Entry-Deterring Agency*

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Abstract

We provide a model in which an intermediary can choose between wholesale or agency. The possibility that buyers and sellers transact directly limits his market power and, thus, creates incentives for him to deter the emergence of bilateral exchanges. In equilibrium, the intermediary chooses agency and thereby pre-empts the emergence of a competing bilateral exchange if the matching technology of the competing exchange is sufficiently efficient. For symmetric Pareto distributions, whenever agency is chosen in equilibrium, consumer and social surplus decrease while listing and transaction prices tend to increase. The predictions of our model are broadly consistent with empirical evidence.

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1 Introduction

Merchants, middlemen, and market makers have always assumed important economic roles. For just as long, they have been viewed with suspicion for doing so.[1] Traditional examples of these intermediaries are real estate brokers, employment agencies, and used car dealers. Following the entry into the 21st century, the emergence of the Internet and e-commerce has witnessed the creation of intermediaries such as Amazon, eBay, Apple with its i-products and services, AirBnB, Expedia, Uber, and Booking.com. Resonating with traditional suspicions, these Internet platforms are viewed as controversial by policy makers and the public, with a major concern being that they extract excessive rents from the two sides of the market.[2,3]

[1] See, for example, Karl Marx (Capital, Vol.1, Chapter 29, p.744), who wrote that "it is evident here how in all spheres of social life the lion's share falls to the middleman. In the economic domain, e.g., financiers, stock-exchange speculators, merchants, shopkeepers skim the cream...".

[2] In Europe, online hotel booking sites Booking.com and Expedia have come under the scrutiny of antitrust authorities, in the wake of which these platforms have abandoned their "favorite nation" clauses. The Apple e-books case is another prominent example of an intermediary – Apple as the provider of the iPad, contracting with publishing companies – alleged of and fined for aiming to foreclose an alternative exchange. Most recently, it has been argued that Uber is trying to "corner" capital required to subsidize drivers in emerging markets like India and China; for example, see http://www.nytimes.com/2016/06/21/business/dealbook/why-uber-keeps-raising-billions.html?_r=0

[3] While new Internet intermediaries have been particularly prominent, even the modern-day debate on excessive rent extraction by intermediaries goes back much longer, through the investigations of real-estate brokers by the Department of Justice and the conviction of Sotheby's and Christie's for collusion in the 2000s. Competitive concerns were also at the source of investigations of the US Department of Justice on real-estate brokerage in 1983 and 2007 (see, e.g., DOJ, 2007), of the "The Toronto Real Estate Board" case brought up by the Competition Tribunal in Canada (CT, 2011), and the basis of allegations of and convictions for collusion by the auction houses Sotheby's and Christie's (see, e.g., Ashenfelter and Graddy, 2005). The International Labor Organization's call for a ban of private fee-charging employment agencies, to the referendum on banning private labor market intermediaries in Washington State in 1914. The International Labor Organization of the United Nations passed a convention in 1949 that banned private fee-charging employment agencies, to be revoked by a second convention by the International Labor Organization only as late as 1997. See conventions C96 and C181 of the International Labor Organization, C96 Fee-Charging Employment Agencies Convention (Revised), 1949, http://www.ilo.org/ilolex/cgi-lex/convde.pl?C096, C181 Private Employment Agencies Convention, 1997, http://www.ilo.org/ilolex/cgi-lex/convde.pl?C181. Similar developments were also present in

Unlike monopoly producers, an intermediary faces the competitive threat of being circumvented by its customers even if it is a monopoly: in principle, buyers and sellers can always trade directly with each other. From a consumer surplus perspective, this competitive threat is welcome because, all else being equal, it reduces the intermediary's market power.[4] However, it also provides the intermediary with incentives to (potentially inefficiently) deter the emergence of a competing bilateral exchange.

In this paper, we focus on two widely used models of intermediation, called *wholesale* and *agency*, which feature prominently in recent antitrust debates and cases (see, e.g., Johnson, 2017). We show that a monopoly intermediary optimally uses agency if and only if the competitive threat is sufficiently strong. He does so to pre-empt the emergence of a bilateral exchange. For symmetric Pareto distributions, this reduces both social and consumer surplus while increasing the listing prices buyers face.

A sketch of our model and the forces at work is as follows. We assume a continuum of buyers and sellers with heterogeneous valuations and costs for a homogeneous good. Buyers have single-unit demand and sellers have single-unit capacities. This implies that, in the absence of a bilateral exchange, wholesale with appropriately chosen prices for buyers and sellers is the optimal mechanism for the intermediary. However, because it induces a spread between buyers' and sellers' prices, traders with values and costs in between these prices cannot trade with the intermediary but can, to their mutual benefit, trade with each other. Wholesale is thus associated with an active bilateral exchange that improves the outside option of not trading with the intermediary even for traders who in equilibrium join the intermediary. Consequently, the bilateral exchange puts downward pressure on the spread the intermediary can charge, and the more so, the more efficient is the bilateral exchange. In contrast, under agency the intermediary does not take a position but rather has the sellers set prices, randomly matching them to buyers and charging a percentage fee on the price whenever a transaction occurs. Because sellers

the US: in 1914 a referendum in Washington State banned private labor market intermediaries, a law that was later overruled by the US Supreme Court. See Adams v. Tanner, 244 U.S. 590 (1917) and a description of the controversy in Foner (1965, p. 177-185).

[4] See, for example, Gehrig (1993).

are heterogeneous with respect to their costs, agency is associated with a non-degenerate distribution of transaction prices, and more importantly, with overlapping supports in the valuations and costs of buyers and sellers who join the intermediary. Because of this overlap, no buyers and sellers are left who could trade to their mutual benefit in a bilateral exchange. Consequently, the intermediary can use agency to deter the emergence of the bilateral exchange.

In the main model, where we assume that transactions in the bilateral exchange occur at a fixed price while buyers and sellers draw their types from general distributions satisfying Myerson's regularity assumptions, we show that entry deterrence via agency is always possible, and profitable if and only if the competing exchange would be sufficiently efficient if it emerged. For symmetric Pareto distributions, we also show that whenever entry deterrence pays off, social surplus and consumer surplus (defined as the aggregate of buyers' and sellers' surplus) decrease, and buyers' prices tend to increase. We also show these findings are robust to alternative bargaining protocols such as take-it-or-leave-it offers, double-auctions, and Nash bargaining for which we can solve for the equilibrium outcomes in closed form when both distributions are uniform.

A policy implication of our results is that banning the agency model can in some cases be welfare improving. This has indeed been the court's decision in the Apple e-books case. In other instances, for example, in real estate brokerage, mandating the wholesale model may not be practical or feasible. An alternative solution is to require flat fees paid independent of whether there is a transaction, which can be shown to be equivalent to a wholesale model of intermediation.[5] The Department of Justice's 2007 settlement with the Realtor's Association was a milder form of this alternative: while the agency model was not banned, the Realtor's Association was required to abandon practices that discriminated against flat-fee agents.

[5]The equivalence between flat participation fees and the wholesale model have been shown e.g. in Niedermayer and Shneyerov (2014) for arbitrary distributions of traders' types. See also (Belleflamme and Peitz, 2015, Chapter 22) for equivalence between the wholesale model and a flat *transaction* fee (i.e. a fee that is only paid in case of a transaction occurring), however, this equivalence only holds for uniform distributions.

To the best of our knowledge, ours is the first paper to analyze an intermediary's incentives and options to deter entry by an alternative exchange. The emergence of the Internet and e-commerce has led to new intermediaries and to an upsurge of research on two-sided markets. Starting with the pioneering work by Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003, 2006), Anderson and Coate (2005) and Armstrong (2006), this literature has primarily focused on monopoly platforms or competition between platforms, abstracting from the exact mechanisms the platforms employ to generate surplus, from traders' options to circumvent the platform, and the platform's incentives to prevent them from so doing.[6] Two notable exceptions that explicitly analyze the platform's trading mechanism are Gomes (2014) and Niedermayer and Shneyerov (2014). Competition between wholesale intermediaries and alternative exchanges fares prominently in the works of Rubinstein and Wolinsky (1987), Stahl (1988), Gehrig (1993), Spulber (1996), Bloch and Ryder (2000), Rust and Hall (2003), Loertscher (2007) and Neeman and Vulkan (2010), which, however, do not address intermediaries' incentives and options to drive out the competing exchanges.

Our paper builds on the existing literature. Participation fees, which are equivalent to the wholesale model, have been analyzed by Niedermayer and Shneyerov (2014), while Loertscher and Niedermayer (2017a,b) analyze brokers' fee structures from an optimal pricing perspective for the agency model. These papers assume that there is no competing bilateral exchange.[7]

Our paper also shares features with the Industrial Organization literature on predation, in particular with the strand of literature where predatory pricing is made credible, without invoking differential access to financing, by the presence of learning-by-doing as analyzed by Cabral and Riordan (1994, 1997), Besanko et al. (2010), and Besanko et al. (2014). In this strand of literature, the cumulative effects of learning-by-doing render predatory pricing credible. In our model it is the threat of the emergence of an alterna-

[6] That circumventing the intermediary is an option will not come as a surprise to any academic who has ever hired a research assistant from a developing country using an online intermediary.

[7] Equivalently, these papers can be viewed as having a bilateral exchange, for which the payoffs are exogenously given and cannot be influenced by the intermediary.

tive exchange between buyers and sellers that makes agency credible for deterring entry. However, the incentives and methods used for entry deterrence in the literature mentioned above and the present paper are different and complementary. A novel feature of our model is that a strategic player, the intermediary, may predate the emergence of an alternative exchange who is not a player.

Recently, Edelman and Wright (2015) developed a model that exhibits excessive intermediation in that more agents are active than would be under Walrasian conditions, but the mechanisms in the two papers are quite different. In their paper, it is price coherence that induces inefficient outcomes whereas in our model it is the opposite – the freedom of sellers to set prices and the non-degenerate price distributions this induces. The two papers thus complement each other. Studying credit card fees and pricing of ancillary goods, Bourguignon et al. (forthcoming) respectively Gomes and Tirole (2018) analyze alternative regulatory interventions in a setting with one-sided incomplete information. Biglaiser and Li (2018) study a moral hazard model in which the presence of a middleman who uses (in our terminology) wholesale can increase or decrease welfare. In contrast, we study an adverse selection model in which the intermediary is always present but in equilibrium chooses an inefficient mechanism to reduce traders' outside options. While the themes of our paper and theirs are similar, the setups, mechanisms, and outcomes are thus quite different, which renders the papers complementary. Our analysis has a similar flavor as Johnson (2017) because both analyze agency versus wholesale. However, the setup and the question are very different: our focus is on the entry deterrence effect of the agency model, which is an effect that cannot occur in Johnson (2017) because in his setup it is, by assumption, impossible for the supplier to circumvent the retailer.

Our paper also contributes to the literature on intermediation and specifically on real estate brokerage such as the papers by Yavas (1996), Hsieh and Moretti (2003), Rutherford et al. (2005), Levitt and Syverson (2008), Hendel et al. (2009) and Loertscher and Niedermayer (2017a,b). The agency model is difficult to reconcile with a principal-agent perspective, but consistent with an optimal pricing perspective.[8] In the companion paper (Loertscher and Niedermayer, 2017b), we also take the theory to the data and estimate demand and supply under the assumption that payoffs from a bilateral exchange market are exogenous (or that no bilateral exchange market exists). Yavas (1996) has investigated whether real-estate brokers have an incentive to induce inefficiently many trades by matching buyers and sellers "horizontally" rather than "vertically" (i.e. high-cost sellers to high-value buyers rather than inducing trades among the most efficient pairs) because brokers earn commissions that depend on the price and not on the surplus trades generate. While the specifics of the models and mechanisms are different, we obtain a similar result: that under agency horizontal matchings occur with positive probability. This contrasts with the wholesale model, which induces trade among the most efficient buyers and sellers with probability 1. In a wider sense, this paper also relates to Fudenberg and Tirole (2000)'s results of entry deterrence in a dynamic market with network externalities.[9]

The remainder of this paper is organized as follows. Section 2 describes the setup. In Section 3, we derive the equilibrium under wholesale and agency and equilibrium entry deterrence for general distributions assuming fixed-price bargaining. In Section 4, we derive normative implications of our model for social and consumer surplus and predictions for equilibrium prices, assuming symmetric Pareto distributions for buyers and sellers. Section 5 shows that our results are robust insofar as the key insights—entry deterrence via agency if the competing exchange is sufficiently efficient, reduction of social and consumer surplus and price increases when entry deterrence occurs—also hold for the alternative bargaining protocols of take-it-or-leave-it offers, Nash bargaining and double-auctions. For tractability, this section assumes uniform distributions. Section 6 contains discussion of extensions and policy implications, and Section 7 concludes. The Appendix contains omitted proofs and an alternative model with binary types, which lends itself to

[8]For a discussion, see Loertscher and Niedermayer (2017a,b).

[9]The key difference is that in Fudenberg and Tirole (2000) the focus is on initial low pricing by a monopolist in the presence of network externalities, which results in entry deterrence in later periods due to an installed base. In our paper, the focus is not the level of pricing, but the type of pricing: wholesale versus agency.

a dynamic random matching extension.

2 Setup

We consider a one-period model with a continuum of buyers and a continuum of sellers, each with mass 1. Buyers have single-unit demand and sellers have single-unit capacities. All agents are risk neutral, have quasilinear preferences, and outside options with value 0. Buyers draw their valuations v independently from the distribution $F(v)$ with support $[0,1]$ and density $f(v) > 0$ for all $v \in [0,1]$ and sellers draw their costs c independently from the distribution $G(c)$ with support $[0,1]$ and density $g(c) > 0$ for all $c \in [0,1]$. Throughout the paper, we assume that the virtual type functions

$$\Phi(v) = v - \frac{1-F(v)}{f(v)} \quad \text{and} \quad \Gamma(c) = c + \frac{G(c)}{g(c)}$$

are increasing in their argument. These conditions are satisfied by a wide range of distributions and captures the standard assumption that marginal revenue is decreasing and marginal cost is increasing. For example, if F and G are uniform, we have $\Phi(v) = 2v - 1$ and $\Gamma(c) = 2c$, which are obviously increasing.

Consider a market in which demand stems from all buyers with $v \leq \overline{v}$ and supply from all sellers with $c \geq \underline{c}$ for some $\overline{v}, \underline{c} \in [0,1]$, and let p_W be the market clearing (or *Walrasian*) price in this market. If $\overline{v} > \underline{c}$, p_W is the unique number such that

$$F(\overline{v}) - F(p_W) = G(p_W) - G(\underline{c}). \tag{1}$$

If $\overline{v} \leq \underline{c}$, any price $p \in [\overline{v}, \underline{c}]$ will be market clearing. In this case, we let $p_W = (\overline{v} + \underline{c})/2$ without loss of generality. We denote by $\overline{p}_W$ the market clearing when there is no truncation, that is, when $\overline{v} = 1$ and $\underline{c} = 0$. Thus, $\overline{p}_W$ is defined as

$$1 - F(\overline{p}_W) = G(\overline{p}_W). \tag{2}$$

Using (1) and (2), observe that whenever $\overline{v}$ and $\underline{c}$ are such that $1 - F(\overline{v}) = G(\underline{c})$, we have

$p_W = \overline{p}_W$.

Our analysis is motivated by the prevalence of two different trading modes platforms use in the real world, sometimes called *wholesale* and *agency*.[10] Under *wholesale*, the intermediary takes a position and stands ready to trade with buyers and sellers. In contrast, under *agency* the intermediary does not take a position but rather has the seller set a price, on which he charges a percentage fee $b \in [0, 1]$.[11] Following the two-sided markets literature, we assume that the intermediary determines the trading mode.

We model wholesale as posting two prices, a price $p_B \in [0, 1]$ for buyers and a price $p_S \in [0, 1]$ for sellers. We assume that the intermediary stands ready to trade at p_B and p_S, rationing the long side of the market randomly. Consequently, the quantity the intermediary trades under wholesale is the minimum between the quantity suppliers are willing to seller and the quantity buyers demand.

Equilibria under wholesale have a single crossing (or monotone sorting) property of the following form. If, given p_B and p_S, a buyer of type $\overline{v}$ is indifferent between joining the intermediary and his next best choice, which is either to remain inactive or to participate in the competing exchange, all buyers with $v > \overline{v}$ prefer to go to the intermediary. Analogously, if a seller of type $\underline{c}$ is indifferent between selling to the intermediary and his best alternative option, all sellers with $c < \underline{c}$ will prefer selling to the intermediary. Accordingly, the intermediary's profit-maximization problem under wholesale is

$$\max_{(p_B, p_S) \in [0,1]^2} (p_B - p_S) \min\{1 - F(\overline{v}), G(\underline{c})\}. \tag{3}$$

Observe that the problem (3) depends on $\overline{v}$ and $\underline{c}$, which for the general case we will derive after introducing the description of the competing exchange.

In the special case when there is no competing exchange, the best alternative for all buyers and sellers is to remain inactive. Assuming all buyers with $v > p_B$ and all sellers with $c < p_S$ play their weakly dominant strategies of joining the intermediary, we have

[10]This language was used by the court in *United States of America v Apple*. It has also found its way into the economics literature; see, for example, Johnson (2017) or De Los Santos and Wildenbeest (2017).

[11]What is called agency here is referred to as *fee-setting* in Loertscher and Niedermayer (2017a,b).

$\overline{v} = p_B$ and $\underline{c} = p_S$. In this case, the intermediary's problem (3) is simply to maximize $(p_B - p_S)\min\{1 - F(p_B), G(p_S)\}$ over p_B and p_S. As there is no point in choosing p_B and p_S such that $1 - F(p_B) < G(p_S)$, the intermediary's problem under wholesale and no outside options for buyers and sellers other than being inactive can equivalently be cast as solving the quantity-setting problem of maximizing over $Q \in [0, 1]$ the function

$$\Pi_W(Q) = (F^{-1}(1 - Q) - G^{-1}(Q))Q.$$

The solution to this problem, stated in terms of prices p_B and p_S, is such that $\Phi(p_B) = \Gamma(p_S)$ and $1 - F(p_B) = G(p_S)$.[12] Let $\Pi_W^0 = \max_{Q\in[0,1]} \Pi_W(Q)$. This is positive under our assumptions because $\Phi(1) = 1$ and $\Gamma(0) = 0$ and continuity of the various functions imply that the positive quantity Q^* is traded at a positive spread $p_B^0 - p_S^0$ with $p_B^0 = F^{-1}(1-Q^*)$ and $p_S^0 = G^{-1}(Q^*)$. As an example, if F and G are uniform, we have $p_B^0 = 3/4$, $p_S^0 = 1/4$, $Q^* = 1/4$ and thus $\Pi_W^0 = 1/8$.

When the intermediary chooses *agency*, buyers and sellers are matched one-to-one at random at the intermediary.[13] Agency also means that the intermediary sets a percentage fee $b \in [0, 1]$ that is levied on the price the seller sets and due if and only if a transaction occurs. Thus, if a seller sets a price p and the buyer he is matched to accepts it, the seller receives $(1 - b)p$. Traders on the long side (if there is a long side) are randomly rationed. Thus, letting ω_B (ω_S) denote the mass of buyers (sellers) joining the intermediary, under agency buyers and sellers are matched at the intermediary with probability $\min\{1, \omega_S/\omega_B\}$ and $\min\{1, \omega_B/\omega_S\}$, respectively.

[12]To see this, notice that the first order condition $\Pi_W'(Q^*) = 0$ is $F^{-1}(1 - Q^*) - G^{-1}(Q^*) - Q^*/f(F^{-1}(1 - Q^*)) + Q^*/g(G^{-1}(Q^*)) = 0$. Substituting $p_B = F^{-1}(1 - Q^*)$, $p_S = G^{-1}(Q^*)$, and the definitions of Φ and Γ yields $\Phi(p_B) = \Gamma(p_S)$ and $1 - F(p_B) = G(p_S)$. Because Φ and Γ are monotone, the first-order condition is sufficient for a maximum.

[13]As will become clear, wholesale can equivalently be interpreted as random matching of the buyers and sellers who join the intermediary and trade at the prices p_B and p_S if a match occurs. The fundamental difference between agency and wholesale is thus not the matching process, but rather who sets the transaction prices and how the intermediary earns his profit. Moreover, the assumption that matching is one-to-one can easily be relaxed in favor of random matchings that allow many buyers to be matched to one seller. In this case, the percentage fee the intermediary charges is levied on the transaction price that results from a second-price auction in which the seller sets the reserve; see Loertscher and Niedermayer (2017a) for more details.

The intermediary faces potential competition from an alternative exchange. For the purposes of making our point as simply as possible, we assume, for now, that in the competing exchange trade occurs at a fixed price equal to the Walrasian price for the set of traders who do not join the intermediary, that is, at p_W as defined in and after (1). We refer to this as *fixed-price bargaining*. Buyers and sellers who join the competing exchange trade with probabilities $\lambda \min\{1, \mu_S/\mu_B\}$ and $\lambda \min\{1, \mu_B/\mu_S\}$, respectively, where μ_B (μ_S) is the mass buyers (sellers) active in the exchange and $\lambda \in [0, 1]$ is common knowledge and the same for all agents.[14] Note that in equilibrium, p_W is fixed and does not adjust after a unilateral deviation. Nevertheless, if $\mu_S = 0$, even buyers with values above p_W have no incentive to go to the competing exchange simply because they would not find a trading partner.

The assumption of fixed-price bargaining in the competing exchange may be a reasonable approximation to decentralized trading when the prime friction agents face is finding a trading partner, and there is a common understanding of what the "right" price is, that is, if the price discovery problem is dwarfed by the problem of finding a trading partner. The assumption of fixed-price bargaining allows us to establish both the *motive* and the *means* for entry deterrence using agency without imposing parameteric restrictions on the distributions F and G. We discuss in more detail what makes the fixed-price bargaining assumption useful for our purposes after Proposition 1 below. We will show in Section 5 that neither motive nor means for entry deterrence via agency hinge on this particular assumption, which permits a closed form solution for the payoff from participating in the competing exchange for any F and G.

Here we offer four additional, non-technical justifications for this assumption. First, the competing exchange may be thought of as an efficient market that only emerges with some probability (such as the asymptotically efficient, dynamic random matching markets analyzed by Satterthwaite and Shneyerov (2007, 2008)). Second, there may be a competing "fringe" of two or more platforms $i = 1, 2, \ldots$ who compete in prices for

[14] If one interprets wholesale as random matching as suggested in footnote 13, the matching probability for a buyer (seller) who joins the intermediary under wholesale is $\min\{1, \omega_S/\omega_B\}$ ($\min\{1, \omega_B/\omega_S\}$).

buyers and sellers. These platforms are inferior in that their matching probability is only λ whereas the matching probability at the intermediary we focus on is 1. Because of Bertrand competition, for given $\overline{v}$ and $\underline{c}$, these platforms set the buyer and seller prices $p_B^i = p_S^i = \overline{p}_W$ for $i = 1, 2, \ldots$. Third, λ may denote the probability that a competing intermediary develops a technology that gives it the possibility to enter the market. This new intermediary could enter using wholesale like the incumbent intermediary and has the same matching technology as the incumbent. Under wholesale, Bertrand competition between the two intermediaries will drive its spread to 0 and its prices to p_W. Last, as we show in Section 6.1, the fixed price(s) offered by the competing exchange may also and largely equivalently be interpreted as the prices of a competing platform in a setup in which, under wholesale, the intermediary we focus on competes à la Bertrand with a platform whose matching technology has the parameter λ. As λ increases, the intermediary's (and its rival's) equilibrium profit decreases, and goes to 0 as λ goes to 1.

For given $\overline{v}$ and $\underline{c}$, the expected payoff of a buyer with value v from the competing exchange will be $V_B(v) = \lambda \max\{v - p_W, 0\}$ and the expected payoff for a seller with cost c will be $V_S(c) = \lambda \max\{p_W - c, 0\}$. We say that the bilateral exchange is *inactive* in equilibrium if $V_B(v) = 0 = V_S(c)$ for all $v, c, \in [0, 1]$. Otherwise, we say that it is *active*. We also assume that that if agents of multiple types are indifferent between participating in the bilateral exchange and in the intermediated market, tie-breaking is *uniform*, that is, all agents break ties in the same way, either in favour of the bilateral exchange or in favor of the intermediated market.

The timing of the game is as follows. In Stage 1, the intermediary chooses the mode of operation (wholesale or agency) and, conditional on that, the optimal prices and the optimal percentage fee, respectively. The mode of operation and the prices respectively the fee are observed by all agents. In Stage 2, all buyers and sellers simultaneously decide whether to join the intermediary, the competing exchange, or to remain inactive. These choices are mutually exclusive. We assume that agents who trade with probability 0 at a given market (i.e. at the intermediary's exchange or in the competing exchange) do not enter this market.[15] We defer further discussions of equilibrium (non)-uniqueness to Subsection 6.3. The equilibrium concept we employ is *perfect Bayesian equilibrium*.

3 Equilibrium

We now analyze the model laid out above, beginning with equilibrium under wholesale, where our focus will be on the equilibrium in which for $\lambda > 0$ the bilateral exchange is active, which seems to be the plausible equilibrium.[16]

Equilibrium under wholesale Because for any $\lambda < 1$, the derivative of $V_B(v)$ with respect to v, provided it exists, is less than 1, and the derivative of $V_S(c)$ is greater than -1 where defined, it follows that for any given p_B and p_S there is single crossing: If a seller of type $\underline{c}$ and a buyer of type $\overline{v}$ are indifferent between joining the intermediary or the competing exchange, all buyers with higher values and all sellers with lower costs will strictly prefer to join the intermediary.[17] Thus, the indifference condition for the buyer with value $\overline{v}$ is $\overline{v} - p_B = V_B(\overline{v})$, and likewise, the indifference condition for a seller with cost $\underline{c}$ is $p_S - \underline{c} = V_S(\underline{c})$. Observe that $\underline{c} \leq p_S \leq p_W \leq p_B \leq \overline{v}$ since the intermediary makes non-negative profits and market participants' utilities are non-negative in the bilateral exchange. Therefore, $V_B(v)$ simplifies to $\lambda(v - p_W)$ and $V_S(c)$ simplifies to $\lambda(p_W - c)$. Substituting the expressions for V_B and V_S, one obtains

$$\overline{v} = \frac{p_B - \lambda p_W}{1 - \lambda} \quad \text{and} \quad \underline{c} = \frac{p_S - \lambda p_W}{1 - \lambda}. \tag{4}$$

[15] This can be justified as the limit of a slightly richer model in which a positive fixed cost of market participation goes to 0.

[16] For any positive spread $p_B - p_S > 0$, traders with values and costs between p_W and p_B respectively p_S and p_W cannot trade at the intermediary but would benefit from trading with each other; see Subsection 6.3 for a detailed discussion of multiplicity and uniqueness of equilibrium.

[17] This conclusion does not hinge on an assumption that buyers are not rationed at the intermediary: If buyers were served with probability q at the intermediary, single crossing will still hold provided $q > \lambda$. If $q \leq \lambda$, no buyer will join the intermediary if $p_W \leq p_B$.

Put differently, for given $\overline{v}$ and $\underline{c}$, the spread the intermediary earns per unit traded is

$$p_B - p_S = (1-\lambda)(\overline{v} - \underline{c}). \tag{5}$$

As we show in the proof of the following proposition, it will not be in the intermediary's interest to induce rationing of buyers. Consequently, if the intermediary trades the quantity Q of buyers and sellers, we have $\overline{v} = F^{-1}(1-Q)$ and $\underline{c} = G^{-1}(Q)$, implying that the spread in (5) becomes

$$p_B - p_S = (1-\lambda)(F^{-1}(1-Q) - G^{-1}(Q)).$$

In turn, this implies that the intermediary's profit as a function of the quantity he trades and of the efficiency parameter λ of the competing exchange, denoted $\Pi_W(Q,\lambda)$, satisfies $\Pi_W(Q,\lambda) = (1-\lambda)\Pi_W(Q)$. Consequently, under wholesale the equilibrium quantity traded by the intermediary, Q^*, is independent of λ. Denoting by $p_B^*(\lambda)$ and $p_S^*(\lambda)$ the equilibrium prices under wholesale, we have

$$p_B^*(\lambda) = (1-\lambda)p_B^0 + \lambda\overline{p}_W \quad \text{and} \quad p_S^*(\lambda) = (1-\lambda)p_S^0 + \lambda\overline{p}_W, \tag{6}$$

where p_B^0 and p_S^0 are the equilibrium prices introduced in Section 2 when there is no competing exchange. Note that $p_B^*(\lambda)$ decreases in λ and $p_S^*(\lambda)$ increases in λ, satisfying $p_B^*(0) = p_B^0$ and $p_S^*(0) = p_S^0$ and $p_B^*(1) = \overline{p}_W$ and $p_S^*(1) = \overline{p}_W$. Denoting by $\Pi_W^*(\lambda) = \max_{Q\in[0,1]} \Pi_W(Q,\lambda)$ the maximum profit under wholesale given λ, this implies in particular[18]

$$\Pi_W^*(\lambda) = (1-\lambda)\Pi_W^0.$$

We summarize this in the first part of the following proposition. The profit-maximizing mechanism in the second part refers to an environment in which the buyers and sellers

[18] It also means that our statement in Section 2 that the model can equivalently be interpreted as one in which, with probability λ, a competitor with the same matching technology enters and induces equilibrium prices of $\overline{p}_W$ because of Bertrand competition is correct: With probability λ, the prices will be $\overline{p}_W$ and the intermediary will earn profits of 0. With probability $1-\lambda$, no entry occurs, and the intermediary will set the prices p_B^0 and p_S^0 and earn Π_W^0. Thus, his expected profit is $\Pi_W^*(\lambda) = (1-\lambda)\Pi_W^0$.

are privately informed about their values and costs, which are independent draws from the commonly known distributions F and G.

Proposition 1. *The maximum profit under wholesale $\Pi_W^*(\lambda)$ is a decreasing function of λ satisfying $\Pi_W^*(0) = \Pi_W^0$ and $\Pi_W^*(1) = 0$. Moreover, for $\lambda = 0$, wholesale is the profit-maximizing mechanism for the intermediary, subject to traders' incentive compatibility and individual rationality constraints.*

The first part of Proposition 1 shows that, under wholesale, increases in the efficiency of the competing exchange provide the intermediary with a motive to deter the emergence of the competing exchange. The intuition is simple. Profit maximization under wholesale involves posting a positive spread between buyer and seller prices. Buyers with slightly smaller values than the price posted by the intermediary cannot possibly benefit from trade with the intermediary but may benefit from joining the competing exchange. Likewise, sellers with costs slightly above the intermediary's seller price may benefit from joining the competing exchange. Because this outside option also exists for buyers and sellers who, in equilibrium, trade with the intermediary, improving the efficiency of the competing exchange puts downward pressure on the spread the intermediary can charge. Ultimately, as λ approaches 1, the intermediary's spread (and hence profit) goes to 0. For example, when F and G are uniform, the equilibrium prices are $p_B^*(\lambda) = (3 - \lambda)/4$ and $p_S^*(\lambda) = (1 + \lambda)/4$, which converge from above respectively below to $1/2$ as $\lambda \to 1$, and $\Pi_W^*(\lambda) = (1 - \lambda)/8$.

The second part of Proposition 1 is proved in Appendix B.1. The fact that wholesale is optimal for $\lambda = 0$ will imply, by continuity and the suboptimality of agency (see Corollary B1 in the Appendix), that for λ sufficiently small, the intermediary will prefer wholesale to agency and entry deterrence. The intuition for this result is the following: in the absence of a bilateral exchange, the intermediary can separate the problem of selling to buyers and buying from sellers since there is a continuum of buyers and sellers with an exogenous outside option going to the intermediary. We know that the optimal mechanism to sell to buyers is a posted price by Myerson (1981). Analogously, the optimal mechanism to buy from sellers is a posted (procurement) price. Therefore, two posted prices, i.e. wholesale is the optimal mechanism.

Let us now briefly discuss what makes the fixed-price bargaining assumption convenient for our purposes. It permits us to conclude that $\Pi_W^*(\lambda)$ will be less than the profit under agency for λ close enough to 1 with a fair degree of generality regarding the distributions. In light of Proposition 1, all we need to show to reach this conclusion is that the intermediary's profit under agency is positive. If we assumed an alternative bargaining protocol, such as take-it-or-leave-it offers, while maintaining the assumption of general distributions F and G, it would be difficult to establish the same conclusion. Although one would expect the intermediary's profit under wholesale to decrease in λ, there is no reason to assume that it would go to 0 as λ approaches 1. (As a matter of fact, as shown in Section 5, the intermediary's profit under wholesale remains strictly positive with take-it-or-leave-it offers when F and G are both uniform; however, the conclusion that entry deterrence via agency occurs for λ sufficiently large is still valid because we can show that the agency profits are higher than the wholesale profits for λ sufficiently large. We manage to show this in Section 5 because we get closed form solutions for profits for uniform distributions.) Because the intermediary's profit under agency depends, in general, in intricate ways on F and G, this would render the analysis intractable.

Equilibrium under agency Next, we analyze equilibrium under agency and show that agency provides a means to deter entry of the competing exchange. We first state a result that proves useful for what follows:

Lemma 1. *Consider the subgame that ensues after the intermediary chooses agency. If the bilateral market is active, then the intermediary earns zero profits.*

The intuition for Lemma 1 is that there is a hold-up problem that causes the unraveling of the intermediated market if the bilateral exchange is active. To see this, take any value for the marginal buyer $\overline{v}$ who is indifferent between the intermediary and the bilateral exchange. Once the marginal buyer $\overline{v}$ chose to enter the intermediated market, he abandoned the option of going to the bilateral exchange and therefore is willing to accept any price offer weakly below $\overline{v}$. Therefore, a seller will never set a price below $\overline{v}$. Therefore, the buyer $\overline{v}$ gets zero utility from going to the intermediary, which is less than the positive utility he would get at the bilateral exchange. Therefore, the intermediated market unravels and no one goes to the intermediary.

Lemma 1 implies that the intermediary only chooses agency in equilibrium if it induces the bilateral market to be inactive.

If the intermediary chooses agency and charges a percentage fee $b \in [0, 1]$, the maximum price a seller can set in the hope of reaching a deal is 1. Given a percentage fee b, this implies that no seller with a cost $c > 1-b$ can profitably join the intermediary under agency. Our assumptions therefore imply that sellers with costs exceeding $1-b$ do not go to the intermediary under agency.

Consider next a seller with cost $c \leq 1-b = \underline{c}$. Assuming that buyers' values are drawn from the prior distribution F, given the percentage fee b, the seller with cost $c \leq 1-b$ maximizes his profit in case of being matched and optimally sets the price[19]

$$p(b, c) = \arg\max_{p}((1-b)p - c)(1 - F(p)) = \Phi^{-1}\left(\frac{c}{1-b}\right).$$

Denote the lowest price set in equilibrium as $\underline{p}$. Observe that $\underline{p} = \Phi^{-1}(0)$ is independent of b for any F and G. Since $\overline{v} = \underline{p}$, it also holds that $\overline{v}$ is independent of b. As an example, if F is uniform, we have $p(b, c) = \frac{1}{2} + \frac{c}{2(1-b)}$, and hence $\overline{v} = 1/2 = \Phi^{-1}(0) = \underline{p}$.

Of course, the expected payoff from joining the intermediary under agency for buyers with values $v < \Phi^{-1}(0)$ is 0.[20] Our assumptions imply that these buyers will not join the intermediary. Importantly, this does not affect the optimal price a seller of type $c \leq 1-b$ sets given b, since $\Phi^{-1}(c/(1-b))$ is also the maximizer of $((1-b)p - c)(1 - F(p))/(1 - F(\underline{p}))$, which is this seller's expected payoff when setting the price $p \in [\underline{p}, 1]$

[19]To see this, notice that the first-order condition for the problem $\max_p((1-b)p - c)(1 - F(p))$ is $0 = -(1-b)f(p(b, c))\,(\Phi(p(b, c)) - c/(1-b))$. Because Φ is increasing, it can be inverted to yield $p(b, c) = \Phi^{-1}(c/(1-b))$. Moreover, the fact that Φ is increasing also implies that the second-order condition is satisfied whenever the first-order condition is. In other words, the problem is quasi-concave.

[20]Note that for $v < 1$, we have $\Phi(v) < v$, which implies $\Phi^{-1}(0) > 0$.

when the buyers' values are drawn from the distribution $F(v)$ conditional on $v \geq \underline{p}$, provided $\underline{p} \leq \Phi^{-1}(0)$.[21] Consequently, the prices sellers set in equilibrium are not affected if only buyers who expect positive gains from participation if the sellers price according to the prior distribution F on $[0, 1]$.

Because the competing exchange is active as soon as $\overline{v} > \underline{c}$, the condition

$$\Phi^{-1}(0) \leq 1 - b \tag{7}$$

is necessary for agency to attract all buyers with $v \geq \Phi^{-1}(0) = \overline{v}$ and all sellers with costs $c \leq 1 - b = \underline{c}$. In other words, if all agents who under agency with a fee $b \leq 1 - \underline{p}$ get a positive expected surplus from joining the intermediary go to the intermediary, the competing exchange is inactive regardless of λ simply because there are no agents left who can trade to their mutual benefit there. Conversely, if (7) is not satisfied, there is no equilibrium in which the intermediary attracts all buyers with $v \geq \overline{v}$ and all sellers with $c \leq \underline{c}$. Therefore, by Lemma 1 the intermediary would make zero profits.

Given this, the intermediary's profit under agency with fee $b \leq 1 - \Phi^{-1}(0)$ is

$$\Pi_A(b) = b \min\{1 - F(\overline{v}), G(\underline{c})\} \frac{\int_0^{1-b} p(b,c)(1 - F(p(b,c)))dG(c)}{(1 - F(\overline{v}))G(\underline{c})},$$

where $f(v)/(1 - F(\overline{v}))$ and $g(c)/G(\underline{c})$ are the conditional densities of active traders, integrated out in the case of buyers, and $\min\{1 - F(\overline{v}), G(\underline{c})\}$ is the mass of matches. Let $\Pi_A^* = \max_{b \in [0, 1 - \Phi^{-1}(0)]} \Pi_A(b)$ be the maximum profit under agency subject to the constraint that b does not exceed $1 - \Phi^{-1}(0)$. $\Pi_A^* > 0$ because at $b = 1 - \Phi^{-1}(0)$ the constraint $b \leq 1 - \Phi^{-1}(0)$ is satisfied and $\Pi_A(b) > 0$. Notice also that Π_A^* is, by construction, independent of λ since the bilateral exchange is not active. If F and G are both uniform, we have $b^* = 1/2$ and $\Pi_A^* = 1/12$.[22]

[21] Technically, this is a reflection of the fact that virtual valuation functions are invariant to truncation from below.

[22] To see this, recall that for F uniform, $\Phi^{-1}(c/(1-b)) = 1/2 + c/(2(1-b))$. Integrating one obtains $\int_0^{1-b} p(b,c)(1 - F(p(b,c)))dG(c) = (1-b)/6$. Because $1 - b \geq \Phi^{-1}(0) = 1/2$ is equivalent to $b \leq 1/2$, we have, for $b \leq 1/2$, $\Pi_A(b) = b/6$, which is strictly increasing in b. For $b \geq 1/2$, $\Pi_A(b) = b(1-b)/3$ is maximized at $b = 1/2$, yielding $\Pi_A^* = 1/12$.

Profitability of entry deterrence via agency We now compare the intermediary's profits under wholesale and under agency and derive the necessary and sufficient condition for entry deterrence via agency to occur.

From Proposition 1 we know that wholesale is optimal for $\lambda = 0$, $\Pi^*_W(\lambda) = (1-\lambda)\Pi^0_W$ is decreasing in λ and equal to 0 for $\lambda = 1$. Together with $\Pi^*_A > 0$ and the suboptimality of agency (see Corollary B1 in the Appendix), this implies that there is a number $\lambda_\Pi \in (0,1)$ such that $\Pi^*_W(\lambda_\Pi) = \Pi^*_A$. Using the formula for $\Pi^*_W(\lambda)$, we obtain

$$\lambda_\Pi = \frac{\Pi^0_W - \Pi^*_A}{\Pi^0_W}, \tag{8}$$

meaning that agency is more profitable than wholesale if and only if $\lambda > \lambda_\Pi$. λ_Π is the relative loss from agency compared to wholesale absent a bilateral exchange. The intermediary is willing to incur this loss if the relative loss from competition by the bilateral exchange (λ) is greater than λ_Π. For example, for uniform distributions, we have $\lambda_\Pi = 1/3$ because, as noted, $\Pi^*_A = 1/12$ and $\Pi^0_W = 1/8$.

Proposition 2. *In equilibrium, the intermediary chooses wholesale for $\lambda < \lambda_\Pi$ and agency for $\lambda > \lambda_\Pi$, where $\lambda_\Pi \in (0,1)$ for any F and G satisfying our assumptions.*

The family of *(generalized) Pareto distributions*[23] $F(v) = 1-(1-v)^\sigma$ and $G(c) = c^\sigma$ provides a tractable specification that satisfies monotonicity of virtual types, where σ measures the elasticity of supply and demand.[24] Indeed, the virtual types $\Phi(v) = (\sigma + 1)v/\sigma - 1/\sigma$ and $\Gamma(c) = (\sigma+1)c/\sigma$ are linear. The linearity of virtual types together with the symmetry assumption that demand and supply have the same elasticity permits closed form solutions, which is analytically convenient. When F and G are symmetric Pareto distributions, we have $p_W = 1/2$ and the Walrasian quantity traded is $(1/2)^\sigma$. In this case, we denote the function λ_Π by $\lambda_\Pi(\sigma)$, which is displayed in Figure 1 (a). All the

[23] We are simplifying terminology here. Technically speaking, F is a finite support generalized Pareto distribution and G is a mirrored finite support generalized Pareto distribution.

[24] The elasticity of supply is $\eta_s(c) = cg(c)/G(c) = \sigma$ and the elasticity of demand $\eta_d(v) = (1-v)f(v)/(1-F(v)) = \sigma$. While the elasticity of supply is the standard definition, the elasticity of demand is mostly defined as $vf(v)/(1-F(v)) = \sigma v/(1-v)$, but for dealing with markets with intermediaries, the above definition is more convenient.

formulas for the symmetric Pareto distributions are collected in Appendix A.2.

The non-monotonicity of $\lambda_\Pi(\sigma)$ deserves a brief discussion. Let $\hat{\Pi}_A = \max_{b\in[0,1]} \Pi_A(b)$ be the maximum profit under agency if one neglects the constraint $1 - b \geq \Phi^{-1}(0)$. For symmetric Pareto distributions, $\arg\max_{b\in[0,1]} \Pi_A(b) = 1/(1+\sigma) = \Phi^{-1}(0)$. This implies that for $\sigma \geq 1$, $\Pi_A^* = \hat{\Pi}_A$. Hence, for $\sigma \geq 1$, the functions $\lambda_\Pi(\sigma)$ and $\hat{\lambda}_\Pi(\sigma) = (\Pi_W^0 - \hat{\Pi}_A)/\Pi_W^0$ coincide as illustrated in Figure 1. In contrast, when $\sigma < 1$, the constraint $1 - b \geq \Phi^{-1}(0)$ becomes binding, implying $b = \sigma/(1+\sigma)$ and $\hat{\Pi}_A > \Pi_A^*$ and thus $\lambda_\Pi(\sigma) > \hat{\lambda}_\Pi(\sigma)$.[25] Moreover, the smaller is σ, the costlier is the constraint, which explains the increasing difference between $\lambda_\Pi(\sigma)$ and $\hat{\lambda}_\Pi(\sigma)$ as σ becomes smaller on $[0, 1]$.

As observed by Loertscher and Niedermayer (2017a), the intermediary's profit-maximizing percentage fee is independent of the *demand* side if the seller's distribution is of the form $G(c) = c^\sigma$ and if entry deterrence is not a concern for the intermediary. Here, we note that the fee is independent of supply side factors, which in our model are captured by G, as long as the constraint $1 - b \geq \Phi^{-1}(0)$ is binding. Thus, the entry-deterrence motive of agency adds a new element of fee invariance, the twist being that the fee may be independent of the supply side if entry deterrence renders the constraint $1 - b \geq \Phi^{-1}(0)$ binding.

In equilibrium, with symmetric Pareto distributions and $\sigma \geq 1$, the price set by a seller under agency is $p(1/(1+\sigma), c) = c + 1/(1+\sigma)$. Buyers with values greater than $p(1/(1+\sigma), 0) = 1/(1+\sigma)$ and sellers with costs less than $\sigma/(1+\sigma)$ trade with positive probability under agency. Because $p(1/(1+\sigma), c)$ decreases in σ, buyers with lower values and sellers with higher costs will be active when the elasticity increases. Nevertheless, the probability that trade actually occurs decreases in σ. We provide intuition for this result after analyzing the welfare effects.

[25] Using the terminology that is widely used in Industrial Organization, the case where the constraint $1 - b \geq \Phi^{-1}(0)$ is not binding (for symmetric Pareto distributions, that is the case for $\sigma \geq 1$) may be thought of as corresponding to a situation in which entry of the competing exchange is *blockaded* as the optimal fee absent the competing exchange also prevents its emergence. In contrast, if the constraint is binding, entry is *deterred*. Note, however, that the very fact that the intermediary chooses agency is already driven by an intention to deter entry, so the analogy is incomplete and the terminology potentially misleading.

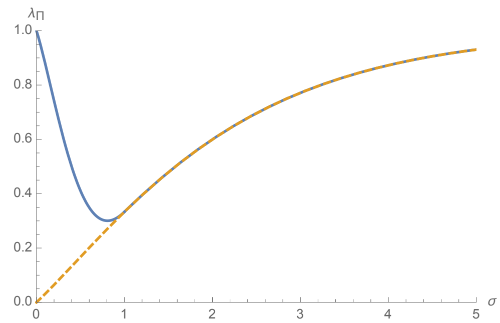

Figure 1: $\lambda_\Pi(\sigma)$ (solid) and $\hat{\lambda}_\Pi(\sigma)$ (dashed) as functions of σ for symmetric Pareto distributions

4 Welfare and Price Effects

We now consider the effects of agency on social surplus, consumer surplus, and on equilibrium prices and price distributions. In this analysis, we focus on (symmetric) Pareto distributions because of tractability.

4.1 Social Surplus and Consumer Surplus Effects

Social surplus is defined as the equally weighted gains from trade of all buyers and sellers and of the profit of the intermediary. Let $S_W(\sigma, \lambda)$ and $S_A(\sigma)$ denote social surplus under wholesale and agency, respectively.[26] Because, as noted, the equilibrium quantity the intermediary trades does not vary with λ and the volume of trade increases with λ in the bilateral market, social surplus under wholesale increases with λ and is therefore at its minimum at $\lambda = 0$. Figure 2 (a) plots the ratio $S_A(\sigma)/S_W(\sigma, 0)$ as a function of σ. It can be seen from the figure that this ratio is less than 1 for all values of σ and decreases with σ. This implies that the wholesale model generates higher social surplus than the agency model for all values of λ.

Proposition 3. *Social surplus under wholesale exceeds social surplus under agency. That is, for all $\sigma > 0$ and all $\lambda \in [0, 1]$, we have $S_W(\sigma, \lambda) > S_A(\sigma)$.*

The proof is in Appendix A.2.

[26]See Appendix A.2 for the precise formulas.

(a) $S_A(\sigma)/S_W(\sigma, 0)$ (b) $\lambda_\Pi(\sigma) > \lambda_{CS}(\sigma)$

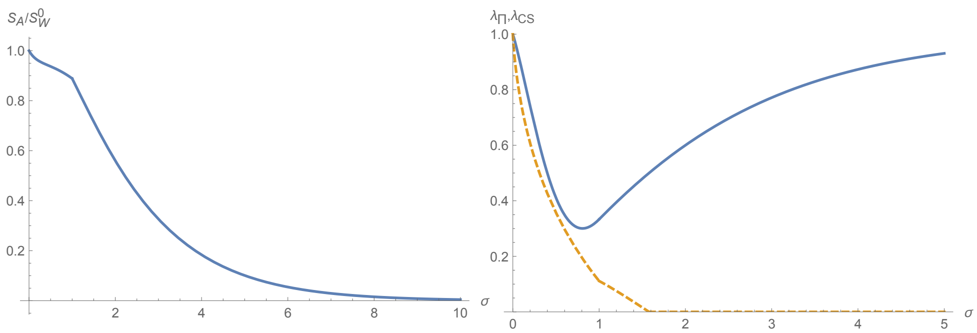

Figure 2: Panel (a): $S_A(\sigma)/S_W(\sigma, 0)$ is less than 1 and decreasing in σ; panel (b): λ_Π (solid) exceeds λ_{CS} (dashed).

The proposition has an intuitive explanation. For simplicity, let us focus on the case $\sigma \geq 1$. Under agency, the inefficiency due to random matching becomes worse as the elasticity σ increases because the fee $b^* = 1/(1+\sigma)$ decreases in σ. This induces ever more agents to become active as σ increases whereas the efficient quantity $(1/2)^\sigma$ becomes smaller as σ increases. In the limit, social surplus (relative to Walrasian welfare) under agency goes to 0. In contrast, under wholesale, the equilibrium prices converge to the Walrasian price of $1/2$ as σ goes to infinity, and the deadweight loss vanishes.

This explanation also provides the intuition for why $\lambda_\Pi(\sigma)$ increases in σ. Both under wholesale and under agency the intermediary's ability to extract rents decreases in σ, which is intuitive. However, as the intermediary extracts a strictly positive fraction of social surplus under wholesale while social surplus under agency goes to 0, it follows that the ratio $\Pi_A^*(\sigma)/\Pi_W^*(\sigma, \lambda)$ goes to 0 for any $\lambda < 1$ as σ goes to ∞.

Proposition 3 has the following corollary.

Corollary 1. *Agency decreases social welfare.*

Another question of interest concerns effects on consumer surplus, which are at the center of attention for many antitrust authorities around the globe. Defining consumer surplus as social surplus minus the intermediary's profit, consumer surplus under agency, denoted $CS_A(\sigma)$, is independent of λ and given as $CS_A(\sigma) = S_A(\sigma) - \Pi_A^*(\sigma)$. Under wholesale, consumer surplus, denoted $CS_W(\sigma, \lambda)$, is given as $CS_W(\sigma, \lambda) = S_W(\sigma, \lambda) - \Pi^*_W(\lambda)$. This strictly increases in λ because λ does not affect social surplus generated by traders who join the intermediary, decreases $\Pi^*_W(\lambda)$ (and thus increases the consumer surplus of those agents who trade via the intermediary) and strictly increases social surplus generated in the matching market. For example, for the uniform case we have $CS_W(1, \lambda) = (1 + 3\lambda)/16$ and $CS_A(1) = 1/12$. Let $\lambda_{CS}(\sigma)$ be the smallest value of $\lambda \in [0, 1]$ such that $CS_W \geq CS_A$.[27] For the uniform, one obtains $\lambda_{CS}(1) = 1/9$. Because $CS_W(\sigma, \lambda)$ increases in λ while $CS_A(\sigma)$ is independent of λ, it follows that consumer surplus under wholesale exceeds consumer surplus under agency if and only if $\lambda > \lambda_{CS}(\sigma)$.

Figure 2 (b) plots $\lambda_\Pi(\sigma)$ and $\lambda_{CS}(\sigma)$ as functions of σ. Because $\lambda_\Pi(\sigma) > \lambda_{CS}(\sigma)$ for all values of σ, it follows that consumer surplus under agency is always lower than consumer surplus under wholesale whenever the intermediary's profits under agency are higher than under wholesale. Because consumer surplus is social surplus minus profits, Proposition 3 has another corollary.

Corollary 2. *Consumer surplus under wholesale exceeds consumer surplus under agency if the intermediary's profits under agency exceed those under wholesale.*

Put differently, Corollary 2 says that entry deterrence decreases consumer surplus.

4.2 Price-Increasing Agency

An important issue in the policy debates pertaining to agency and wholesale concern the effects of the different modes on prices. For example, in the Apple e-books case, agency was associated with higher prices faced by buyers. As we show next, our model also sheds light on these questions.

Under agency one needs to distinguish between the average listing price, which we denote by $p^L(\sigma)$, and the average transaction price, which we denote by $p^T(\sigma)$. The listing price of a seller with cost c who faces a fee b is $p(b, c)$. Because higher listing

[27] Formally, λ_{CS} is defined as $\lambda_{CS} := \min_{\lambda \in [0,1]}\{\lambda | CS_W - CS_A \geq 0\}$. For sufficiently large values of σ there will be no $\lambda \geq 0$ such that CS_W does not exceed CS_A, which explains the somewhat complicated definition.

prices will garner a smaller market share, the average transaction price under agency will be below the average listing price. Depending on the empirical application, either $p^L(\sigma)$ or $p^T(\sigma)$ (or the distributions of transaction prices) may be observed. Observing posted prices on online platforms corresponds to observing listing prices. In real estate research, typically transaction prices are observed and analyzed. The average (or expected) listing price under agency $p^L(\sigma)$ given a percentage fee b is independent of b, that is,

$$p^L(\sigma) \equiv \mathbb{E}_c[p(b,c)|c \leq 1-b] = 1 - \sigma/(1+\sigma)^2, \tag{9}$$

where the equality can be established by straightforward integration.[28] Of course, $p^L(\sigma)$ is independent of λ.

Under wholesale, there is no difference between the listing and the transaction price because all transactions occur at the uniform listing price. This price is

$$p_B^*(\sigma, \lambda) = 1 - (\sigma + \lambda)/(2(1+\sigma)),$$

which is decreasing in σ and λ. Observe that the average listing price under wholesale and agency is the same ($p_B^*(\sigma,\lambda) = p^L(\sigma)$) if and only if $\lambda = \sigma(\sigma-1)/(1+\sigma)$. Let

$$\lambda_L(\sigma) = \max\{\sigma(\sigma-1)/(1+\sigma), 0\}$$

denote the threshold such that $\lambda \geq \lambda_L(\sigma)$ implies that the average listing price is larger under agency than under wholesale. One can show that for all $\sigma > 0$,

$$\lambda_L(\sigma) < \lambda_\Pi(\sigma). \tag{10}$$

This requires a little work for $\sigma < 1$ but is immediate for $\sigma \geq 1$ because then $\lambda_L(\sigma) = 0$ while we know that $\lambda_\Pi(\sigma) \in (0,1)$. Figure 3 (a) illustrates this. Inequality (10) implies:

Proposition 4. *The average listing price under agency, $p^L(\sigma)$, exceeds the listing price*

[28] It is an implication of Proposition 4 in Loertscher and Niedermayer (2017b), which uses the insight that the distribution of listing prices does not vary with b.

under wholesale, $p_B^*(\sigma, \lambda)$, if the intermediary prefers agency to wholesale.

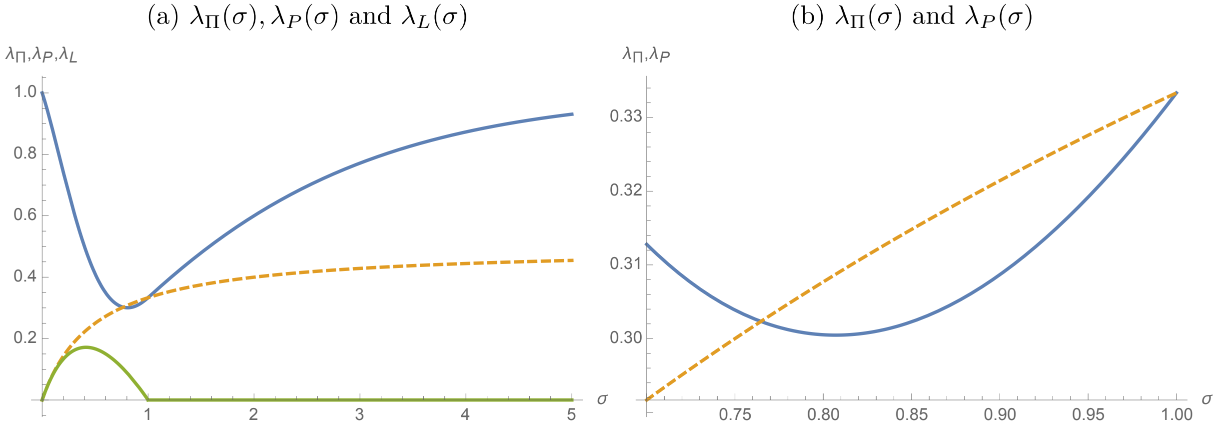

Figure 3: Panel (a):$\lambda_\Pi(\sigma)$ (upper solid),$\lambda_P(\sigma)$ (dashed) and $\lambda_L(\sigma)$ (lower solid); panel (b): zooms in on $\lambda_\Pi(\sigma)$ (solid) and $\lambda_P(\sigma)$ (dashed), $\lambda_\Pi(\sigma) < \lambda_P(\sigma)$ for some $\sigma < 1$

Thus, with regards to listing prices our model implies that agency leads to price increases.

As mentioned, in some cases analysts may have access to transaction prices. This is, for example, the case for the real-estate data set compiled and analyzed by Genesove and Mayer (2001). Under agency, the average transaction price $p^T(\sigma)$ is given as[29]

$$p^T(\sigma) = \frac{(1+\sigma)\text{Gamma}(1/2+\sigma)}{2\text{Gamma}(3/2+\sigma)}, \tag{11}$$

where $\text{Gamma}(z) = \int_0^\infty x^{z-1}e^{-x}dx$ is the Gamma function. Defining $\lambda_P(\sigma)$ as the unique number such that $p^T(\sigma) = p_B^*(\sigma, \lambda_P(\sigma))$, one can show numerically that for all $\sigma \geq 1$

$$\lambda_P(\sigma) \leq \lambda_\Pi(\sigma), \tag{12}$$

with strict inequality for $\sigma > 1$. This is illustrated in Figure 3 (a) and implies that, for $\sigma \geq 1$, agency increases average transaction prices whenever it occurs in equilibrium. In contrast, for $\sigma < 1$, as illustrated in Figure 3 (a) and highlighted in Figure 3 (b), there

[29]Appendix A.2 provides details concerning the derivation of $p^T(\sigma)$.

is a small parameter region such that $\lambda_P(\sigma) > \lambda_\Pi(\sigma)$, implying the transaction-price decreasing agency is an (albeit rare) possibility.

Summarizing, we have the following: If agency is observed empirically if and only if it is more profitable than wholesale, then agency is associated with higher listing prices. For $\sigma \geq 1$, which seems to be the empirically most relevant case, agency also increases average transaction prices.[30] This is in line with recent empirical evidence; see, for example, De Los Santos and Wildenbeest (2017), who find that the change from agency to wholesale led to a decrease of prices for e-books of 18 percent on average.

Figure 3 displays $\lambda_\Pi(\sigma), \lambda_P(\sigma)$ and $\lambda_L(\sigma)$ in panel (a) and zooms in on the (small) parameter region for which $\lambda_\Pi(\sigma)$ is slightly smaller than $\lambda_P(\sigma)$. For $\lambda \in (\lambda_\Pi(\sigma), \lambda_P(\sigma))$, the intermediary chooses agency in equilibrium and thereby reduces the expected transaction price compared to wholesale.

5 Take-It-Or-Leave-It Offers and Other Bargaining Protocols

We now demonstrate that our insights based on the assumption of fixed-price bargaining are robust. Specifically, we now assume that the bilateral exchange has one of three alternative, widely used bargaining protocols—random proposer take-it-or-leave-it offers, Nash bargaining, and double-auctions—while assuming that the distributions F and G are uniform.[31] We will show that entry deterrence via agency is an equilibrium phenomenon that occurs when λ is large enough under each of these alternative bargaining protocols, and that agency, when it is chosen in equilibrium, decreases social surplus and consumer surplus and increases the average listing price and the average transaction price relative to wholesale. We take this as a corroboration that our results do not hinge on the specific

[30] The optimal fee absent the need to deter entry is $b = 1/(1+\sigma)$. For $\sigma <$, this would correspond to a fee of more than fifty percent, which are rarely observed. For example, even Apple "only" charges 25 percent fees.

[31] As is well known, deriving equilibrium predictions is notoriously difficult for bargaining under incomplete information with general distributions.

assumptions concerning bargaining.

Under wholesale, we focus on the equilibrium that has the same monotone sorting structure as the above. That is, given (p_B, p_S) all buyers with values above some $\overline{v}$ and all sellers with costs below some $\underline{c}$ go to the intermediary, all buyers with values $v \in [\underline{v}, \overline{v}]$ and all sellers with $c \in [\underline{c}, \overline{c}]$ go to the bilateral exchange, and all other agents remain inactive. Under agency with a fee $b \leq 1 - \Phi^{-1}(0)$, as before, the bilateral exchange is inactive if all buyers with $v \geq \Phi^{-1}(0)$ and all sellers with costs $c \leq 1 - b$ go to the intermediary.

We begin by briefly describing the standard bargaining protocols. With *random proposer take-it-or-leave-it-offers*, the buyer makes an offer p_b with probability α, which the seller can either accept or reject. With probability $1 - \alpha$, the seller makes the take-it-or-leave-it-offer p_s, which the buyer can either accept or reject. Under *generalized Nash bargaining*, the buyer with value v gets α of the joint surplus $\max\{v - c, 0\}$ when matched to a seller with cost c while the seller gets $1 - \alpha$ times this surplus. In the $k = 1/2$ *double-auction*, a buyer and a seller submit bids p_b and p_s simultaneously and trade at the price $p = (p_b + p_s)/2$ if and only if $p_b \geq p_s$; for examples, see Chatterjee and Samuelson (1983); Rustichini et al. (1994), and Satterthwaite and Williams (2002). With buyers' values and sellers' costs uniformly distributed on $[\underline{v}, \overline{v}]$ and $[\underline{c}, \overline{c}]$, respectively, the $k = 1/2$ double-auction has a Bayes Nash equilibrium in which the buyer with value $v \in [\underline{v}, \overline{v}]$ and the seller with cost $c \in [\underline{c}, \overline{c}]$ bid

$$p_b(v) = \frac{1}{4}\underline{c} + \frac{1}{12}\overline{v} + \frac{2}{3}v \quad \text{and} \quad p_s(c) = \frac{1}{4}\overline{v} + \frac{1}{12}\underline{c} + \frac{2}{3}c,$$

respectively, inducing trade whenever $v \geq c + (\overline{v} - \underline{c})/4$.[32]

We defer the technical derivation of the following proposition to the Appendix.

Proposition 5. *Assume F and G uniform. Under each of the three standard bargaining protocols, (i) entry deterrence occurs in equilibrium for λ sufficiently large, and (ii) entry deterrence, when it occurs, decreases social surplus and consumer surplus. Moreover, for*

[32] See, for example, Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983), who also observe that this is a second-best mechanism.

any $\lambda > 0$ the average listing price under agency, $p^L(1)$, is larger than the listing price under wholesale, $p_B^*(\lambda) = (3-\lambda)/4$ for any of the three standard bargaining protocols.

Proposition 5 provides a robustness check for and remarkable corroboration of the results derived from the model with fixed-price bargaining. In particular, both the channel through which entry deterrence can occur and the motive for entry deterrence remain the same under the standard bargaining protocols, and both the normative implications of entry deterrence via agency and the empirical predictions regarding average listing prices are, qualitatively, the same.

Moreover, under each of the three standard bargaining protocols agency also increases the average transaction price when it occurs in equilibrium. This follows from the fact that the average transaction price under agency is $p^T(\sigma)$ as given in (11), regardless of the bargaining protocol (simply because the competing exchange is inactive under agency). As shown in the proof of Proposition 5,

$$\lambda_\Pi(1) = \frac{1}{3} < \min\{\lambda_\Pi^{TOL}, \lambda_\Pi^{DA}, \lambda_\Pi^{NB}\},$$

where λ_Π^{TOL} $(\lambda_\Pi^{DA}, \lambda_\Pi^{NB})$ is the threshold value under take-it-or-leave-it offers (double-auction, Nash bargaining) above which the intermediary chooses agency in equilibrium.[33]

[33] The proof of Proposition 5 shows that $\lambda_\Pi^{TOL} = \lambda_\Pi^{NB} = 2/3 > \lambda_\Pi^{DA} = 4/9$. The payoff under agency being fixed because the distribution is uniform, this ranking occurs because the intermediary's profit under wholesale is the same with Nash bargaining and take-it-or-leave-it offers and larger than with a double-auction. Both the equality and the ranking are interesting in light of the differences in assumptions regarding information and the (in)efficiency of bargaining. Even though Nash bargaining is ex post efficient while the other two bargaining protocols are not, the extra entry of inefficient traders that Nash bargaining (and take-it-or-leave-it offers) induce relative to a double-auction makes the bilateral matching market less attractive from the perspectives of the indifferent traders with values $\overline{v}$ and cost $\underline{c}$, whose outside options determines the intermediary's spread under wholesale. Thus, the less efficient information structure and bargaining that is associated with double-auctions is better from the perspective of buyers and sellers than Nash bargaining. (Strictly speaking, the preceding argument shows that this is true for those buyers and sellers who under wholesale trade via the intermediary; however, the proof also shows that, using self-explanatory notation, $CS_W^{DA} > CS_W^{NB} > CS_W^{TOL}$, where the first inequality is due to excessive entry under Nash bargaining and the second because of the inefficiency of take-it-or-leave-it offers). This is reminiscent of the efficiency-enhancing aspect of incomplete information bargaining observed by Shneyerov and Wong (2011) and Lauermann (2013) and reflective of some of the same forces (that is, excessive entry with complete information bargaining).

As mentioned, desirable properties of the double auction given the uniform distribution have been noted before (see e.g. Myerson and Satterthwaite, 1983). The present finding adds a new element to this

Because $\lambda_{\Pi}(1) \geq \lambda_P(1)$ as stated in (12), the same numerical analysis that underlies Figure 3 (a) at $\sigma = 1$ implies that agency increases average transaction prices when it occurs.

6 Discussion

We now discuss a number of aspects of our analysis and its implications and provide extensions.

6.1 Bertrand Competition Deterring Agency

As mentioned, our assumption of fixed-price bargaining has the additional benefit that it can rather directly be extended to accommodate a competing exchange that posts prices $\tilde{p}_S$ for sellers and $\tilde{p}_B$ for buyers with $\tilde{p}_B \geq p_W \geq \tilde{p}_S$, with the matching technology of the competing exchange being parametrized by λ. We refer to this competing exchange as the *weak* platform and to the intermediary we have focused on so far, whose matching parameter is 1, as the *strong* platform. The analysis we have performed up to now corresponds to the special case that the weak platform posts a spread of zero, that is, $\tilde{p}_B = p_W = \tilde{p}_S$. Assuming temporarily that the prices $\tilde{p}_B$ and $\tilde{p}_S$ are exogenously given and, for simplicity, are still such that $1-F(\tilde{p}_B) = G(\tilde{p}_S)$, one obtains for $v \geq \tilde{p}_B$ and $c \leq \tilde{p}_S$ an expected payoff from participating in the competing exchange of $\tilde{V}_B(v) = \lambda(v - \tilde{p}_B)$ and $\tilde{V}_S(c) = \lambda(\tilde{p}_S - c)$, respectively. Thus, the expected profit of the intermediary who

list. As it accounts for private information held by the two parties, it cannot be as efficient as Nash bargaining because of the impossibility theorem of Myerson and Satterthwaite. But exactly because it induces bid shading by the buyer and the seller, it prevents the most inefficient buyers and sellers − the buyers with values below 3/8 and the sellers with costs above 5/8 − from entering the matching market, thereby improving sorting and surplus in the matching market relative to Nash bargaining and take-it-or-leave-it offers.

Somewhat ironically, once one accounts for the intermediary's incentive to deter entry of the competing exchange, buyers and sellers may be better off with Nash bargaining or take-it-or-leave-it offers in the bilateral exchange than with a double-auction. This is the case for all $\lambda \in (4/9, 2/3)$, which are such that the intermediary chooses agency given a double-auction and wholesale given any of the other two bargaining protocols.

sets prices $p_B > \tilde{p}_B$ and $p_S < \tilde{p}_S$ to trade the quantity $Q < 1 - F(\tilde{p}_B) = G(\tilde{p}_S)$ is

$$\Pi_W^{\overline{B}}(Q, \lambda) = [(1-\lambda)\left(F^{-1}(1-Q) - G^{-1}(Q)\right) + \lambda(\tilde{p}_B - \tilde{p}_S)]Q.$$

Because $Q < 1 - F(\tilde{p}_B)$ is equivalent to $F^{-1}(1-Q) - G^{-1}(Q) > \tilde{p}_B - \tilde{p}_S$, it follows that $\Pi_W^{\overline{B}}(Q, \lambda)$ decreases in λ. For $\tilde{p}_B - \tilde{p}_S$ small, as λ goes to 1, we have $\sup_Q \Pi_W^{\overline{B}}(Q, \lambda) = (1 - F(\tilde{p}_B))(\tilde{p}_B - \tilde{p}_S)$, which will be less than Π_A^* if $\tilde{p}_B - \tilde{p}_S$ is small enough.

This idea—that agency is used to prevent Bertrand competition with a price-posting rival—resonates with Apple's use of agency for e-books with the rival being Amazon when Apple launched the iPad, and with Apple and Amazon nowadays using agency (see, e.g., De Los Santos and Wildenbeest, 2017), possibly to deter competition with their price-posting rival Barnes & Noble.

Of course, the assumption that the competitor's prices are fixed is not ideal. Fortunately, it can easily be dispensed with as we show next by modeling the weak platform (i.e. the competing exchange) as a strategic player. For expositional ease, we assume that F and G are uniform. Assuming that only the strong platform can choose agency, the equilibrium outcome under agency is well understood and as described above because it does not depend on the nature of the competing exchange, which is inactive under agency. At the end of this subsection we provide a brief discussion of and a justification for the assumption that only the strong platform is active under agency.

For now, we focus on wholesale competition between the two platforms. Let (p_B, p_S) and $(\tilde{p}_B, \tilde{p}_S)$ be the prices of the strong and the weak platform, respectively. The equilibrium structure of the market is such that buyers with values above a threshold $\overline{v}$ and sellers with costs below a threshold $\underline{c}$ join the strong platform, buyers with values $v \in [\tilde{p}_B, \overline{v}]$ and sellers with costs $c \in [\underline{c}, \tilde{p}_S]$ join the weak platform, and all other traders remain inactive,[34] where $\overline{v} = (p_B - \lambda\tilde{p}_B)/(1-\lambda)$ and $\underline{c} = (p_S - \lambda\tilde{p}_S)/(1-\lambda)$ are the same as

[34] Of course, under wholesale the weak platform opens itself scope for a bilateral matching market as long as it charges a positive spread $\tilde{p}_B - \tilde{p}_S$. We abstract from this possibility as it would only reinforce the point to be made: The matching market would put downward pressure on the equilibrium spreads of both platforms, thereby making wholesale less attractive than agency for a larger set of λ.

in (4) with p_W replaced by $\tilde{p}_B$ and $\tilde{p}_S$, respectively. The equilibrium profits of the two platforms as functions of λ, denoted $\Pi_W^B(\lambda)$ and $\tilde{\Pi}_W^B(\lambda)$, are[35]

$$\Pi_W^B(\lambda) = \frac{2(1-\lambda)}{(4-\lambda)^2} \quad \text{and} \quad \tilde{\Pi}_W^B(\lambda) = \frac{\lambda(1-\lambda)}{2(4-\lambda)^2}.$$

Observe that $\Pi_W^B(\lambda)$ decreases in λ and satisfies $\Pi_W^B(0) = 1/8$ and $\Pi_W^B(1) = 0$ just as with fixed-price bargaining. Moreover, $\Pi_W^B(\lambda) < \Pi_A^* = 1/12$ if and only if $\lambda > 2(2\sqrt{2} - 4) \approx 0.48$. Thus, the strong platform prefers entry deterrence via agency to competition under wholesale whenever λ exceeds $2(2\sqrt{2} - 4)$.

Let us now briefly discuss, and in some way justify, the assumption that only the strong intermediary chooses agency. A natural conjecture is that, if both platforms were given the opportunity the choose agency, Bertrand-style competition will drive both fees down to 0. We now argue that there are plausible reasons to believe that this is not the case. In Loertscher and Niedermayer (2017b, Prop.4), we show that if $G(c)$ is a Pareto distribution, then the distribution of listing prices does not vary with the fee b (see also (9) and footnote 28). Thus, conditional on being matched, buyers have no preferences over platforms based on their fees, assuming all active sellers join the same platform. Hence, under these assumptions, buyers will prefer the platform with the higher matching probability irrespective of its fee (and the difference to the fee of the rival platform). Anticipating that all buyers join the strong platform, all sellers with costs less than $1 - b$ will follow suit and join the strong platform as well. Consequently, with uniform distributions and two platforms with matching probabilities 1 and λ and fees b and $\tilde{b}$, respectively, there is a plausible equilibrium under agency with $b = 1/2$ and $\tilde{b} = 0$ in which all buyers with values above 1/2 and all sellers with costs less than 1/2 join the

[35] To see this, notice first that for given prices of the rival, each platform optimally chooses prices such that the demand it faces is equal to the supply it attracts. Consequently, for given $(\tilde{p}_B, \tilde{p}_S)$ the strong platform's problem is to choose (p_B, p_S) to maximize $(1 - F(\overline{v}))p_B - G(\underline{c})p_S$ subject to the constraint that $1 - F(\overline{v}) = G(\underline{c})$. Likewise, given (p_B, p_S) the problem of the weak platform is to choose $(\tilde{p}_B, \tilde{p}_S)$ to maximize $\lambda[(F(\overline{v}) - F(\tilde{p}_B))\tilde{p}_B - (G(\tilde{p}_S) - G(\underline{c}))\tilde{p}_S]$ subject to $F(\overline{v}) - F(\tilde{p}_B) = G(\tilde{p}_S) - G(\underline{c})$. Plugging in the formulas for $\overline{v}$ and $\underline{c}$, making use of the uniform distributions, and solving the linear system of first-order conditions gives the equilibrium prices $p_B^*(\lambda) = 3(2-\lambda)/(2(4-\lambda))$, $p_S^*(\lambda) = (2+\lambda)/(2(4-\lambda))$, $\tilde{p}_B^*(\lambda) = (5 - 2\lambda)/(2(4 - \lambda))$ and $\tilde{p}_S^*(\lambda) = 3/(2(4 - \lambda))$. Plugging these values back into the objective functions gives $\Pi_W^B(\lambda)$ and $\tilde{\Pi}_W^B(\lambda)$.

strong platform and all other traders remain inactive. The resulting equilibrium profits are $\Pi_A^* = 1/12$ and $\tilde{\Pi}_A^* = 0$.

6.2 Collusion, Self-Enforcing and Optimal Mechanisms

There are alternative ways the intermediary can implement agency. For example, he could employ a large number of brokers, each with the capacity to execute one trade, each of them charging the same percentage fee b and buyers and sellers being pairwise matched across brokers (e.g. using something like a multiple listing service).

It is remarkable that such a market structure, which, consisting of a large number of active brokers each of whom has a small market share, at face value looks very competitive, may have entry-deterring effects and be the result of collusion. For the purpose of this discussion, it is useful to restrict attention to symmetric Pareto distributions with $\sigma \geq 1$.[36]

Keeping fixed the random matching technology across brokers and assuming symmetric Pareto distributions with $\sigma \geq 1$, the fee $b^* = 1/(1 + \sigma)$ is optimal in the sense of implementing the broker-optimal, incentive compatible, individually rational mechanism derived by Myerson and Satterthwaite (1983) (as we show in Appendix B.3). It remains optimal when these distributions are truncated to the sets of buyer- and seller-types that can trade with positive probability under agency.[37] In this sense, agency with the percentage fee b^* is *self-enforcing*—if all other brokers set the same fee, holding fixed the matching technology and the behaviour of all buyers and sellers, no individual broker has an incentive to deviate.

[36] In some sense, the restriction to Pareto distributions is without loss of generality here while the restriction to $\sigma \geq 1$ is not. Let us briefly elaborate. Loertscher and Niedermayer (2017a) show the following: A Pareto distribution on the seller side implies that agency with a percentage fee $b = 1/(1+\sigma)$ is an optimal mechanism for a broker facing one buyer and one seller drawing their types from $F(v)$ and $G(c) = c^{\sigma}$, respectively. If G is an arbitrary distribution with increasing virtual cost $\Gamma(c)$, agency with a fee $\omega(p) = p - \mathbb{E}_v[\Gamma^{-1}(\Phi(v))|v \geq p]$ levied on the transaction price p set by the seller is optimal for the broker; indeed, for $G(c) = c^{\sigma}$, we have $\omega(p) = p/(1 + \sigma)$. Thus, to the extent that percentage fees are replaced by the general fee function $\omega(p)$, the arguments generalize immediately, provided only the constraint that $\Phi^{-1}(0) \leq 1 - b^*$ is not binding. In general, this constraint will not bind if and only if $\Phi^{-1}(0) \leq \Gamma^{-1}(1)$ while for symmetric Pareto distributions it will not bind if and only if $\sigma \geq 1$.

[37] See also Tirole (2016) for the importance of time consistency and selection effects for optimal mechanisms.

If one thinks of the brokers as colluding to deter entry by a competing exchange using agency, symmetric Pareto distributions with $\sigma \geq 1$ thus correspond to a case in which the incentive compatibility problem typically associated with collusive agreements can be resolved even in a one-shot game. Moreover, while collusive behaviour is often associated with a restriction of quantity, the number of brokers (or the intermediary's capacity) used to deter entry of the competing exchange is $G(1-b^*) = (\sigma/(1+\sigma))^\sigma$, which is larger even than the Walrasian quantity of $(1/2)^\sigma$. In other words, neither the absence of incentive problems nor a large number of brokers are necessarily indicating the absence of collusive behaviour among middlemen.

Wholesale is an optimal mechanism for a large intermediary facing a deep market with a continuum of buyers and sellers when there is no competing bilateral exchange, with "optimal" meaning that there is no other incentive compatible and individually rational mechanism that generates higher profits, which we show in Appendix B.1. An open question concerns the optimal mechanism for an intermediary who faces an active competing exchange. This is challenging because it corresponds to a mechanism design problem with endogenous and type dependent participation constraints. Even *exogenous* type dependent participation constraints are known to be difficult (see e.g., Jullien, 2000).

6.3 Multiplicity and Uniqueness of Equilibrium

As is well known, models of market making are prone to exhibit multiple equilibria simply because not joining a given platform or exchange is often a best response if no one else joins it. For example, in our setup there is always an equilibrium under wholesale in which no one joins the bilateral exchange.

Similarly, under wholesale a multiplicity of equilibria can occur that differ with respect to the mass of buyers and sellers who join the intermediary as noted, for example, by Gehrig (1993). In our setting with fixed-price bargaining, the only multiplicity of this kind that can obtain is that no traders join the intermediary under wholesale. However, if we assume that under wholesale the intermediary is committed to buying from sellers at price p_S, there is no equilibrium under wholesale in which the intermediary attracts no traders. This is stated in the following proposition. The proposition also states that within each of the two classes of equilibria (both intermediary and bilateral exchange active; only intermediary active) there is a unique equilibrium.

Proposition 6. *There are two equilibria in the subgame following the intermediary's choice of wholesale given prices* (p_B, p_S)*:*

(i) The intermediary and the bilateral exchange are both active.

(ii) The intermediary is active and the bilateral exchange is not active.

In either equilibrium, buyers with values above $\overline{v}$ *and sellers with costs below* $\underline{c}$ *go to the intermediary, and* $\overline{v}$ *and* $\underline{c}$ *are uniquely determined.*

As argued, equilibrium (ii) is not compelling if the intermediary charges a positive spread $p_B - p_S$ because traders with values and costs between p_W and p_B respectively p_S and p_W cannot trade at the intermediary but would benefit from trading with each other. Discarding this equilibrium as implausible gives a unique (plausible) equilibrium under wholesale with prices $p^*_B(\lambda)$ and $p^*_S(\lambda)$ given by (6).

We now consider agency. From Lemma 1 we know that under agency the bilateral exchange and the intermediated market cannot simultaneously be active, which contrasts with wholesale (as per (i) in Proposition 6). In addition, if $b < 1 - \Phi^{-1}(0)$, there is a second kind of multiplicity of equilibria, indexed by j. These equilibria differ with respect to the lowest price $\underline{p}_j$ sellers set in equilibrium and that constitutes the lowest valuation of a buyer joining the intermediary under agency: $\underline{p}_j$ can be supported as such a lower bound if and only if $\underline{p}_j \in [\Phi^{-1}(0), 1-b]$. To see the "if"-part, notice that if no seller sets a price below $\underline{p}_j$, no buyer with a value below $\underline{p}_j$ has an incentive to join the intermediary, and if no buyer with a value below $\underline{p}_j$ joins the intermediary, no seller has an incentive to set a price below $\underline{p}_j$. To see that $\underline{p}_j < \Phi^{-1}(0)$ is not consistent with equilibrium, recall that for a seller of cost 0, the profit-maximizing price $\Phi^{-1}(0)$ does not vary with the lowest valuation of a buyer joining the intermediary, provided this valuation is $\Phi^{-1}(0)$ or less. That no $\underline{p}_j > 1 - b$ is consistent with an equilibrium in which trade occurs via the intermediary is an implication of Lemma 1.

The equilibria that differ with respect to $\underline{p}_j$ do not invalidate our main point: for any equilibrium under agency with $\underline{p}_j \in [\Phi^{-1}(0), 1 - b]$, the profit of the intermediary exceeds the wholesale profit $\Pi_W^*(\lambda)$ for λ sufficiently large. (Of course, our comparative statics analysis in Section 4 rests on the assumption that $\underline{p}_j = \Phi^{-1}(0)$.)

In contrast, entry deterrence obviously fails in the equilibrium in which no trade occurs via the intermediary. As we argue next, this equilibrium is eliminated if we assume that under agency the intermediary can also commit to standing ready to buy from sellers.

In equilibrium under wholesale, no seller will remain unmatched at the intermediary. This means that the assumption that the intermediary stands ready to buy at p_S can be recast as saying that under wholesale the intermediary stands ready to buy from any unmatched seller (at the best price available to any seller, which under wholesale is simply p_S). Imposing the same assumption under agency would mean that the intermediary stands ready to buy from any unmatched seller at the price $1 - b$.

For symmetric Pareto distributions with $\sigma \geq 1$, that is, when the constraint $b \leq 1 - \Phi^{-1}(0)$ is not binding, we now show that under this alternative assumption under agency there is no equilibrium in which the intermediated market is inactive. To see this, it suffices to recall that the equilibrium fee for this case is $b^* = 1/(1 + \sigma)$ while the Walrasian price is $p_W = 1/2$. Thus, for any $\sigma \geq 1$ and any $\lambda < 1$, stipulating a strategy profile according to which no one joins the intermediary, any seller with a cost $c < 1 - b^*$ has a unilateral incentive to join the intermediated market because $1 - b^* - c > \lambda \max\{p_W - c, 0\}$. Moreover, if we assume that in addition the intermediary stands ready to sell any items he buys from unmatched sellers at the price $\Phi^{-1}(0)$, all buyers with $v \geq \Phi^{-1}(0)$ will join the intermediary, implying that the lowest price sellers set in equilibrium is $\Phi^{-1}(0)$, which further implies that the mass of buyers and sellers joining the intermediary is the same and that the intermediary never has to take a position. To see the latter, notice that $\Phi^{-1}(0) = 1/(1 + \sigma)$ for $F(v) = 1 - (1 - v)^\sigma$. Consequently,

$G(1-b^*) = (\sigma/(1+\sigma))^\sigma = 1 - F(\Phi^{-1}(0))$. This means that we obtain as the unique equilibrium outcome under agency the one we had focused on.

For $\sigma < 1$, the constraint $b \leq 1 - \Phi^{-1}(0)$ is binding and we have $b^* = \sigma/(1+\sigma)$. Without adjusting the fee, there would be more sellers than buyers at the intermediary (i.e. $G(1-b^*) > 1 - F(\Phi^{-1}(0))$), which makes the commitment to buy potentially costly. More generally, if F and G are not symmetric, the amendments just discussed need not be sufficient to induce as the unique equilibrium outcome the one we have focused on. Although this raises a number of interesting questions, we think this analysis is best left for future research.

6.4 Policy Implications

Policy-makers and antitrust authorities concerned with the competitiveness of a brokerage market that uses agency may quite naturally look at the level of the fees. Intuition suggests that lowering fees will enhance consumer surplus. However, in our model the anticompetitive effects of, say, percentages fees do not stem so much from their level as from their very nature: if a given percentage fee is used to deter the emergence of a competitive exchange, then so will any lower percentage fee. Therefore, if agency is used for the purpose of entry deterrence, standard regulatory approaches will not be effective. A policy implication of our results is thus that in some cases, banning the agency model can be welfare enhancing.

For some markets, such as real-estate brokerage, the rather drastic intervention of banning the agency model and requiring the wholesale model may not be feasible for a variety of reasons. For one thing, requiring real estate brokers to buy and sell houses, like retailers buy and sell all sorts of storable goods, is not going to be feasible, simply because of the liquidity constraints these brokers face. However, a more moderate policy intervention that might pre-empt entry-deterring behavior by brokers would be to require brokers to use flat-fee agency; that is, a fixed fee, which is paid when listing the property and which is paid irrespective of whether the property was sold. It can be shown that flat-fee agency is equivalent to posted prices (i.e., the wholesale model) in our setup. Suppose, for example, that, like in Niedermayer and Shneyerov (2014), the intermediary charges upfront *participation fees* τ_B from buyers and τ_S from sellers and then allows buyers and sellers to randomly match and bargain by a random proposer game in which the seller makes an offer with probability α and the buyer with probability $1-\alpha$. In equilibrium, any seller that makes an offer offers p_B; any buyer that proposes offers p_S; and only buyers with $v \geq p_B$ and sellers with $c \leq p_S$ enter. The optimal fixed participation fees $\tau_S = \alpha(p_B - p_S)$ and $\tau_B = (1-\alpha)(p_B - p_S)$ extract the same rents as wholesale and provide the same incentives, so that the outcome is equivalent to wholesale.

The agreement of real estate brokerage associations with the Department of Justice to stop practices which have been seen as discriminatory toward flat-fee brokers can be seen as a step in this direction (see DOJ, 2007).[38]

At a basic level, a key implication of our analysis is that collusion among brokers may look different from collusion in product markets that has been the more traditional focus in economics. As discussed in Subsection 6.2, collusion by brokers to prevent the emergence of a competing exchange may involve an aggregate capacity in excess of the Walrasian quantity and a percentage fee that is self-enforcing even in a one-shot setup.

6.5 Dynamic Random Matching

In Appendix C we derive results for a variant of our model with a binary type space, in which buyers' valuations are drawn from $\{v_1, v_2\}$ and sellers' costs are drawn from $\{c_1, c_2\}$. The main results on entry deterrence go through in this alternative setting.

The binary type space has the advantage that some of the calculations become more tractable. In particular, this tractability allows us to address dynamics: consider a model in which a trader that does not trade today has the chance to trade in the future. Such traders wait in the market to try to get a future trade opportunity. In such a dynamic

[38] The allegations were that flat-fee brokers were put at a disadvantage in the Multiple Listing Services, e.g., in cases where a buyer's broker and a seller's broker would usually cooperate, cooperation was refused for flat-fee brokers.

random matching setup, the relevant distributions of valuations and costs in the market are different from the primitive distributions for two reasons. First, traders have the option value of future trade which should be added to the seller's cost and subtracted from the buyer's valuation. Second, low valuation buyers and high cost seller trade with a lower probability, wait longer until they get to trade, and hence are overrepresented in the pool of traders in the market compared to the distribution of entrants in every period.

Dealing with these two changes of the type distributions in a dynamic model is known to be a very hard problem, see Satterthwaite and Shneyerov (2007, 2008).[39] For binary type spaces, a dynamic model is more tractable, see Duffie et al. (2005). We analyze entry-deterring agency in a dynamic random matching with a binary type space in Appendix C and find that our results go through qualitatively. In a dynamic model, traders' concern about a lower probability of trade becomes less important, but in exchange, traders worry about a longer time on market, which has a similar effect.

6.6 Incomplete Foreclosure

We have stated our results on entry-deterring agency in the clearest possible way by assuming that entry deterrence leads to bilateral exchange shutting down completely. However, our results also hold for a less extreme version of our model: if entry deterrence does not lead to the bilateral exchange shutting down completely, but only to operating at an inefficiently low scale.

Consider a bilateral market that operates at an inefficiently low scale if only a small mass of buyers and sellers enter (e.g. if $\min\{1 - F(\overline{v}), G(\underline{c})\} < T$ for some threshold T), so that the probability of meeting a trading partner is λ_0. Above the threshold T, the probability of meeting a trading partner is $\lambda > \lambda_0$. Further assume that there is a small mass ϵ of both buyers and sellers who always go to the bilateral market, possibly because they are sophisticated at searching and do not need an intermediary or

[39]It can be seen from these articles how complex the analysis of dynamic random matching with continuous type spaces is, even if one considers the limit as search frictions vanish. Away from the limit the analysis typically becomes even harder.

because they cannot afford the intermediary's fees. As long as $\epsilon < T$, entry deterrence leads to the bilateral exchange operating at an inefficiently low scale, which reduces (but does not completely eliminate) the competitive threat the bilateral exchange poses to the intermediary's rent extraction.

This less extreme incomplete foreclosure interpretation of our results lends itself well to empirical analysis: we observe e.g. for the real-estate market in the US that the bilateral market is not completely inactive, but rather operates at an inefficiently low scale. We discuss how our paper relates to empirical research in Appendix D.

7 Conclusions

In this paper, we present a model in which the operator of one market can successfully deter the emergence of a competing exchange. Entry deterrence is the more profitable the more efficient is the matching technology in the competing exchange whose emergence is deterred. The paper thus brings to light a relevant and novel possibility of entry deterrence. For symmetric Pareto distributions, entry deterrence is always harmful to social welfare, and when it occurs in equilibrium, to consumer surplus defined as social welfare less the intermediary's profit.

Policy-makers and antitrust authorities concerned with the competitiveness of a brokerage market that uses agency quite naturally look at the level of the fees charged. Intuition suggests that lowering fees will enhance consumer surplus. However, in our model the anticompetitive effects of, say, percentages fees do not stem so much from their level as from their very nature. If agency is used for the purpose of entry deterrence, standard regulatory approaches will not be effective as any lower fee will also deter entry. The first-order welfare gains in our model are achieved by inducing intermediaries to use the wholesale model or equivalent mechanisms (such as flat-fee agency).

Our paper also brings to light novel and perhaps surprising aspects of collusion in intermediated markets. To prevent the emergence of a competing exchange, collusion by brokers may involve an aggregate capacity in excess of the Walrasian quantity and a percentage fee that is self-enforcing even in a one-shot setup.

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Online Appendix

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A Proofs and Derivations

A.1 Proofs

Proof of Lemma 1. To prove the lemma, we show that a coexistence between the intermediated market and the bilateral exchange under agency leads to a contradiction, with coexistence meaning that they both attract positive measures of buyers and sellers who trade. This then implies that if the bilateral exchange is active, the intermediated market cannot be active, implying that the intermediary makes zero profits.

Denote the set of valuations of buyers and costs of sellers who go to the intermediary by I_B and I_S, respectively. Further, denote the boundaries of these sets by $\underline{v}_I = \inf I_B$, $\overline{c} = \sup I_S$, $\overline{v}_I = \sup I_B$. (Note that, according to the notation used in the text, we have $\overline{v} = \underline{v}_I$.) The proof consists of four steps. Denote by $U_B(v)$, $U_S(c)$ the utility from going to the intermediary and by $V_B(v)$, $V_S(c)$ the utility from going to the bilateral exchange. All of these functions are defined for a given equilibrium. Let $U_S(p, c)$ be the utility of a seller with cost c who sets the price p and goes to the intermediary. (Thus, $U_S(c) = \max_p U_S(p, c)$.) Denote by H the distribution of prices set by sellers who go to the intermediary. (If no seller goes to the intermediary, we define $H(p) = 0$ for all $p \leq 1$.)

Step 1: We show that $\overline{v}_I = 1$.

The buyer's utility can be written as $U_B(v) = \int_{\underline{p}}^{v} (v - p)dH(p)$, where $\underline{p}$ is the lowest price set at the intermediary. This implies $U_B'(v) = H(v)$.

In equilibrium no seller would set a price $p > \overline{v}_I$, therefore $H(v) = 1$ for all $v \geq \overline{v}_I$. Consequently, $U_B'(v) = 1$ for all $v \geq \overline{v}_I$.

Next, recall that, for $v \geq p_W$, $V_B(v) = \lambda(v - p_W)$. Hence, $V_B'(v) = \lambda$. If $\overline{v}_I < 1$, we have $U_B(\overline{v}_I) = V_B(\overline{v}_I)$ and for all $v > \overline{v}_I$, $U_B(v) > V_B(v)$ because for $v \geq \overline{v}_I$ we have

$U'_B(v) = 1 > \lambda = V'_B(v)$. This is a contradiction to $\overline{v}_I = \sup I_B < 1$. Therefore $\overline{v}_I = 1$.

Step 2: We show that I_B is convex.

Assume to the contrary that I_B is not convex and that there is at least one interval $[v_1, v_2]$ missing from I_B, formally $[v_1, v_2] \cap I_B = \emptyset$ and $\underline{v}_I < v_1 < v_2 < \overline{v}_I$.

A seller would never set a price in $[v_1, v_2]$ since setting such a price is dominated by setting the price v_2. Therefore, for $v \in [v_1, v_2]$, we have

$$\begin{aligned} U_B(v) &= \int_{\underline{p}}^{v} (v-p)dH(p) \\ &= \int_{\underline{p}}^{v_1} (v-v_1)dH(p) + \int_{\underline{p}}^{v_1} (v_1-p)dH(p) + \int_{v_1}^{v} (v-p)dH(p) \\ &= (v-v_1)H(v_1) + U_B(v_1) + 0 \end{aligned}$$

where the second equality comes from splitting the integral at v_1 and adding and subtracting $\int_{\underline{p}}^{v_1} v_1 dH(v)$ and the third equality from the definition of $U_B(v_1)$ and the fact that H has no mass in $[v_1, v_2)$. The above implies $U'_B(v) = H(v_1)$ for all $v \in [v_1, v_2)$. The same holds for v_2, even if H has a mass point on v_2, because even if the probability of trade jumps up discontinuously for the buyer at v_2, the additional trades that occur are at the price v_2, hence they give zero utility to the buyer of type v_2. Next, observe that $U_B(v_1) > 0$ since U_B increases in v and buyers of some types $v < v_1$ are willing to go to the intermediary. For the same reason, $U_B(v_2) > 0$.

Therefore, the marginal types v_1 and v_2 are indifferent between the intermediary and the bilateral exchange and not between the intermediary and not trading. That is,

$$U_B(v_1) = V_B(v_1), \tag{13}$$

$$U_B(v_2) = V_B(v_2). \tag{14}$$

Also, observe that, for all $v > p_W$, $V_B(v) = \lambda(v - p_W) = V_B(v_1) + \lambda(v - v_1)$ and for all $v \in [v_1, v_2]$, $U_B(v) = U_B(v_1) + (v - v_1)H(v_1)$. Hence, (14) can be rewritten as

$$U_B(v_1) + (v - v_1)H(v_1) = V_B(v_1) + \lambda(v - v_1).$$

Combining this with (13) implies $H(v_1) = \lambda$. This implies that all buyers on $[v_1, v_2]$ are indifferent between the intermediated market and the bilateral exchange.

From this point on, we can take two different approaches, each using a different assumption on the tie-breaking rule.

Approach I: Assume a tie-breaking rule according to which agents choose to go to the intermediary if indifferent.

Then, since buyers on $[v_1, v_2]$ are indifferent, they would go to the intermediary. This is a contradiction to $[v_1, v_2] \cap I_B = \emptyset$. Therefore, I_B is convex.

Approach II: Assume a tie-breaking rule according to which agents choose to go to the bilateral exchange if indifferent.

Let $\tilde{F}$ be the distribution of v of buyers going to the intermediary. Then $U_S(p, c) = \gamma((1-b)p - c)(1 - \tilde{F}(p))$, where $\gamma \in (0, 1]$ is the probability of being matched. Since $[v_1, v_2] \cap I_B = \emptyset$, it holds that $\tilde{F}(v) = \tilde{F}(v_1)$ for all $v \in [v_1, v_2]$. This implies $U_S(v_2, c) - U_S(p, c) = \gamma(1-b)(v_2 - p)(1 - \tilde{F}(v_1))$ for all $p \in [v_1, v_2]$. This means that there is a discrete increase of U_S when a seller increases his price from v_1 to v_2. This also implies that $U_S(v_2, c) > U_S(v_1 - \epsilon, c)$ for all $c \in I_S$ for a sufficiently small ϵ. Therefore, no seller sets a price in the interval $[\overline{p}, v_2]$ for some $\overline{p} < v_1$. This also means that no price is set in the interval $[\overline{p}, v_2]$. This implies that all buyers with $v \in [\overline{p}, v_2]$ are indifferent between going to the intermediary and the bilateral exchange, but buyers on $[\overline{p}, v_1]$ go to the intermediary and buyers on $[v_1, v_2]$ go to the bilateral exchange. This is a contradiction to the tie-breaking rule. Therefore, it is not possible that there is an interval $[v_1, v_2]$ missing from I_B.

There is still the case left that there is no interval missing from I_B, but there is a positive mass of buyers between $\underline{v}_I$ and $\overline{v}_I$ that do not go to the intermediary. An example would be $I_B = [\underline{v}_I, \overline{v}_I] \cap \mathbb{Q}$, that is all buyers whose valuation is an irrational number would not go to the intermediary. However, this is not possible for the following reasons. First, observe that $U_B(v)$ is convex and $V_B(v)$ is linear in v. This implies that $U_B(v)$ and $V_B(v)$ can intersect at most twice. The only way to have a positive mass of buyers in

I_B missing from $[\underline{v}_I, \overline{v}_I]$ is that on some interval $[v_1, v_2]$ with $[\underline{v}_I, \overline{v}_I] \cap [v_1, v_2] \neq \emptyset$ buyers are indifferent between the intermediary and bilateral exchange, i.e. $U_B(v) = V_B(v)$, and some choose the intermediary and other the bilateral exchange. However, this contradicts our assumption on the tie-breaking rule that all indifferent agents make the same choice.

We are left with the case in which the curves $U_B(v)$ and $V_B(v)$ touch only once. However, in this case, there is only mass zero of buyers who go to the bilateral exchange, which means that the bilateral exchange is inactive.

Since all possibilities of I_B lead to contradictions, I_B has to be convex.

Step 3: $p_W > \underline{v}_I$.

Assume to the contrary that $p_W < \underline{v}_I$. Then, $V_B(\underline{v}_I) = \lambda(\underline{v}_I - p_W) > 0$. Observe that a seller will never set a price below $\underline{v}_I$ at the intermediary. Therefore, $U_B(\underline{v}_I) = 0$. However, this is a contradiction to $\underline{v}_I$ going to the intermediary since $V_B(\underline{v}_I) > U_B(\underline{v}_I)$.

Step 4: The bilateral exchange cannot be active if the intermediated market is active.

Since $I_B = [\underline{v}_I, 1]$ for some $\underline{v}_I < p_W$, the bilateral exchange cannot exist because there is no buyer who would trade at the bilateral exchange at price p_W.

Putting together steps 1 to 4 shows that having the bilateral exchange and the intermediated market coexist leads to a contradiction. Therefore, if the bilateral exchange is active, no one goes to the intermediary and the intermediary makes zero profits. □

Proof of Proposition 5. For each of the three standard bargaining protocols, we denote by $\hat{V}_B(q)$ and $\hat{V}_S(q)$ the expected payoff from participating in the bilateral exchange for the buyer and seller who are indifferent between joining the intermediary and the bilateral exchange, conditional on being matched in the bilateral exchange, when the intermediary trades the quantity q. Consequently, given $\hat{V}_B(q)$ and $\hat{V}_S(q)$ the intermediary's profit maximization problem is to choose q to maximize

$$\left(F^{-1}(1-q) - G^{-1}(q) - \lambda(\hat{V}_B(q) + \hat{V}_S(q))\right) q.$$

Random proposer take-it-or-leave-it-offers The marginal buyer who is indifferent between trading with the intermediary and going to the bilateral market has valuation

$\overline{v} = 1 - q$, the marginal seller cost $\underline{c} = q$. Buyers with a valuation below $\underline{c}$ and sellers with costs above $\overline{v}$ do not enter the bilateral market. Hence, in the bilateral market, buyer valuations and seller costs are uniformly distributed on $[\underline{c}, \overline{v}]$. Denote this distribution as $\tilde{G}(x) = \tilde{F}(x) = (x - \underline{c})/(\overline{v} - \underline{c})$.

The optimal price offer $p_b(v)$ of a buyer with value $v \in [\underline{c}, \overline{v}]$ maximizes $(v - p_b)\tilde{G}(p_b)$, yielding $p_b(v) = (v + \underline{c})/2$. Likewise, the optimal offer $p_s(c)$ of a seller with cost $c \in [\underline{c}, \overline{v}]$ is the p_s that maximizes $(p_s - c)(1 - \tilde{F}(p_s))$, yielding $p_s(c) = (\overline{v} + c)/2$. Observe that the offers of the marginal traders – of the buyer with value $\overline{v}$ and the seller with cost $\underline{c}$ – are accepted with probability 1/2. Setting $\underline{c} = q$ and $\overline{v} = 1 - q$, the expected payoff for the marginal seller who is matched to a buyer therefore is

$$\hat{V}_S(q) = (1 - \alpha)(p_s(\underline{c}) - \underline{c})(1 - \tilde{F}(p_s(\underline{c}))) + \alpha \int_{\underline{c}}^{\overline{v}} (p_b(v) - \underline{c}) d\tilde{F}(v) = \frac{\overline{v} - \underline{c}}{2} = \frac{1 - 2q}{4}.$$

Because this is independent of α, it follows by symmetry that $\hat{V}_B(q) = \hat{V}_S(q)$, yielding $\hat{V}(q) = 2\hat{V}_S(q) = (1 - 2q)/2$.

Plugging this $\hat{V}(q)$ into the wholesale intermediary's profit maximization problem $(F^{-1}(1 - q) - G^{-1}(q) - \lambda\hat{V}(q))q$, we get the maximizer $q^* = 1/4$ and the maximum

$$\Pi_W^{TOL} = \frac{2 - \lambda}{16}.$$

As derived before, under agency, the intermediary's profit is $\Pi_A^* = 1/12$. Hence, the intermediary prefers agency to wholesale ($\Pi_A > \Pi_W^{TOL}$) if

$$\lambda > \frac{2}{3} = \lambda_\Pi^{TOL}.$$

Welfare under wholesale generated by trade via the intermediary is 3/16.[40] The surplus

[40] A simple geometric argument can be used. Social welfare under efficiency is 1/4 while Harberger's triangle due to the intermediary's monopoly power is 1/16.

generated in the matching market is

$$(1-2q)\lambda\left[\alpha\frac{\int_{1/2}^{1-q}\int_{q}^{2v-(1-q)}(v-c)dcdv}{(1-2q)^2}+(1-\alpha)\frac{\int_{q}^{1/2}\int_{2c-q}^{1-q}(v-c)dvdc}{(1-2q)^2}\right]=\lambda\frac{1-2q}{8}|_{q=1/4}=\frac{\lambda}{32},$$

where the term $1-2q$ is the mass of traders in the matching market. Therefore, social welfare under wholesale is

$$S_W^{TOL}=\frac{3}{16}+\frac{\lambda}{32}.$$

Because welfare under agency is only 1/6, it follows that entry deterrence always decreases social welfare.

Under wholesale, consumer surplus, defined as the surplus of buyers and sellers and denoted CS_W^{TOL},

$$CS_W^{TOL}=S_W^{TOL}-\Pi_W^{TOL}=\frac{2+3\lambda}{32}=\frac{12+18\lambda}{192}$$

while consumer surplus under agency is $CS_A=1/12$. Thus, $CS_W^{TOL}>CS_A$ is equivalent to

$$\lambda>\frac{2}{9}=\lambda_{CS}^{TOL},$$

implying that agency is always detrimental to consumer surplus when it is preferred by the intermediary.

Nash bargaining Under generalized Nash bargaining, where the buyer with value v obtains α of the joint surplus $\max\{v-c,0\}$ when matched to a seller with cost c, the expected payoff of a buyer of type v conditional on being matched in the random matching market is $\alpha\int_q^v(v-c)dc/(1-2q)=\alpha(v-q)^2/(2(1-2q))$.

The marginal buyer with value $v=1-q$ has an expected payoff from participating in the matching market, conditional on being matched, of $\hat{V}_B(q)=\alpha(1-2q)/2$. Analogously, $\hat{V}_S(q)=(1-\alpha)(1-2q)/2$ can be established under generalized Nash bargaining where the seller gets the share $(1-\alpha)$ of the surplus. Just like with take-it-or-leave-it offers, with generalized Nash bargaining, α does not affect $\hat{V}(q)=\hat{V}_B(q)+\hat{V}_S(q)=(1-2q)/2$, but in contrast to random take-it-or-leave-it offers, α affects the division of the sum of the marginal utilities $\hat{V}(q)$ into $\hat{V}_B(q)$ and $\hat{V}_S(q)$. Consequently, α will affect the equilibrium prices under generalized Nash bargaining but neither the bid-ask spread nor the quantity traded. Moreover, because $V(q)$ is the same under generalized Nash bargaining and under take-it-or-leave-it offers, the intermediary's equilibrium profits will be the same, that is, $\Pi_W^{NB} = (2-\lambda)/16$ and so will be the conditions under which entry deterrence occurs; that is, for

$$\lambda > \frac{2}{3} = \lambda_{\Pi}^{NB}.$$

The only payoff-relevant differences regard the surplus that is generated in the random matching market. With generalized Nash bargaining, this surplus is

$$(1-2q)\lambda \frac{\int_q^{1-q}\int_q^v (v-c)dcdv}{(1-2q)^2}|_{q=1/4} = \frac{\lambda}{24},$$

implying that social welfare under wholesale and generalized Nash bargaining is

$$S_W^{NB} = \frac{3}{16} + \frac{\lambda}{24}.$$

A fortiori, entry-deterring agency will reduce social welfare. Consumer surplus with wholesale and Nash bargaining is

$$CS_W^{NB} = S_W^{NB} - \Pi_W^{NB} = \frac{3+5\lambda}{48} = \frac{12+20\lambda}{192},$$

which for any

$$\lambda > \frac{1}{5} = \lambda_{CS}^{NB}$$

is larger than the consumer surplus under agency of 1/12.

Double Auction We focus on the linear equilibrium described in the text. The marginal traders who will be indifferent between participating and being inactive are the seller with cost $\overline{c}$ equal to $p_b(\overline{v})$; that is, $\overline{c} = \overline{v} - (\overline{v} - \underline{c})/4$, and the buyer with value $\underline{v}$ equal to $p_s(\underline{c})$;

that is, $\underline{v} = \underline{c} + (\overline{v} - \underline{c})/4$. Consequently, the masses of active buyers and sellers will be

$$\overline{v} - \underline{v} = \frac{3}{4}(\overline{v} - \underline{c}) = \overline{c} - \underline{c}.$$

The expected payoff of a buyer of type $v \in [\underline{v}, \overline{v}]$ upon being matched is therefore

$$V_B(v, \overline{v}, \underline{c}) = \frac{1}{\overline{c} - \underline{c}} \int_{\underline{c}}^{p_s^{-1}(p_b(v))} [v - (p_b(v) + p_s(c))/2] dc.$$

Substituting $\underline{c} = q$ and $\overline{v} = 1 - q$, we obtain

$$\hat{V}_B(q) = V_B(1 - q, 1 - q, q) = \frac{3}{8}(1 - 2q).$$

By symmetric arguments, $\hat{V}_S(q) = \hat{V}_B(q)$, so that for the $k = 1/2$ double-auction,

$$\hat{V}(q) = \frac{3}{4}(1 - 2q).$$

The profit maximizing quantity under wholesale is still $q^* = 1/4$. The equilibrium prices are

$$p_B^* = \frac{3}{4} - \frac{\lambda}{16} \quad \text{and} \quad p_S^* = \frac{1}{4} + \frac{3\lambda}{16}, \tag{15}$$

yielding and equilibrium spread of $(4 - 3\lambda)/8$ and thus an equilibrium profit of

$$\Pi_W^{DA} = \frac{4 - 3\lambda}{32}.$$

Therefore, entry deterrence occurs in equilibrium if and only if

$$\lambda > \frac{4}{9} = \lambda_\Pi^{DA}.$$

Equilibrium welfare under wholesale is

$$S_W^{DA} = \frac{3}{16} + \lambda \frac{3}{64},$$

where

$$\lambda\frac{3}{64}=\lambda\frac{3(1-2q)}{4}\frac{\int_{q+(1-2q)/4}^{1-q}\int_{q}^{v-(1-2q)/4}(v-c)dcdv}{(3(1-2q)/4)^2}|_{q=1/4}$$

is the expected social surplus from the matching market.[41] Note that $S_W^{DA}>1/6=S_A$.

Consumer surplus under wholesale is then easily seen to be

$$CS_W^{DA}=\frac{4+7\lambda}{64}=\frac{12+21\lambda}{192},$$

which is larger than consumer surplus under agency for any

$$\lambda>\frac{4}{21}=\lambda_{CS}^{DA}.$$

□

Proof of Proposition 6. We first prove that for case (i) of the Proposition (both the intermediary and the bilateral exchange are active) the equilibrium is unique.

The proof for case (i) proceeds in four lemmas.

Lemma A1. *There is single-crossing. That is, if any buyers and sellers go to the intermediary then it is buyers in the interval $[\overline{v},1]$ for some $\overline{v}$ and sellers in the interval $[0,\underline{c}]$ for some $\underline{c}$.*

Proof. The utility from going to the intermediary is

$$U_B(v)=\min\left\{1,\frac{\int_{v\in I_B}dF(v)}{\int_{c\in I_S}dG(c)}\right\}(v-p_B),$$

for buyer of type v, where I_B and I_S are the sets of buyers and sellers going to the intermediary, respectively. The utility of a seller of type c from going to the intermediary is $U_S(c)=p_S-c$. The utility from going to the bilateral exchange is $V_B(v)=\lambda(v-p_W)$

[41] To see this, notice that the density of buyers (and of sellers) in the matching market is 1 divided by the mass of active buyers; that is $1/(3(1-2q)/4)$, which explains the term $1/(3(1-2q)/4)^2$. The surplus of a trade being $v-c$ and trade occurring, as noted, whenever $v\geq c+(1-2q)/4$ explains the upper bounds of the inner integral while the lower bound in the outer integral reflects the fact that $\underline{v}=q+(1-2q)/4$. Multiplying by the mass of active traders $3(1-2q)/4$ gives the result.

for the buyers and $V_S(c) = \lambda(p_W - c)$. Observe that

$$U'_B(v) = \min\left\{1, \frac{\int_{v\in I_B} dF(v)}{\int_{c\in I_S} dG(c)}\right\} > \lambda = V'_B(v),$$

for all v since otherwise no buyer would go to the intermediary. Therefore, buyers with $v \in [\overline{v}, 1]$ (if any) will go to the intermediary. Also, $U'_S(c) = -1 < -\lambda = V'_S(c)$ for all c. Therefore, sellers with $c \in [0, \underline{c}]$ (if any) will go to the intermediary. □

Lemma A2. *If* $1 - F(\overline{v}) = G(\underline{c})$ *and* $\overline{v} > \underline{c}$, *then* p_W *is independent of* $\overline{v}$ *and* $\underline{c}$.

Proof. If buyers with $v > \overline{v}$ and sellers with $c < \underline{c}$ go to the intermediary, the residual demand at price p at the bilateral exchange is $F(\overline{v}) - F(p)$, the residual supply is $G(p) - G(\underline{c})$. Plugging $1 - F(\overline{v}) = G(\underline{c})$ in the market equilibrium condition for the bilateral exchange, $F(\overline{v}) - F(p_W) = G(p_W) - G(\underline{c})$ reveals that p_W is independent of $\overline{v}$, $\underline{c}$ and satisfies $1 - F(p_W) = G(p_W)$. □

Lemma A3. *Given prices* p_S *and* p_B, p_W *is pinned down uniquely. Moreover,* $1-F(\overline{v}) = G(\underline{c})$ *holds.*

Proof. Given that buyers in $[\overline{v}, 1]$ and sellers in $[0, \underline{c}]$ go to the intermediary, the indifference conditions for buyer $\overline{v}$ and seller $\underline{c}$ are

$$q(\overline{v})(\overline{v} - p_B) = \lambda(\overline{v} - p_W), \tag{16}$$

$$p_S - \underline{c} = \lambda(p_W - \underline{c}), \tag{17}$$

where $q(\overline{v}) = \min\{1, G(\underline{c})/(1 - F(\overline{v})\}$. Taking total derivatives with respect to $\overline{v}$ and p_W in (16), we get

$$d\overline{v}\,[q'(\overline{v})(\overline{v} - p_B) + q(\overline{v}) - \lambda] = -\lambda dp_W.$$

Observe that the expression in square brackets on the left-hand-side is positive since $q(\overline{v})$ is weakly increasing, $\overline{v} > p_W$ and $q(\overline{v}) \geq \lambda$ (if $\lambda > q(\overline{v})$, no buyer would go to the intermediary). Since $-\lambda$ on the right-hand-side is negative, we have $d\overline{v}/dp_W < 0$. By an analogous reasoning, a total differentiation of (17) reveals that $d\underline{c}/dp_W < 0$.

Next, consider the market equilibrium condition for the bilateral exchange

$$F(\overline{v}) - F(p_W) = G(p_W) - G(\underline{c}). \tag{18}$$

Observe that it is satisfied for $\overline{p}_W$ if $1 - F(\overline{v}) = G(\underline{c})$, where $\overline{p}_W$ is given by $1 - F(\overline{p}_W) = G(\overline{p}_W)$. Further, rearranging (18) yields

$$F(\overline{v}(p_W)) + G(\underline{c}(p_W)) = F(p_W) + G(p_W),$$

where the functions $\overline{v}(p_W)$ and $\underline{c}(p_W)$ are defined such that the equations (16) and (17) are satisfied.

The left-hand-side of the above equation is weakly decreasing in p_W, the right-hand-side is increasing in p_W. Therefore, there is a unique solution for the price in the bilateral exchange: $p_W = \overline{p}_W$. □

Lemma A4. *Given p_B and p_S, we have unique thresholds $\overline{v}$ and $\underline{c}$.*

Proof. Given that p_W is uniquely pinned down as $\overline{p}_W$, $\overline{v}$ and $\underline{c}$ are unique solutions of the equations (16) and (17). To see that (16) has a unique solution, observe that the derivative of the left-hand-side is $q'(\overline{v})(\overline{v} - p_W) + q(\overline{v})$ and the derivative of the right-hand-side is λ. The former is greater than the latter since $q(\overline{v}) > \lambda$ and $q'(\overline{v}) > 0$ and $\overline{v} - p_B > 0$. By an analogous reasoning, there is a unique solution to (17). For certain values of p_B and p_S there is no solution $\overline{v}$, $\underline{c}$ since no one would go to the intermediary. However, the intermediary would never choose such prices p_B, p_S in equilibrium. Therefore, there exists a solution the equation system (16) and (17) and the solution is unique. □

Lemmas A1 to A4 together imply that there is a unique equilibrium for case (i) of the Proposition, in which all buyers with $v \in [\overline{v}, 1]$ and all sellers with $c \in [0, \underline{c}]$ go to the intermediary and the values $\overline{v}$ and $\underline{c}$ are uniquely determined.

For case (ii), the proof proceeds analogously, with the difference that $V_B(v)$ and $V_S(c)$ should be both replaced by 0 since market participants do not have the outside option of the bilateral exchange. Since the proofs of Lemmas A1 to A4 only relied on $V_B'(v)$ being below and $V_S'(c)$ being above a certain value, all the proofs go through when replacing $V_B'(v)$ and $V_S'(0)$ (the latter being negative) with 0.

□

A.2 Derivation of Results for Symmetric Pareto Distributions

In this part of the Appendix, we state and derive various formulas used for symmetric Pareto distributions in the text. The following results are derived by plugging in the functional forms of the Pareto distributions to the definitions of the various expressions and some algebra.

Profits The profit $\Pi_A^*(\sigma)$ under agency is given as

$$\Pi_A^*(\sigma) = \begin{cases} \frac{\sigma^2 \text{Gamma}(\sigma)\text{Gamma}(1+\sigma)}{(1+\sigma)^\sigma \text{Gamma}(2+2\sigma)} & \text{if} \quad \sigma < 1 \\ \left(\frac{\sigma}{1+\sigma}\right)^\sigma B\left(1+\sigma, 1+\sigma\right)\right) & \text{if} \quad \sigma \geq 1 \end{cases},$$

where $\text{Gamma}(z) = \int_0^\infty x^{z-1} e^{-x} dx$ is the Gamma function and $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ is the Beta function. The profit under wholesale, denoted $\Pi_W^*(\lambda, \sigma)$, is

$$\Pi_W^*(\lambda, \sigma) = (1-\lambda)\left(\frac{\sigma}{2}\right)^\sigma \left(\frac{1}{1+\sigma}\right)^{1+\sigma}.$$

Consequently, $\lambda_\Pi(\sigma)$ is given as

$$\lambda_\Pi(\sigma) = \begin{cases} 1 - 2^\sigma \sigma^{2-\sigma} \text{Gamma}(\sigma)\text{Gamma}(2+\sigma)/\text{Gamma}(2+2\sigma) & \text{if} \quad \sigma < 1 \\ 1 - 2^\sigma (1+\sigma)\sigma B(1+\sigma, 1+\sigma) & \text{if} \quad \sigma \geq 1 \end{cases}.$$

Surplus The general expression for social surplus under wholesale is

$$S_W = \int_0^{Q^*} (F^{-1}(1-q) - G^{-1}(q))dq + \lambda \int_{Q^*}^{G(\overline{p}_W)} (F^{-1}(1-q) - G^{-1}(q))dq,$$

while the one under agency is

$$S_A = \min\{1 - F(\overline{v}), G(\underline{c})\} \frac{\int_0^{1-b^*} \int_{\Phi^{-1}(c/(1-b^*))}^1 (v-c)f(v)g(c)dvdc}{(1-F(\overline{v}))G(\underline{c})}.$$

For symmetric Pareto distributions, these formulas specialize to

$$S_W(\sigma,\lambda) = \frac{\lambda(1+\sigma)^{1+\sigma} + (1-\lambda)\sigma^\sigma(1+2\sigma)}{2^\sigma(1+\sigma)^{2+\sigma}}$$

respectively, for $\sigma \geq 1$,

$$S_A(\sigma) = \sigma^\sigma \frac{1+3\sigma}{(1+\sigma)^{1+\sigma}} B(1+\sigma, 1+\sigma)$$

and, for $\sigma < 1$,

$$S_A(\sigma) = \frac{\sigma^{1+\sigma}(1+\sigma)^{1-2\sigma}\text{Gamma}(\sigma)\text{Gamma}(1+\sigma)}{\text{Gamma}(2+2\sigma)}.$$

For the uniform, these become $S_W(1,\lambda) = (3+\lambda)/16$ and $S_A(1) = 1/6$.

Consequently, for $\sigma \geq 1$, the ratio $S_A(\sigma)/S_W(\sigma,0)$ is given by

$$\frac{S_A(\sigma)}{S_W(\sigma,0)} = \frac{2^\sigma(1+\sigma)(1+3\sigma)\text{Gamma}(1+\sigma)^2}{(1+2\sigma)\text{Gamma}(2+2\sigma)}, \tag{19}$$

while for $\sigma < 1$ it is

$$\frac{S_A(\sigma)}{S_W(\sigma,0)} = \left(\frac{\sigma}{1+\sigma}\right)^\sigma \frac{\text{Gamma}(1+\sigma)\text{Gamma}(2+\sigma)}{\text{Gamma}(2+2\sigma)}. \tag{20}$$

Prices To derive $p^T(\sigma)$, notice that for $\sigma \geq 1$

$$p^T(\sigma) = \frac{\int_0^{\sigma/(1+\sigma)} p(1/(1+\sigma),c)(1-F(p(1/(1+\sigma),c)))dG(c)}{G(\sigma/(1+\sigma))(1-F(1/(1+\sigma)))\mu^T},$$

where μ^T is the share of matches that lead to a transaction given as

$$\mu^T = \frac{\int_0^{\sigma/(1+\sigma)}(1-F(p(1/(1+\sigma),c)))dG(c)}{G(\sigma/(1+\sigma))(1-F(1/(1+\sigma)))}.$$

Plugging in the functions for F and G and simplifying yields the result.

For $\sigma < 1$, the fee is $\sigma/(1+\sigma)$. Denoting by $\hat{p}^T(\sigma)$ and $\hat{\mu}^T$ the average transaction price and share of matches that lead to a transaction in this case, we have

$$\hat{p}^T(\sigma) = \frac{\int_0^{1/(1+\sigma)} p(\sigma/(1+\sigma), c)(1 - F(p(\sigma/(1+\sigma), c)))dG(c)}{G(1/(1+\sigma))(1 - F(1/(1+\sigma)))\hat{\mu}^T}$$

and

$$\hat{\mu}^T = \frac{\int_0^{1/(1+\sigma)}(1 - F(p(\sigma/(1+\sigma), c)))dG(c)}{G(1/(1+\sigma))(1 - F(1/(1+\sigma)))}.$$

Plugging in the functions for F and G and simplifying yields $\hat{p}^T(\sigma) = p^T(\sigma)$.

Proof of Proposition 3. Consider first $\sigma \geq 1$. The duplication formula for the Gamma-function implies that

$$\frac{\text{Gamma}(z)}{\text{Gamma}(2z)} = \frac{2^{1-2z}\sqrt{\pi}}{\text{Gamma}(z + 1/2)}.$$

Setting $z = 1 + \sigma$ and using this fact, (19) is equivalent to

$$\frac{S_A(\sigma)}{S_W(\sigma, 0)} = \frac{(1+\sigma)(1+3\sigma)}{2^\sigma(1+2\sigma)}\frac{\sqrt{\pi}}{2}\frac{\text{Gamma}(1+\sigma)}{\text{Gamma}(3/2+\sigma)}.$$

Because $\frac{(1+\sigma)(1+3\sigma)}{2^\sigma(1+2\sigma)}$ and $\frac{\text{Gamma}(1+\sigma)}{\text{Gamma}(3/2+\sigma)}$ are both decreasing in σ (with the sign of the derivative of Gamma$(1+\sigma)$/Gamma$(3/2+\sigma)$ equal to the sign of the difference between the harmonic number at σ and at $\sigma+1/2$, which is negative), $S_A(\sigma)/S_W(\sigma, 0) \leq S_A(1)/S_W(1, 0) = 8/9$. For $\sigma \in (0, 1)$, analogous arguments show that $S_A(\sigma)/S_W(\sigma, 0) < 1$. □

B Mechanism Design Results

B.1 Optimality of Wholesale

Consider a setup with N buyers and N sellers who draw their values and costs independently from distributions F and G with support $[0, 1]$. Assume that F and G are common knowledge but that each agent's realized type is his private information. Assume that agents can only trade via the mechanism designer's intermediary and that each agent's utility of the outside option of not trading is 0. A direct mechanism is a collection of functions $\langle Q, M\rangle$ with $Q : [0, 1]^{2N} \to [0, 1]^{2N}$ specifying, for all agents i, the probability Q_i that the agent trades and $M : [0, 1]^{2N} \to \mathbb{R}^{2N}$ specifying, for all agents i, the payment

M_i makes to the mechanism. A mechanism is feasible if $\sum_{i \in \mathcal{B}} Q_i \leq \sum_{j \in \mathcal{S}} Q_j$, where $\mathcal{B}$ ($\mathcal{S}$) is the set of buyers (sellers). By the revelation principle (see e.g. Myerson, 1981), the focus on direct mechanisms is without loss of generality. A direct mechanism $\langle Q, M \rangle$ is (Bayes-Nash) incentive compatible if, knowing his own type and the functions $\langle Q, M \rangle$ and the distributions F and G, each agent's expected payoff is maximized when reporting his type truthfully, with expectations taken with respect to F and G, assuming all other agents report truthfully. The mechanism is individually rational when this expected payoff is not less than 0.

Let $\Phi(v) := v - (1 - F(v))/f(v)$ and $\Gamma(c) := c + G(c)/g(c)$ denote the virtual value and virtual cost functions associated with F and G, respectively. We assume that the functions $\Phi(F^{-1}(1-q))$ and $\Gamma(G^{-1}(q))$ intersect once for $q \in [0, 1]$. The regularity condition that $\Phi(v)$ and $\Gamma(c)$ are monotone functions is sufficient without being necessary for this. The assumption of a unique point of intersection guarantees that the design problem in the limit is quasiconcave. Let (p_B, p_S) be the unique solution to

$$1 - F(p_B) = G(p_S) \tag{21}$$

and

$$\Phi(p_B) = \Gamma(p_S). \tag{22}$$

With a continuum of buyers and sellers each with mass 1, the following result is true:

Proposition B1. *Consider an intermediary who faces a continuum of buyers and sellers who draw their types independently from distributions F and G. The optimal mechanism for the intermediary that respects agents' incentive compatibility and individual rationality constraints is the wholesale model with prices p_B and p_S satisfying (21) and (22).*

Proof of Proposition B1. As is well known, the revenue equivalence theorem implies that up to additive constants payments and expected revenue are pinned down by the allocation rule of a mechanism (see e.g. Myerson, 1981; Riley and Samuelson, 1981; Krishna, 2002). With a profit-maximizing mechanism, the additive constants are pinned down by the agents' individual rationality constraints. The allocation of the optimal mechanism with N buyers and N sellers who draw their types $\boldsymbol{v} = (v_1, .., v_N)$ and $\boldsymbol{c} = (c_1, ..c_N)$ independently from the distributions F and G induces the q buyers with the highest values and the q sellers with the lowest costs to trade, where q is such that

$$\Phi(v_{(q)}) \geq \Gamma(c_{[q]}) \quad \text{and} \quad \Phi(v_{(q+1)}) < \Gamma(c_{[q+1]})$$

with $v_{(q)}$ denoting the qth highest element of $\boldsymbol{v}$ and $c_{[q]}$ denoting the qth lowest element of $\boldsymbol{c}$ (and with $c_{[0]} = 0 = v_{(N+1)}$ and $c_{[N+1]} = 1 = v_{(0)}$ to make sure q is well defined). This is a conceptually straightforward generalization of the broker-optimal mechanism derived by Myerson and Satterthwaite (1983) for the case $N = 1$.

In the dominant strategy implementation of the optimal mechanism, trading buyers pay $\max\{v_{(q+1)}, \Phi^{-1}(\Gamma(c_{[q]}))\}$ and trading sellers are paid $\min\{c_{[q+1]}, \Gamma^{-1}(\Phi(v_{(q)}))\}$.

We now show that as N goes to infinity, the buyers' payments converge to p_B^0 and the sellers' payments converge to p_S^0 and the quantity traded converges to q^*.

$$\operatorname*{plim}_{N\to\infty} v_{(q)} = \operatorname*{plim}_{N\to\infty} v_{(q+1)} = \operatorname*{plim}_{N\to\infty} \Phi^{-1}(\Gamma(c_{[q]})) =: p_B. \tag{23}$$

Similarly,

$$\operatorname*{plim}_{N\to\infty} c_{[q]} = \operatorname*{plim}_{N\to\infty} c_{[q+1]} = \operatorname*{plim}_{N\to\infty} \Gamma^{-1}(\Phi(v_{(q)})) =: p_S, \tag{24}$$

while the fraction of buyers and sellers who trade satisfy

$$\operatorname*{plim}_{N\to\infty} \frac{q}{N} = \operatorname*{plim}_{N\to\infty} \frac{\max\{i|v_{(i)} \geq p_B\}}{N} = 1 - F(p_B), \tag{25}$$

$$\operatorname*{plim}_{N\to\infty} \frac{q}{N} = \operatorname*{plim}_{N\to\infty} \frac{\max\{i|c_{[i]} \geq p_S\}}{N} = G(p_S). \tag{26}$$

(23), (24), (25), and (26) imply that the optimal mechanism converges to wholesale with p_B and p_S that satisfy $\Phi(p_B) = \Gamma(p_S)$ and $1 - F(p_B) = G(p_S)$.

□

B.2 Nonoptimality of Agency in Thick Markets

We now show that with a continuum of traders, agency with a fee b satisfying $1 - b \geq \Phi^{-1}(0)$ is strictly suboptimal in the sense that the per trader profit of intermediary is strictly smaller under agency than under wholesale.

Corollary B1. *Assume that Φ and Γ are strictly increasing and otherwise make the same assumptions as in Appendix B.1. Then, profit per trader trader under agency is strictly smaller than under wholesale.*

Corollary B1. Because of the revenue equivalence theorem, it suffices to compare the allocation rules under agency and under wholesale. Observe that under wholesale, agents with types between p_S and p_B with $p_B > p_S$ trade with probability 0 and all other agents trade with probability 1. In contrast, under agency all buyers with values $v \geq \Phi^{-1}(0)$ and all sellers with costs $c \leq 1 - b$ trade with positive probability that is less than 1 (unless $v = 1$ or $c = 0$). Thus, the allocation rules differ. Since wholesale is optimal, it follows that agency is strictly suboptimal. □

B.3 Optimality of Agency in One-to-One Matching Setup

We now show that if buyers and sellers are matched bilaterally and the intermediary is allowed to design an arbitrary, incentive compatible, individually rational mechanism to extract rents from buyers and sellers, then he cannot do better than charge a transaction fee $\omega(p)$ as a function of the transaction price.

Assume in the following that the buyer's valuation is drawn from $[\overline{v}, 1]$ and the seller's costs is drawn from $[0, \underline{c}]$ for some $\overline{v}$, $\underline{c}$ satisfying $\Phi(\overline{v}) \leq \Gamma(0)$ and $\Phi(1) \leq \Gamma(\underline{c})$ (note that this condition is satisfied for Pareto distributions with $\sigma \geq 1$).

Proposition B2. *There is no incentive compatible, individually rational mechanism that generates higher profits for the intermediary than the following: let the seller set the price p and charge a fee $\omega(p)$ in case of a transaction with*

$$\omega(p) = p - \mathbb{E}_v[\Gamma^{-1}(\Phi(v))|v \geq p],$$

for $p \leq 1$ *and some* $\omega(p) \geq 1 - \mathbb{E}_v[\Gamma^{-1}(\Phi(v))|v \geq 1]$ *for* $p > 1$.

Proof. The Proposition is a simplified version of Proposition 2 in Loertscher and Niedermayer (2017a). To make the paper self-contained, we provide a simplified version of the proof of Proposition 2 in Loertscher and Niedermayer (2017a) here. First, observe that by Myerson and Satterthwaite (1983), Theorem 3, we know that the profit maximizing incentive compatible, individually rational mechanism satisfies the following conditions: (i) the good is traded if and only if $\Phi(v) \geq \Gamma(c)$, (ii) the least efficient agents (the seller with cost $\underline{c}$ and the buyer with valuation $\underline{v}$) get zero utility.

We first show that the fee $\omega(p)$ provided in the Proposition induces the optimal allocation rule. Observe that the seller's maximization problem is

$$\max_p (1 - F(p))(p - \omega(p) - c).$$

Substituting yields $\max_p \int_p^1 [\Gamma^{-1}(\Phi(v)) - c] dF(v)$. The first-order condition $0 = -f(p)[\Gamma^{-1}(\Phi(p)) - c]$ has the solution $p = \Phi^{-1}(\Gamma(c))$ and is sufficient given the monotonicity of Φ and Γ. The good is traded if and only if $v \geq p = \Phi^{-1}(\Gamma(c))$, which implies that the optimal allocation rule is induced.

Next, observe that a buyer with $\underline{v}$ trades probability zero since the seller with the lowest cost set the price $\underline{p} = \Phi^{-1}(\Gamma(0))$ and $\underline{p} \geq \underline{v}$ because of $\Phi(\underline{v}) \leq \Gamma(0)$. Further, a seller with cost $\underline{c}$ trades with probability zero since the fee does not allow him to get a higher net price than $\underline{c}$ even if he sets a price of 1. To see this, observe that a seller with cost $\Gamma^{-1}(\Phi(1))$ sets the price 1, pays the fee $\omega(1) = 1 - \Gamma^{-1}(\Phi(1))$, and keeps the net price $\Gamma^{-1}(\Phi(1))$, i.e. he has zero profits. Therefore any seller with $c \geq \Gamma^{-1}(\Phi(1))$ makes zero profits. This inequality holds for $\underline{c}$ because of the assumption $\Gamma(\underline{c}) \geq \Phi(1)$. □

The above fee simplifies to $\omega(p) = bp$ with $b = 1/(\sigma + 1)$ for Pareto distributions $F(v) = 1 - (1 - v)^\sigma$, $G(c) = c^\sigma$.

C Model with Binary Types

In this appendix, we provide a model with discrete – indeed, binary – types, the main purpose being that this specification more readily admits a dynamic extension than a model with a continuum of types. It also allows us to show that our main findings are robust with respect to type distributions and dynamics.

C.1 Setup

We now assume that the type sets are $\mathcal{V} = \{v_1, v_2\}$ for buyers and $\mathcal{C} = \{c_1, c_2\}$ for sellers, and we impose symmetry in the sense that $G(c_1) = h$ and $F(v_1) = 1 - h$, so that $1 - F(v_2) = h$ and $1 - G(c_2) = 1 - h$. We also assume $v_2 > v_1 \geq c_2 > c_1$, which implies that it is efficient for all sellers to produce and for all buyers to buy one unit.

While a binary type space is more tractable for some questions, the specification does come at a cost, since one can get results which are an artefact of the discrete type space: Demand functions are not strictly downward sloping and continuous anymore, supply functions are not strictly upward sloping and continuous. This can lead to multiple prices or multiple quantities being in equilibrium in several specifications. To avoid such issues we assume that the fraction of efficient buyers v_2 and the fraction of efficient sellers c_1 is both h.

Another way of getting rid of artefacts of a discrete type space would be to add small perturbations of the type space, so that buyers' valuation would be either in an ϵ-environment of v_1 or an ϵ-environment of v_2 and the same for the seller. However, this would lead to a considerably more tedious notation.

C.2 Equilibrium analysis

We now derive the equilibrium outcome for a given choice of mechanism by the market maker.

Wholesale If the intermediary wants to induce the Walrasian traded quantity (full trade) using wholesale, he optimally sets $p_B = v_1$ and $p_S = c_2$ and nets a profit of $\Pi^* = v_1 - c_2$, with w standing for Walrasian sets. If there is no random matching market, his profit when trading only with the most efficient set of traders (that is, v_2 and c_1) is the (restricted quantity) monopoly profit $\Pi_m = h(v_2 - c_1)$.

If $\Pi^* \geq \Pi_m$, a profit maximizing intermediary implements first-best. To make sure that there is a deadweight loss of monopoly, we assume $\Pi^* < \Pi_m$, which is equivalent to

$$h > \frac{v_1 - c_2}{v_2 - c_1} =: \mu, \tag{27}$$

where μ is the ratio of markups under full trade and under exclusive trade (i.e. with efficient buyer and seller types only). From here onwards, assume that (27) holds. Note that for the continuous type model studied in the main text, there is always a deadweight loss of monopoly. Consequently, assumption (27) should be seen as avoiding an artefact of a discrete type space.

Wholesale with random matching market Let λ be the matching probability in the random matching market and assume that trade takes place at the expected price $(v_1 + c_2)/2$. This price can be due to any of the bargaining protocols mentioned in the main text, i.e. random proposal take-it-or-leave-it-offers, Nash bargaining, double auctions, or fixed-price bargaining.[42] For a buyer of type v_2 and a seller of type c_1, the expected payoffs of participating in the random matching market, denoted, respectively, as $V_B(v_2)$ and $V_S(c_1)$, are

$$V_B(v_2) = \lambda\left(v_2 - \frac{v_1 + c_2}{2}\right) \quad \text{and} \quad V_S(c_1) = \lambda\left(\frac{v_1 + c_2}{2} - c_1.\right).$$

[42]It does not matter for our results, which price in the interval $[c_2, v_1]$ is chosen, but it is convenient to have a price exactly in the middle of the interval. To get this, for random proposal take-it-or-leave-it-offers, the offer probability has to be $\alpha = 1/2$. For Nash bargaining, the bargaining weight has to be $\alpha = 1/2$. For double auctions and fixed-price bargaining, any price in the interval $[c_2, v_1]$ could be the transaction price, including $(v_1 + c_2)/2$.

The prices (p_B, p_S) the intermediary sets when trading with the efficient types only satisfy

$$v_2 - p_B = V_B(v_2) \quad \text{and} \quad p_S - c_1 = V_S(c_1),$$

or equivalently

$$p_B = (1-\lambda)v_2 + \lambda\frac{v_1 + c_2}{2} \quad \text{and} \quad p_S = (1-\lambda)c_1 + \lambda\frac{v_1 + c_2}{2}.$$

The intermediary's profit with an active random matching market (which occurs if he trades only with the efficient types) is therefore

$$\Pi_W = h(p_B - p_S) = h(1-\lambda)(v_2 - c_1) = (1-\lambda)\Pi_m.$$

Observe that Π_W goes to 0 as λ goes to 1. Notice also that Π^* approaches 0 as μ approaches 0. Therefore, there will be parameter constellations such that entry-deterring agency will be profitable if it leads to a strictly positive profit.

Another relevant comparison is between Π_W and the Walrasian profit Π^*. One can show that

$$h(1-\lambda)(v_2 - c_1) = \Pi_W > \Pi^* = v_1 - c_2$$

is equivalent to

$$\lambda < 1 - \frac{\mu}{h} = \frac{h-\mu}{h} =: \lambda^*.$$

Note that for $\mu \to 0$ (which is equivalent to $v_1 \to c_2$), this condition is always satisfied. $\mu \to 0$ is reasonable to consider, given that it implies zero profits when implementing the Walrasian allocation, which always holds for a continuous type space.

The intermediated market and the bilateral exchange with wholesale are illustrated in Fig. 4.

Agency Assume now that instead of the wholesale model, the intermediary employs a large number of brokers (with mass 1 or larger) and randomly matches buyers and sellers one-to-one across the brokers if the number of buyers and sellers joining the intermediated

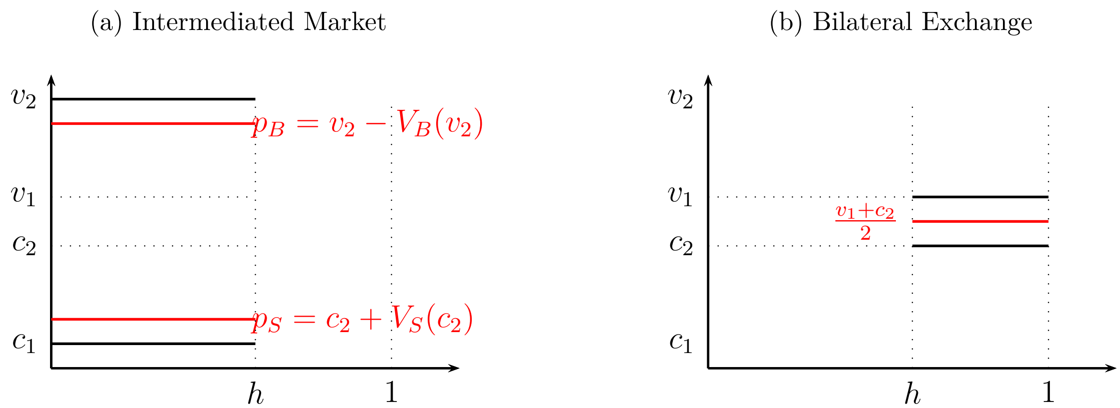

Figure 4: The intermediated and the bilateral market with wholesale.

market is the same. (Otherwise, agents on the long side are rationed randomly). The agency model used by the brokers has the fee $\underline{\omega}$ for the low price $\underline{p}$ and the fee $\overline{\omega}$ upon the high price $\overline{p}$ (and appropriately chosen ω for any other price, which will be off equilibrium). We now derive the optimal structure of this agency model.

The intermediary wants the efficient seller to set $\underline{p}$, which will be accepted by both buyers, and the inefficient seller to set $\overline{p}$, which will only be accepted by the high type buyer. Accordingly, the incentive constraints for the sellers are

$$h(\overline{p} - \overline{\omega} - c_2) \geq \underline{p} - \underline{\omega} - c_2 \tag{28}$$

$$h(\overline{p} - \overline{\omega} - c_1) \leq \underline{p} - \underline{\omega} - c_1. \tag{29}$$

Inequalities (28) and (29) are equivalent to

$$h(\overline{p} - c_2) - \underline{p} + c_2 \geq h\overline{\omega} - \underline{\omega} \tag{30}$$

$$h(\overline{p} - c_1) - \underline{p} + c_1 \leq h\overline{\omega} - \underline{\omega}. \tag{31}$$

As $h(\overline{p} - c_2) - \underline{p} + c_2 \geq h(\overline{p} - c_1) - \underline{p} + c_1$ is equivalent to $c_2 \geq c_1$, which is assumed (with strict inequality), we know that there is a number $h\overline{\omega} - \underline{\omega}$ such that both incentive constraints are satisfied.

The individual rationality constraints for the sellers are

$$\overline{p} - \overline{w} - c_2 \geq 0 \tag{32}$$

$$\underline{p} - \underline{w} - c_1 \geq 0. \tag{33}$$

Making the individual rationality constraint for the inefficient seller type binding, we get $\overline{w} = \overline{p} - c_2$. Using this and making the incentive constraint for the efficient seller type bind, we get $\underline{\omega} = \underline{p} - c_1 - h(c_2 - c_1)$. It is easy (and routine) to verify that the individual rationality constraint for the efficient seller type and the incentive constraint for the inefficient seller type will be satisfied with slack. The intermediary's profit under entry-deterring agency is therefore

$$\Pi_A \quad = \quad h\underline{\omega} + h(1-h)\overline{\omega} = h(\underline{p} + \overline{p} - c_1 - c_2) - h^2(\overline{p} - c_1),$$

since with probability h, the seller is efficient and trades for sure (fee $\underline{\omega}$), and with probability $1 - h$, the seller is inefficient and trades with probability h (fee $\overline{\omega}$). Of course, $\overline{p} = v_2$ and $\underline{p} = v_1$, so that

$$\underline{\omega} = v_1 - c_1 + h(c_2 - c_1) \quad \text{and} \quad \overline{\omega} = v_2 - c_2. \tag{34}$$

Plugging this into Π_A yields

$$\Pi_A = h(1-h)(v_2 - c_1) + h(v_1 - c_2) = h(v_2 - c_1)(1 - h + \mu) = (1 - h + \mu)\Pi_m,$$

which is less than Π_m under condition (27).

Equilibrium mechanisms If we assume that $\mu = 0$, we get

$$\Pi_A \quad = \quad h(1-h)(v_2 - c_1). \tag{35}$$

Therefore, under the assumption that $\mu = 0$, $\Pi_A > \Pi_W$ is equivalent to $\lambda > h$. Under this assumption, the fees simplify to $\overline{w} = v_2 - c_2$ and $\underline{w} = (1-h)(v_1 - c_1)$. In general, $\Pi_A > \Pi_W$ can be rearranged to

$$\lambda > \lambda_\Pi := h - \mu$$

Note that $\lambda_\Pi > 0$ by condition (27), which states that $h - \mu > 0$. Summarizing, we have established the following result.

Proposition C3. *Entry-deterring agency is profitable for the intermediary if and only if the competing exchange were otherwise sufficiently efficient, that is if and only if* $\lambda > h-\mu$*.*

A further comparison worthwhile making concerns agency and wholesale with the Walrasian quantity. We know that for λ sufficiently large, the intermediary prefers Walrasian wholesale (v_1, c_2) to posting a large spread (v_2, c_1). One may wonder whether Walrasian wholesale may be preferred to agency. This is, however, not the case for the following reason. Fee setting generates higher profits than Walrasian wholesale if

$$h(v_2 - c_1)(1 - h + \mu) = \Pi_W > \Pi^* = v_1 - c_2,$$

which can be rearranged to $h > \mu$, which is satisfied by assumption. This means that agency is always preferred to Walrasian wholesale.

Note further that whenever wholesale with a large spread is preferred to agency ($\lambda < \lambda_\Pi$), it is also preferred to Walrasian wholesale, since $\lambda^* > \lambda_\Pi$. (This actually also follows from the fact that agency is always preferred to Walrasian wholesale.)

The agency model is illustrated in Fig. 5.

C.3 Dynamic random matching

Dynamic effects arise naturally in the context of intermediated trade because agents who are not matched or do not trade today have the option of trading tomorrow.[43] We now extend the static model to account for these effects.

[43]See, for example, Spulber (1996), Rust and Hall (2003) and Duffie et al. (2005).

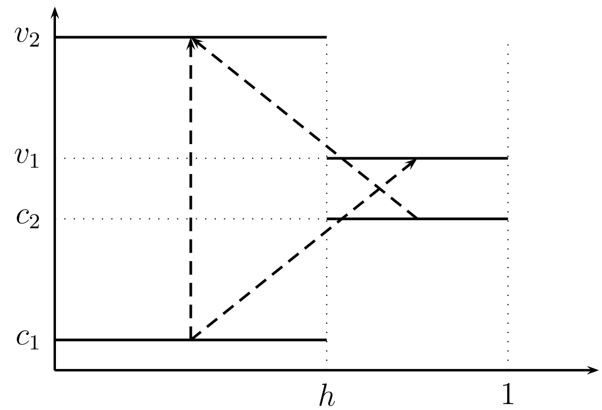

Figure 5: Trade in the intermediated market with agency.

Setup We consider the following dynamic random matching extension of our baseline model. There is an infinite horizon model with periods $t = 0, 1, 2, ...$ Initially both the intermediated and the direct market are empty. In each period mass 1 of buyers and mass 1 of sellers enter the market. If a trader trades, he leaves the market. If he does not trade, he stays in the market with the survival probability δ. With probability $1 - \delta$ a buyer or seller who does not trade drops out for exogenous reasons and gets utility 0. These assumptions are similar to those in Satterthwaite and Shneyerov (2008)) and Shneyerov and Wong (2010). The assumptions on the distribution of types are the same as in the static model, that is every buyer is of type v_2 with probability h and of type v_1 with probability $1 - h$ while every seller is of type c_1 with probability h and of type c_2 with probability $1 - h$, where $v_2 > v_1 \geq c_2 > c_1$.

Before we solve the dynamic random matching model, we briefly discuss the rationale for these assumptions. The literature on dynamic random matching typically uses one of the following three modeling assumptions to model impatience of participants in a market: (i) fixed search costs per period of participation, (ii) discounting, (iii) random drop-out of traders in every period. The third modeling assumption has several advantages in our setup. First, it is a parsimonious way of having both something similar to discounting (as in (ii)) and making sure that no traders stay in the market forever (as in (i)). The assumption also has the effect that the intermediary is also impatient: if a trader does not trade in a certain period, he may drop out in the next, which means a potentially foregone opportunity to extract rents for the intermediary. Another advantage is that a model with drop-out nests the static model we discussed before: if the probability of dropping out after a period without trade is 1, we are essentially in the static setup (repeated indefinitely). A further advantage is that the intermediary can simply focus on the stationary equilibrium of the market.[44] With discounting, one would have to consider both profits in the stationary (limiting) equilibrium and profits on the transition path to this equilibrium. An intermediary whose impatience stems only from drop-outs can ignore the transition path.

One can also think of a larger model, in which all three sources of impatience exists, but search costs converge to zero and the discount factor converges to 1.

Wholesale In the intermediated market, buyers with v_2 and sellers with c_1 enter and trade immediately.[45] In the non-intermediated market, buyers with v_1 and sellers with c_2 enter. Their per period probability of trade is λ. If they do not trade, they stay in the market with probability δ and may trade in any of the subsequent periods, provided they do not drop out. Hence, the ultimate probability of trade is

$$\hat{\lambda} = \sum_{t=0}^{\infty} [(1-\lambda)\delta]^t \lambda = \frac{\lambda}{1-\delta(1-\lambda)}$$

It is easy to see that for $\delta \to 0$, $\hat{\lambda} \to \lambda$, and for $\delta \to 1$, $\hat{\lambda} \to 1$.

Since only λ is replaced by $\hat{\lambda}$ and everything else remains the same, the intermediary's per period profits are

$$\Pi_W = h(1-\hat{\lambda})(v_2 - c_1).$$

Note that the usual subtleties due to per period vs per cohort profits do not occur here,

[44] One has to be careful, when analyzing the stationary equilibrium though, since one has to look at per entering cohort profits rather than per period profits. We will discuss this later on.

[45] For $\lambda = 0$, It can be shown that this is the optimal mechanism, irrespective of δ, see e.g. Niedermayer and Shneyerov (2014), even if one allows for non-stationary, non-anonymous mechanisms.

since traders joining the intermediary's platform trade immediately, so that per period and per cohort profits are the same.

Agency Next, consider agency. We will consider the same type of equilibrium as in the static setup: all traders join the intermediary's platform, sellers with low costs c_1 set a low price $\underline{p}$, sellers with high costs c_2 set a high price $\overline{p}$. In equilibrium, high valuation buyers accept both the high and the low price, low valuation buyer only accept the low price.

The dynamic random matching model differs from the static setup in two ways. First, the distribution of types in the market differ from the entrant population as less efficient traders spend more time in the market. Second, traders have an option value of delaying trade and trading with a potentially more attractive future trading partner. Note that by assuming that all impatience stems from the drop-out probability, we can focus on per cohort profits in the steady state equilibrium rather than on the complicated transition path to the steady state.

Because of symmetry of the buyer and seller probabilities of being efficient, it is sufficient to analyze efficient vs. inefficient traders. The analysis then applies to both the buyer and the seller side. Note that efficient traders (v_2 and c_1) trade immediately, so the mass of efficient traders in the market is equal to the mass of efficient entering traders, h. Denote the mass of inefficient traders in the market as m and the fraction of efficient traders in the market as $\tilde{h}$. The following has to hold:

$$\frac{m}{m+h} = 1 - \tilde{h},$$

which is equivalent to

$$m = h\frac{1-\tilde{h}}{\tilde{h}}.$$

In a stationary equilibrium, the inflow and the outflow of a certain type of agents has to be equal. For the efficient types, this clearly holds, since they trade with probability 1. For the inefficient agents, the inflow has mass $1-h$. The outflow is given by the mass m of agents in the market and the probability of not staying in the market. The probability of staying in the market is given by the probability of not trading $1 - \tilde{h}$ and the probability of not dropping out δ. The inflow-outflow equilibrium equation is hence

$$1 - h = m(1 - (1 - \tilde{h})\delta)$$

Plugging in the expression for m yields a quadratic equation in $\tilde{h}$. Rearranging and solving for $\tilde{h}$ gives the solution[46]

$$\tilde{h} = \frac{2\delta h - 1 + \sqrt{(2\delta h - 1)^2 + 4\delta(1-\delta)h^2}}{2\delta h}$$

It is easy to check th at for $\delta \to 0$, $\tilde{h} \to h$ and for $\delta \to 1$, $\tilde{h} \to 2 - 1/h$. The latter implies

(a) λ_Π as a function of h for $\delta = 0$ (solid), $\delta = 0.5$ (dashed), $\delta = 0.9$ (dash-dotted), and $\delta = 0.99$ (dotted).

(b) $\overline{p}$ as a function of h for $\delta = 0$ (solid), $\delta = 0.5$ (dashed), $\delta = 0.9$ (dash-dotted), and $\delta = 0.99$ (dotted).

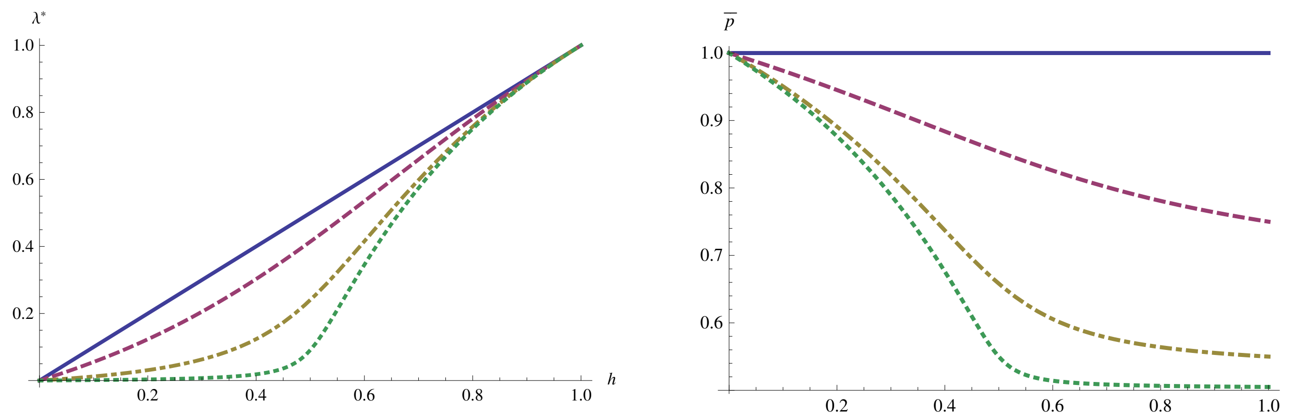

Figure 6: λ_Π and $\overline{p}$.

that the system is stable for $\delta \to 1$ if $h \in (1/2, 1)$. It is also easy to check that for $h \to 0$, $\tilde{h} \to 0$ and for $h \to 1$, $\tilde{h} \to 1$. From here onwards, we therefore assume

$$\max\{1/2, \mu\} < h < 1$$

[46] The two candidates to the quadratic equation are $\tilde{h}_{1,2} = \frac{2\delta h - 1 \pm \sqrt{(2\delta h-1)^2 + 4\delta(1-\delta)h^2}}{2\delta h}$. Since the square root is greater than $2\delta h - 1$, the solution with a minus sign would yield a negative value, contradicting that it is a probability. Therefore, the solution is given by the expression with the plus sign.

Though $\tilde{h}$ is the probability that a trader meets an efficient potential trading partner in a given period, what matters for the decisions of traders is the ultimate probability of meeting an efficient type. This probability is given as

$$\hat{h} = \sum_{t=0}^{\infty} \left[\delta(1-\tilde{h})\right]^t \tilde{h} = \frac{\tilde{h}}{1-\delta(1-\tilde{h})}$$

Plugging in $\tilde{h}$ yields

$$\hat{h} = \frac{2h}{1+\sqrt{1-4(1-h)h\delta}}$$

Observe that for $h \geq 1/2$, $4(1-h)h\delta \leq 1$. Therefore, the square root is always real.

Next, we can derive the incentive compatibility and individual rationality constraints of traders. Note that since the setup is stationary, we only need to check whether a high cost seller has the incentive to set a high price in every period rather than a low price in every period, since the trade-off is the same in every period. Similarly, we only need to check that a low cost seller has the incentive to set a low price in every period rather than a high price in every period. Further, one can simply replace h with $\hat{h}$ for most of the analysis. This yields the incentive compatibility constraints for the sellers

$$\hat{h}(\overline{p} - \overline{w} - c_2) \geq \underline{p} - \underline{w} - c_2 \tag{36}$$

$$\hat{h}(\overline{p} - \overline{w} - c_1) \leq \underline{p} - \underline{w} - c_1 \tag{37}$$

The individual rationality constraints can be written analogously. By the same logic as for the static setup, we get the fees

$$\overline{w} = \overline{p} - c_2, \tag{38}$$

$$\underline{w} = \underline{p} - c_1 - \hat{h}(c_2 - c_1). \tag{39}$$

For the inefficient buyer, the option value of future trade is 0 as he will never get an offer below $\underline{p}$ no matter how long he waits. Hence, by the same logic as before, incentive compatibility and individual rationality constraints imply that $\underline{p} = v_1$.

For the efficient buyer, the situation is somewhat more complicated than in the static setup: an efficient buyer getting a high price offer $\overline{p}$ has the option of delaying trade and potentially getting a low offer $\underline{p}$ in the future. Hence, the efficient buyer's incentive compatibility constraint is

$$v_2 - \overline{p} \geq \delta \hat{h}(v_2 - \underline{p}),$$

where the left-hand side is the utility from accepting a high offer immediately and the right-hand side is the value of waiting a period and then staying in the market until getting a low offer. Again, stationarity makes sure that this condition is sufficient since rejecting a high offer in the current period and accepting a high offer some time in the future cannot be optimal. Rearranging yields

$$\overline{p} = (1 - \delta \hat{h})v_2 + \delta \hat{h} v_1.$$

This allows us to write per cohort profits. Note that considering per cohort profits is the right measure, since the mass of entering traders per period is exogenously given. The mass of efficient sellers entering per period is h, each trading immediately and generating profits $\underline{w}$. The mass of inefficient sellers entering per period is $1 - h$, each generating profit $\overline{w}$ with the ultimate probability $\hat{h}$. Hence, profits are

$$\Pi_A = h\underline{w} + (1-h)\hat{h}\overline{w} = h(\underline{p} - c_1) + \hat{h}((1-h)\overline{p} - (c_2 - hc_1)). \tag{40}$$

Entry-deterrence We now compare profits for wholesale and agency. Note that Π_W decreases in $\hat{\lambda}$ and $\hat{\lambda}$ increases with λ, whereas Π_A is independent of $\hat{\lambda}$ (and λ). Therefore, if $\Pi_W = \Pi_A$ for some λ_Π, then $\Pi_W < \Pi_A$ for all $\lambda > \lambda_\Pi$. One can show that λ_Π exists and is unique, since for $\lambda = 1$, $\Pi_W = 0 < \Pi_A$, for $\lambda = 0$ we know that the optimal mechanism is wholesale, and Π_W is continuous and strictly decreasing in λ.

We can get λ_Π by solving $\Pi_W = \Pi_A$ in closed form:

$$\lambda_\Pi = \frac{2(h-\mu)(1-\delta)}{1 - 2\delta(h-\mu) + \sqrt{1 - 4(1-h)h\delta}}.$$

or

$$\lambda_\Pi = \frac{2(h-\mu)(1-\delta)}{2\delta\mu + \sqrt{(2\delta h - 1)^2 + 4\delta(1-\delta)h^2} - (2\delta h - 1)}.$$

With some algebra, it can be shown that $\lambda_\Pi \in (0,1)$ if $h > \mu$. This also implies that $\Pi_A > 0$, since for $\lambda = 1$, $\Pi_W = 0$.

One can also show that agency is always preferred to Walrasian wholesale, just as for the static setup. Note that for Walrasian wholesale, profits are the same as in the static setup since traders trade immediately. That is, profits are $v_1 - c_2$. Again, it can be shown that $h > \mu$ implies $\Pi_A > v_1 - c_2$.

For the special case $v_1 = c_2$ (or, equivalently, $\mu = 0$), this further simplifies to

$$\lambda_\Pi = \frac{2h(1-\delta)}{1 - 2h\delta + \sqrt{1 - 4(1-h)h\delta}}.$$

We can plot λ_Π as a function of h for different values of δ as depicted in Figure 6. For $\delta = 0$ we are back in the static setup and $\lambda_\Pi = h$ as before. The figure illustrates that as δ increases, λ_Π becomes lower, that is as the market becomes more dynamic (or as frictions become smaller), it is more likely that agency is preferred by the intermediary. This can be shown to hold in general, that is, $\partial\lambda_\Pi/\partial\delta < 0$ holds under the assumption that $h > \mu$.

Additionally, one may wonder how prices and fees change as δ changes. To provide some numerical examples, set $v_2 = 1$, $c_1 = 0$ and $v_1 = c_2 = 1/2$. The high price $\overline{p}$ as a function of h is plotted in panel (b) of Figure 6 for the same values of δ as in panel (a). Note that $\underline{p} = v_1$ for any δ and h. $\overline{w}$ and $\underline{w}$ are plotted in panels (a) and (b) of Figure

C.4 Welfare

We now turn to the analysis of the effects of entry-deterring agency on social welfare and on consumer surplus, and we provide a brief assessment of the quantitative effects of entry-deterring agency in our model. We conclude the section with a discussion of policy implications of our simple model. Throughout this section, we confine attention to the basic (that is, static) model.

(a) $\overline{w}$ as a function of h for $\delta = 0$ (solid), $\delta = 0.5$ (dashed), $\delta = 0.9$ (dash-dotted), and $\delta = 0.99$ (dotted).

(b) $\underline{w}$ as a function of h for $\delta = 0$ (solid), $\delta = 0.5$ (dashed), $\delta = 0.9$ (dash-dotted), and $\delta = 0.99$ (dotted).

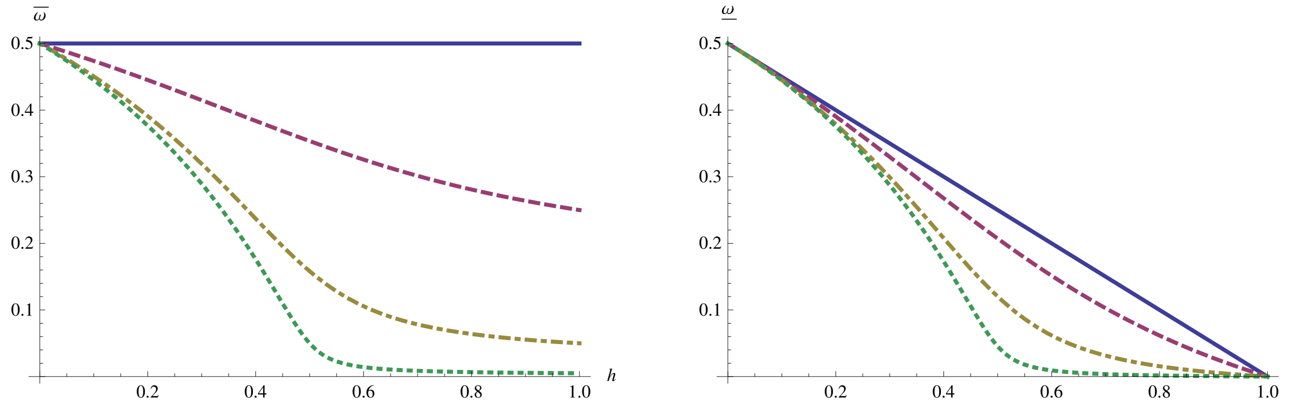

Figure 7: $\overline{\omega}$ and $\underline{\omega}$.

C.4.1 Effects of Agency on Welfare and Consumer Surplus

When the intermediary posts the prices $p_B = (1-\lambda)v_2 + \lambda\frac{v_1+c_2}{2}$ and $p_S = (1-\lambda)c_1 + \lambda\frac{v_1+c_2}{2}$, welfare S_W under wholesale is

$$S_W = h(v_2 - c_1) + \lambda(1-h)(v_1 - c_2) = (v_2 - c_1)[h + \lambda(1-h)\mu]. \tag{41}$$

Under wholesale, consumer surplus, defined as the sum of surplus grasped by buyers and sellers and denoted CS_W, is

$$CS_W = \lambda(v_2 - c_1)h + \lambda(1-h)(v_1 - c_2) = \lambda(v_2 - c_1)(h + (1-h)\mu). \tag{42}$$

Welfare S_A under agency, on the other hand, is

$$\begin{aligned} S_A &= h[hv_2 + (1-h)v_1 - c_1] + (1-h)h[v_2 - c_2] \\ &= h(v_2 - c_1) + h(1-h)(v_1 - c_2) = (v_2 - c_1)[h + h(1-h)\mu]. \end{aligned} \tag{43}$$

The first equality in (43) follows because the fraction h of the sellers have low costs and trade with all buyers, thereby generating a surplus of $hv_2 + (1-h)v_1 - c_1$ while the remaining sellers have high costs and only trade if matched to a buyer with a high valuation. The second equality follows after rearranging terms. Consumer surplus under agency, denoted CS_A and defined like under wholesale as the sum of buyer and seller surplus, is

$$\begin{aligned} CS_A &= h[hv_2 + (1-h)v_1 - c_1 - \underline{\omega}] + (1-h)h[v_2 - c_1 - \overline{\omega}] \\ &= h^2(v_2 - v_1 + c_2 - c_1) = h^2(v_2 - c_1)(1-\mu), \end{aligned} \tag{44}$$

where the second line follows after plugging in the value for $\underline{\omega}$ and $\overline{\omega}$ given in (34) and simplifying.

Observe that for $\mu = 0$, we have $S_W = S_A$. For $\mu > 0$, S_W increases in λ while S_A is independent of λ. At $\lambda = h$, we have $S_W = S_A$ and $S_W > S_A$ for any $\lambda > h$, assuming $v_1 > c_2$.[47] Since the condition for agency to be profitable is $\lambda > h - \mu$, this means that (i) agency is profitable whenever it reduces welfare but also, and somewhat surprisingly, (ii) that profitable and welfare enhancing agency is possible, the latter occurring when $h - \mu < \lambda < h$. The intuition seems to be that if h is large relative to λ, the intermediary under agency is a better match maker than is the random matching market.[48]

Notice also that CS_A is independent of λ while CS_W increases in λ. Since at $\lambda = h$, $CS_W > CS_A$, it follows that agency decreases consumer surplus whenever it decreases total welfare. Because CS_W is continuous in λ, it follows also that the parameter space for which agency is detrimental to consumer surplus is larger than the parameter space for which it is detrimental to welfare. This reflects a theme from Loertscher and Niedermayer (2017a), where agency emerges as a tool to extract rents from buyers and sellers.

[47] At $v_1 = c_2$ welfare is the same with agency and wholesale because both mechanisms always induce trade by the efficient agents.

[48] Conditional on being an inefficient type, an agent's probability of trade in the matching market under wholesale is λ while his probability of trading at the market maker with agency is h. Since the efficient types trade with probability 1 regardless of the mechanism used by the intermediary, the result follows.

Somewhat tedious algebra reveals that $CS_A = CS_W$ at $\lambda = \lambda_{CS}$ with

$$\lambda_{CS} := h^2 \frac{1-\mu}{h + (1-h)\mu}. \tag{45}$$

Since $\lambda_\Pi = h - \mu < \lambda_{CS}$, entry deterrence is profitable and increases consumer surplus for $\lambda \in \left(h - \mu, h^2 \frac{1-\mu}{h+(1-h)\mu}\right)$. Summarizing, we have established the following two propositions, where the comparisons are made with welfare and, respectively, consumer surplus under (selective) wholesale.

Proposition C4. *1. ("Bad wholesale equilibrium outcome") If $\lambda < h - \mu$, agency achieves higher welfare than wholesale, but a profit maximizing intermediary will choose wholesale.*

2. ("Good agency equilibrium outcome") If $\lambda \in [h - \mu, h]$, agency achieves higher welfare and a profit maximizing intermediary will choose agency.

3. ("Entry-deterring agency equilibrium outcome") If $\lambda > h$, wholesale achieves higher welfare, but a profit maximizing intermediary will choose agency.

The possibility of positive equilibrium effects on welfare and consumer surplus of entry deterrence are reminiscent of Cabral and Riordan (1994, 1997), who show that in the presence of a learning curve these effects can go either way. Note, however, that in continuous type space model, the profit generated by choosing the Walrasian quantity is zero, which corresponds to a μ close to 0 in our setup. For μ close to zero, the middle case ("good agency equilibrium outcome") vanishes.

Proposition C5. *For $\lambda \in \left(h - \mu, h^2 \frac{1-\mu}{h+(1-h)\mu}\right)$, entry-deterring agency increases consumer surplus while for $\lambda > h^2 \frac{1-\mu}{h+(1-h)\mu}$ it decreases consumer surplus.*

Note again that for $\mu \to 0$, both λ_Π and λ_{CS} go to h, which means that consumer surplus always decreases when the intermediary prefers agency.

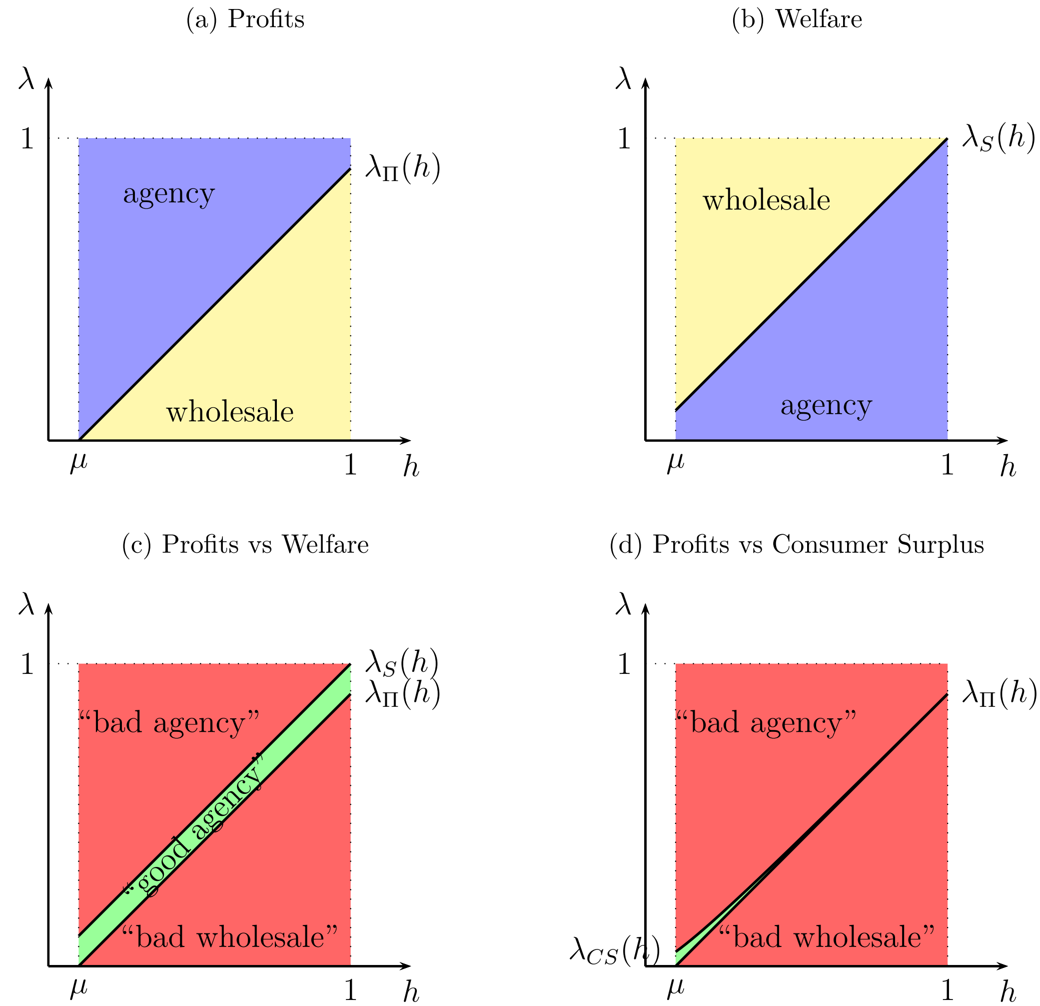

Figure 8: The optimal mechanism in term of profits and welfare (Subfigures 8a and 8b); comparison of profits vs welfare and profits vs consumer surplus (Subfigures 8c and 8d) with $\lambda_\Pi(h) = h - \mu$, $\lambda_S(h) = h$, and $\lambda_{CS}(h) = h - \frac{h\mu}{h(1-\mu)+\mu}$

D Empirics

An empirical analysis is outside of the scope of this article. However, there are a few issues worth discussing.

One is the extent of price dispersion in search markets. This is difficult to measure empirically because differences in sales prices can be attributed to either observed heterogeneity (for real-estate, the number of bedrooms of a property, whether it has air conditioning, etc.), to unobserved heterogeneity (characteristics of a property that are observed by market participants, but not by the econometrician, e.g., whether the view from the apartment is beautiful), or to true price dispersion.

In Loertscher and Niedermayer (2017b), we structurally estimate an agency model, taking payoffs from a bilateral exchange as exogenously given. The structural estimates allow us to disentangle the three effects leading to differences in sales prices. We show how the three effects can be disentangled in Figure 9. It should be noted that not only the analysis of observed and unobserved heterogeneity poses challenges, but also its graphical representation: while sales prices are measured in hundreds of thousands of dollars, the quality-adjusted prices one obtains after correcting for observed heterogeneity and the counterfactual simulations of how much price variation one would see if there were no unobserved heterogeneity are quality-adjusted prices normalized to a mean of 1. We deal with this challenge by computing "denormalized quality-adjusted prices": we multiply the quality-adjusted prices with \$230,000, the average transaction price in the data set in Loertscher and Niedermayer (2017b).

Based on the structural estimate of true price dispersion (solid line in Figure 9), we can look at how the 6% fees charged by real-estate brokers compare to price dispersion. Figure 10 shows the distribution of "denormalized quality-adjusted" gross prices (solid) and "denormalized quality-adjusted" prices net of the 6% fee charged by real-estate brokers. It shows that the two distributions have considerable overlap.

An issue Loertscher and Niedermayer (2017b) do not deal with is the endogeneity of the bilateral exchange market that is the focus of this article. This is the right approach

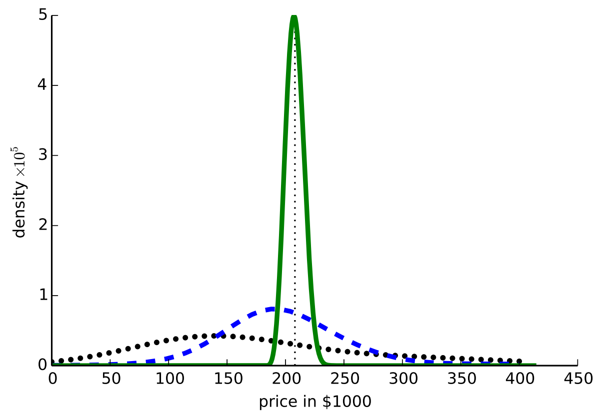

Figure 9: Empirically estimated density of transaction prices without correcting for heterogeneity (dotted, black), correcting for observed heterogeneity (dashed, blue) and correcting for both observed and unobserved heterogeneity (solid, green).

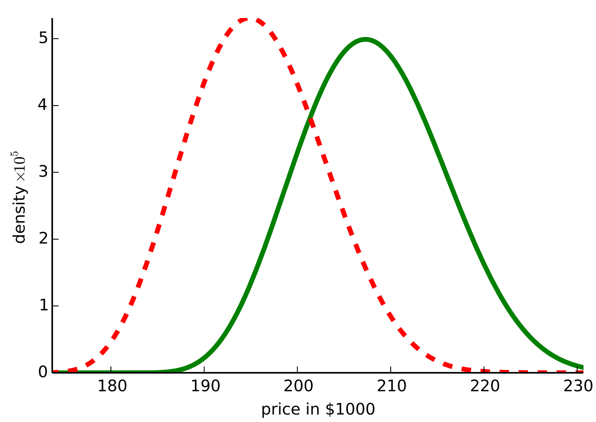

Figure 10: Empirical density of gross (solid, green) and net (dashed, red) transaction prices adjusted by both observed and unobserved heterogeneity. Based on Loertscher and Niedermayer (2017b).

for that paper, since they analyze data from Boston in the early 1990s when the fraction of bilateral transactions was quite small.

An interesting question for future research would be to analyze data from regions and periods of time in which a bilateral exchange came into existence and started growing. One such data set is that analyzed by Hendel et al. (2009), who consider both real-estate broker sales and For-Sale-By-Owner intermediary sales in Madison, Wisconsin, from 1998 to 2004. In this period of time, the For-Sale-By-Owner intermediary grew considerably. They find that properties sold through the For-Sale-By-Owner intermediary took considerably longer to sell than properties sold through real estate brokers, which is consistent with the assumption $\lambda < 1$; that is, that matchings through brokers are more efficient than bilateral matchings.[49] In a dynamic setting, λ determines that probability of sale in a single period and hence the time on market.

From the perspective of our model, the emergence of the For-Sale-By-Owner intermediary in Madison could be interpreted as either entry deterrence by brokers still being successful and the bilateral market still operating at an inefficiently small scale, but the efficiency at this small scale improving due to technological progress such as the Internet (i.e. λ_0 in Section 6.6 increasing). Alternatively, it could be seen as one of the few occasions in which a bilateral market managed to break free from entry deterrence.[50]

Another interesting question relates to the comparison to the used car market. Used car dealers typically choose the wholesale model and the fraction of bilateral transactions is known to be considerably higher for used cars than for real-estate transactions. An empirical study by Gavazza et al. (2014) that analyzes the (bilateral) used car market from a search theoretic perspective finds that there are considerable search frictions in this market, which from the perspective of our model means a low λ.[51] In a market

[49] It should be noted that Hendel et al. (2009) also find that the probability of sale is slightly higher for the For-Sale-By-Owner intermediary than for real-estate brokers; however, the difference is not statistically significant.

[50] We do not have an answer to why a bilateral exchange can break free. Possibly, enough sophisticated buyers and sellers are present in a region, who are efficient at searching for a trading partner even without an intermediary.

[51] There are different ways to model frictions in the used car market. In both Gavazza et al.'s (2014) and our article these frictions are modeled as search frictions; that is, a lower probability of meeting a

with a low λ, intermediaries do not have an incentive to collude to foreclose the bilateral market, since it is a lesser competitive threat than in markets with a larger λ. Hence, intermediaries are more likely to choose wholesale, which is indeed what we observe for used car dealers.

trading partner in a given period. An alternative is to model frictions as adverse selection: a buyer may find a seller, but there is a high probability that the seller has a used car of inferior quality. While we have no formal model, we conjecture that our results should qualitatively go through also in the latter specification, since the basic driving force is that the intermediary has an incentive to predate if the bilateral market is too efficient.