplete information models are necessary for generating non-trivial monopoly pricing problems, as well as a brief discussion of open and emerging issues.

5.1 Necessity of incomplete information models

In Section 2 we introduced a standard monopoly pricing problem involving a monopoly seller that faces a strictly decreasing inverse demand function P with a marginal cost function K' satisfying $P(0) > K'(0)$ and $P(1) < K'(1)$. We also provided a micro-foundation for the inverse demand function P by specifying an incomplete information model in which consumers have *private* values. Adopting a mechanism design approach, we then characterized the monopoly's optimal selling mechanism, without imposing the regularity assumption of Myerson (1981) or any restrictions on the contracting space. As we saw, the optimal selling mechanism either involves setting a single market-clearing price or using a two-price selling mechanism that essentially represents a form of second-degree price discrimination. This model thereby generates a tradeoff between the monopoly firm's profit and social surplus which—among other things—provides scope for the analysis of welfare-increasing price ceilings in Section 4.1. If one were to modify the model by assuming that consumer values are common knowledge, then the optimal selling mechanism would involve first-degree price discrimination and full surplus extraction on the part of the seller. In this case, the

[18]Translated to the product market setting, this result means that a price ceiling sufficiently close to but above the equilibrium price under price-taking behavior eliminates random rationing while expanding total output, consumer and social surplus.

monopoly uses an efficient selling mechanism and, consequently, the complete information model does not generate a tradeoff between profit and social surplus, which is at the heart of much analysis in economics. Generating a tradeoff between profit and social surplus in a complete information monopoly pricing problem therefore requires imposing restrictions on the contracting space (such as uniform pricing or market-clearing pricing) which—as discussed in the introduction—is problematic in light of the Lucas critique, or introducing other constraints that are outside the standard framework. In this sense, incomplete information models are necessary to generate a tradeoff between profit and social surplus.

These ideas are formally summarized in the following theorem. It combines the insight of Coase (1960) that absent transaction costs bargaining will be efficient with the almost contemporaneously published results of Vickrey (1961) that incomplete information can constitute such a transaction cost.[19] We therefore refer to it as the Coase-Vickrey Theorem:

Theorem 1 ("Coase-Vickrey Theorem")**.** *Incomplete information is necessary and sufficient for generating a tradeoff between profit and social surplus, without including any contractual restrictions or other constraints in the standard monopoly pricing problem.*

5.2 Quantity competition

The analysis in this paper has been confined to models with a single firm. However, the monopoly and monopsony analysis without regularity extends directly to a Cournot-style model of quantity competition in which a Walrasian auctioneer aggregates firms' quantities demanded (supplied) and sells (buys) this aggregate quantity using the revenue-maximizing mechanism (cost-minimizing mechanism). As is formally shown Loertscher and Muir (2024b), with symmetric firms this model always has a unique equilibrium. Moreover, this equilibrium is symmetric and the equilibrium quantity traded is increasing in the number of firms.

5.3 Optimal regulation

Our analysis of monopoly and monopsony pricing models without regularity also gives rise to new rationales for and aspects of price regulation. For example, there is scope for price regulation to increase social and consumer surplus even when the quantity sold remains the same because, under laissez-faire, the firm allocates inefficiently. Further, as noticed, price ceilings and minimum wages can have the effect of making rationing and involuntary unemployment uniquely optimal. This can reduce social surplus and consumer or worker

[19] To the best of our knowledge, the two papers were first jointly discussed in the published literature in economics by Sonnenschein (1983).

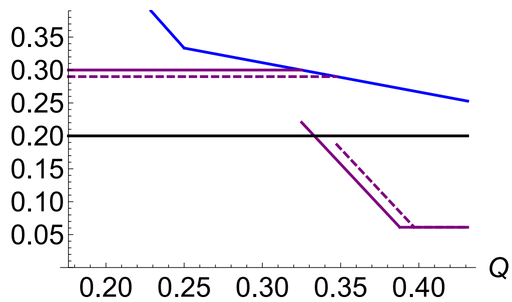

Figure 10: The figure augments the right-hand panel of Figure 8, where the constant marginal cost is 0.2, the price ceiling is 0.3 and $\overline{R}_C'(Q, 0.3)$ is plotted in solid purple, with the marginal revenue $\overline{R}_C'(Q, 0.29)$ corresponding to a price ceiling of $\overline{p} = 0.29$. Given $\overline{p} = 0.3$, the firm's optimal quantity is such it is optimally sold using two prices and rationing.

surplus even while increasing the equilibrium quantity and raises the question of what optimal regulation consists of in environments like these.[20] In environments where $R(Q)$ is concave, a Ramsey regulator who aims to maximize a convex combination of the firm's profit and social surplus can, without loss of generality, confine its instruments to a price ceiling because with or without a price ceiling the firm will optimally set a single price. When R is not concave, this is no longer the case because in the presence of a price ceiling the firm may use two prices and engage in socially *excessive* production. As an illustration, consider a situation like that in Figure 10 and assume that the solution to the regulator's problem is that the quantity produced is $D(\overline{p})$ and that it is sold at the market-clearing price $\overline{p}$ with $\overline{p} = 0.3$. As the figure shows, the firm's best response to this price ceiling is to sell a slightly larger quantity than $D(\overline{p})$ using two prices and rationing. The positive social surplus effect of the quantity increase is second-order to the negative effect on social surplus due to rationing. Consequently, production in the presence of a price ceiling alone can be excessive from the perspectives of social surplus and of a regulator that puts sufficient weight on social surplus. One way to prevent such excessive production is to impose a price floor, in addition to a price ceiling. (In the crudest form, in this example the price floor could be set equal to the price ceiling, thereby eliminating all scope for rationing for the firm.)

5.4 Concluding remarks

The analysis of incomplete information monopoly and monopsony problems without imposing regularity provides a disciplined and unified perspective on when firms find it optimal to set market-clearing prices or wages, and when they prefer to engage in price or wage discrim-

[20]Loertscher and Muir (2024b) provide a step in this direction by analyzing optimal price regulation in the tradition of Ramsey (1927) and Boiteux (1956) without imposing regularity or restricting the firm's mechanism, given the pricing constraints.

ination, and and how they optimally do so. The analysis can also be extended to analyze environments involving resale, as well as price ceiling and minimum wage constraints. Randomization that results from two-price mechanisms can take the form of sequential pricing and selling, which is popular in the events industry where high-priced tickets are sold first and in the fashion industry in the form of sales. In labor markets, randomization takes the form of involuntary unemployment, which can mean that only a fraction of the workers who want to work at a prevailing wage are employed or that workers who would like to work full-time are only provided with part-time work. The analysis reveals novel aspects, including the possibility that minimum wages can eliminate involuntary unemployment and that, in the presence of a price ceiling, a firm may sell some units at below cost. Much more work remains to be done.

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A Omitted details from proof of Proposition 1

In the body of the paper we explicitly constructed a two-price mechanism that achieves revenue $\overline{R}(Q)$ from selling the quantity Q. To complete the proof of Proposition 1, it remains to show that the seller cannot do any better than this. To prove this result, we need to introduce notation for more general selling mechanisms. Without loss of generality, we can restrict attention to direct selling mechanisms $\langle x, p\rangle$, where $x : [0,1] \to [0,1]$ is an allocation rule that maps consumer values v to an allocation probability $x(v)$ and $p : [0,1] \to \mathbb{R}$ is a transfer rule that maps consumer values v to a price $p(v)$ that is paid *conditional* on that consumer successfully securing a unit of the good. Incentive compatibility then requires, for all $v, \hat{v} \in [0,1]$,

$$x(v)(v - p(v)) \geq x(\hat{v})(v - p(\hat{v})) \tag{IC}$$

and individual rationality requires, for all $v \in [0,1]$,

$$v - p(v) \geq 0. \tag{IR}$$

Let $U(v) := x(v)(v - p(v))$ denote the equilibrium payoff of a worker of type v under an arbitrary incentive compatible mechanism $\langle x, p\rangle$. By the envelope theorem the incentive compatibility constraints are equivalent to requiring that $x(v)$ is increasing in v and $U'(v) = x(v)$ holds almost everywhere. We can therefore write $U(v) = U(0) + \int_0^v x(t)\, dt$. From this expression we can see that $U(v)$ is increasing is in v and the individual rationality constraints will also hold if and only if $U(0) \geq 0$. The expected payment made to the seller by consumers of type v is then given by

$$x(v)p(v) = x(v)v - \int_0^v x(t)\, dt - U(0),$$

and from this last expression we can also see that $U(0) = 0$ must hold under any optimal selling mechanism. Integrating, we then obtain the seller's total revenue

$$\int_0^1 \left(x(v)v - \int_0^v x(t)\, dt \right) f(v)\, dv,$$

where $F(v) = 1 - P^{-1}(v)$ represents the distribution of consumer values and $f(v)$ is the corresponding density. Note that this implies that we also have $P(Q) = F^{-1}(1 - Q)$. Using

$\int_0^1 \int_0^v x(t)\,dt\,dF(v) = \int_0^1 (1-F(v))x(v)\,dv$ this becomes

$$\int_0^1 x(v)\left(v - \frac{1-F(v)}{f(v)}\right) f(v)\,dv.$$

We now make a change of variables and introduce a quantile allocation rule $y(z) = x \circ F^{-1}(z)$ that maps quantiles $z \in [0,1]$ to the corresponding allocation for consumers of type $v = F^{-1}(z)$. Incentive compatibility now requires that the quantile allocation rule y is increasing. Making this change of variables $v \mapsto z$ we can rewrite the seller's revenue as

$$\int_0^1 y(z)\left(F^{-1}(z) - \frac{1-z}{f(F^{-1}(z))}\right) dz.$$

For any $Q \in [0,1]$, we now make use of the relation $P(Q) = F^{-1}(1-Q)$, which implies that $R(Q) = QF^{-1}(1-Q)$ and $R'(Q) = F^{-1}(1-Q) - \frac{Q}{f(F^{-1}(1-Q))}$. This allows us to simplify the seller's revenue to

$$\int_0^1 y(z)R'(1-z)\,dz.$$

Without loss of generality we can restrict attention to right-continuous allocation rules that we can view as probability measures on $[0,1]$ by setting $y(1) = 1$. Integrating by parts and using $R(0) = R(1) = 0$ we have

$$\int_0^1 R(1-z)\,dy(z).$$

Fixing the number of units for sale at Q, the seller's revenue maximization problem is given by

$$\max_{y:[0,1]\to[0,1]} \int_0^1 R(1-z)\,dy(z) \tag{8}$$
$$\text{s.t.} \quad y \text{ increasing} \quad \text{and} \quad \int_0^1 y(z)\,dz = Q.$$

The set of extreme points of the set of probability measures on $[0,1]$ is simply given by the set of step functions on $[0,1]$. That is, we can think of any increasing quantile allocation rule y as a convex combination of step functions on $[0,1]$. Moreover, given a quantity $Q' \in [0,1]$, the step function $H(z, Q') := \mathbf{1}(z \geq 1 - Q')$ on $z \in [0,1]$ is simply the allocation rule corresponding to the posted-price mechanism that sells Q' units at the market-clearing price $P(Q')$. By the payoff equivalence theorem, we can therefore compute revenue under the revenue-maximizing implementation of any increasing quantile allocation rule y by expressing it as a convex combination of step functions, and then taking a corresponding convex combination of the revenues generated by the posted price mechanisms associated with each step function in the decomposition. This is precisely how the seller's objective function is computed in (8), and from this it immediately follows that

$$\max_{\substack{y:[0,1]\to[0,1] \\ y \text{ increasing}}} \int_0^1 R(1-z)\,dy(z) \leq \max_{\substack{y:[0,1]\to[0,1] \\ y \text{ increasing}}} \int_0^1 \overline{R}(1-z)\,dy(z).$$

Combining this last expression with the quantity constraint $\int_0^1 y(z)\,dz = Q$, we have that an upper bound on the designer's revenue from selling Q units is given by $\overline{R}(Q)$. However, as we just saw from our previous argument, this upper bound is achieved by the optimal two-price mechanism.