Coordinated Effects*

Simon Loertscher† Leslie M. Marx‡

June 23, 2020

Abstract

Coordinated effects are harms that arise because a merger increases the risk of coordination. A firm is called a "maverick" if its acquisition puts an otherwise competitive market at risk of coordination. The notions of coordinated effects and mavericks fare prominently in antitrust but have proved elusive to formalize. Modeling coordination as selection schemes, we provide clear-cut definitions of and tests for both that only require pre-merger data. Risk for coordinated effects varies with the price formation process. A maverick-based approach is useful for Cournot, but requires refinements for procurement-based markets. HHI increases do not reliably predict coordinated effects.

Keywords: merger review, buyer power, mavericks, unilateral effects, neutral for rivals spread

JEL Classification: D44, D82, L41

*We thank Eric Emch, Joe Farrell, Nicholas Hill, Marc Ivaldi, Louis Kaplow, Scott Kominers, Patrick Rey, Tom Ross, Yossi Spiegel, Mike Whinston, Ralph Winter, Christoph Wolf and seminar participants at Columbia University, Cornell University, Swiss Competition Commission, University of Melbourne, U.S. Department of Justice, U.S. Federal Trade Commission, Crowell & Moring, 4th Asia Pacific IO Conference, Hal White Antitrust Conference, MaCCI Conference on Mergers and Antitrust, 11th Annual Searle Center Conference on Antitrust Economics and Competition Policy, 2018 Econometric Society European Meeting, 2018 Jiangxi University of Finance and Economics Conference on Competition Policy, and 9th Annual Federal Trade Commission Microeconomics Conference for helpful comments. We thank Bing Liu for excellent research assistance. We gratefully acknowledge support from the European Research Council under the Grant Agreement no. 340903, the Samuel and June Hordern Endowment, a University of Melbourne Faculty of Business & Economics Eminent Research Scholar Grant, and the Australian Research Council under Discovery Project Grant DP200103574.

†Department of Economics, Level 4, FBE Building, 111 Barry Street, University of Melbourne, Victoria 3010, Australia. Email: simonl@unimelb.edu.au.

‡Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, USA: Email: marx@duke.edu.

1 Introduction

Competition authorities regularly review proposed mergers. They oppose those that they determine will have sufficiently detrimental effects, recognizing that one source of detrimental effects is that a merger can change "the nature of competition in such a way that firms that previously were not coordinating their behaviour, are now significantly more likely to coordinate and raise prices or otherwise harm effective competition."[1] Adverse competitive effects of mergers that arise in this way are referred to as "coordinated effects" and play a prominent role in antitrust thinking and practice. For example, Kolasky (2002, p. 1) observes: "Concern over what we now call coordinated effects has long been at the core of U.S. merger policy." Likewise, in evaluating a hospital merger, Judge Richard Posner wrote that "the ultimate issue is whether the challenged acquisition is likely to facilitate collusion" and, in particular, "whether the challenged acquisition is likely to hurt consumers, as by making it easier for the firms in a market to collude, expressly or tacitly, and thereby force price above or farther above the competitive level."[2]

Closely related to coordinated effects is the notion of a *maverick* firm, which plays a role of similar prominence in merger review and merger decisions.[3] Broadly and vaguely, a maverick is a firm whose acquisition will put a market at risk for coordination when the market is not at risk for coordination with the maverick firm present. Baker (2002, pp. 140–141, 197) argues for a "maverick-centered approach" to coordinated effects, stating that "the identification of a maverick that constrains more effective coordination is the key to explaining ... which particular changes in market structure from merger or exclusion are troublesome, and why" and that "In many settings, regulators reliably can identify an industry maverick that prevents or limits coordination." Kolasky (2002) argues that the elimination of a maverick may be necessary for coordinated effects. Notwithstanding their prominence and relevance, thus far, coordinated effects of mergers and maverick firms have proven elusive to define, let alone to analyze or test for.[4] In light of the far-reaching implications of decisions that are

[1]European Commission's *Guidelines on the Assessment of Horizontal Mergers* (*EC Guidelines*), para. 22(b). Similarly, the U.S. *Horizontal Merger Guidelines* (*U.S. Guidelines*) recognize that a merger "can enhance market power by increasing the risk of coordinated, accommodating, or interdependent behavior among rivals" (p. 2). Similar guidance is provided by the Australian Competition and Consumer Commission's *Merger Guidelines*.

[2]*Hospital Corp. Of America v. FTC*, 807 F.2d 1381, 1386 (7th Cir. 1986), paras. 1 and 7.

[3]The particular concerns raised by mergers involving mavericks are discussed in, for example, the U.S., EC, and Australian merger guidelines, and they arise in many merger cases. Examples include the proposed acquisition of maverick T-Mobile by AT&T, the acquisition of maverick Northwest Airlines by Delta Airlines, and the proposed acquisition of maverick baby food maker Beech-Nut by Heinz. On mavericks in EC merger decisions, see Bromfield and Olczak (2018).

[4]"One particular problem is that neither the theoretical nor empirical literature tells us much at all about whether the disappearance of a single firm through merger will increase the likelihood of coordination, other

based on coordinated effects and mavericks, this state of affairs is, mildly put, problematic.

In this paper, we provide a model of coordination that allows us to define what it means for a market to be at risk for coordination, to quantify the risk, and to define and test for maverick status. Both our measure of coordinated effects and our definition of a maverick are operational based only on pre-merger observables. Specifically we model coordination by suppliers as participation in a selection scheme, whereby each coordinating supplier is chosen with some probability to be the only supplier from the group of coordinators to participate in the market. Selection schemes are appealing because they do not require transfers or the communication of any private, production-relevant information. As noted by Fellner (1950, p. 58), colluding firms "frequently try to obtain maximum market shares with no direct compensation from other firms rather than maximum profits via direct compensation from the most efficient participants."[5]

For participation in a selection scheme to pay off for each candidate participant, each supplier has to be selected to be the active coordinator with sufficiently high probability. Thus, any selection scheme is defined by a set of *critical* shares or selection probabilities. Of course, each supplier's critical share is strictly less than one because coordination eliminates competition. However, for a market to be at risk for coordination by a given set of suppliers, the critical shares of all these suppliers need to sum up to less than one. Because the scheme can be inefficient (for example, because the suppliers have different marginal costs or because the reaction of non-coordinating outsiders erodes the benefits from coordination accruing to insiders), there is no a priori reason for this sum of critical shares to be less than one. Thus, for a given set of candidate coordinators, the sum of their critical shares provides a natural way to define whether a market is at risk for coordination by these firms—if the sum is less than one, then it is at risk for coordination. This is the definition for being *at risk for coordination* that we use in this paper. Relative to a given set of candidate coordinators,

than, perhaps, in the extreme case where a merger reduces the number of firms in a market from three to two" (Kolasky, 2002, p. 7). Antitrust officials have described a maverick as "a firm that declines to follow the industry consensus and thereby constrains effective coordination" (Kolasky, 2002, p. 7), while the *U.S. Guidelines* (p. 4) describe a maverick as "a firm that has often resisted otherwise prevailing industry norms to cooperate on price setting or other terms of competition." Ivaldi et al. (2007, pp. 224, 228) define a maverick as "a firm that has a drastically different cost structure, production capacity or product quality, or that is affected by different factors than the other market participants" and "is thus unwilling to participate to a collusive action." Kwoka (1989, p. 410) identifies a maverick as the relatively "more rivalrous" firm, and Ivaldi and Lagos (2017) take the view that a maverick is a small firm, while in de Roos and Smirnov (2019) a maverick is a fringe firm that disrupts coordination.

[5] Selection schemes were used by cartels in, for example, Choline Chloride, Copper Plumbing Tubes, Electrical and Mechanical Carbon and Graphite Products, Food Flavour Enhancers, Industrial and Medical Gases, Industrial Bags, Industrial Tubes, Methylglucamine, Monochloroacetic Acid, and Zinc Phosphate (Marshall and Marx, 2012, Table 6.1). We remain agnostic as to whether the coordination that we consider is explicit or tacit collusion or whether it might be found to violate antitrust laws.

we say that some other firm m outside this set is a *maverick* if the market is not at risk for coordination by these firms with m present and is at risk for coordination without m.

Let us briefly summarize and discuss our main findings, beginning with results for the setting of efficient procurement markets in which the buyer uses a second-price auction. This setting not only provides an important benchmark because the dominant strategies of the second-price auction eliminate complications due to strategic complements or substitutes, but also is of interest in itself given the quantitative importance of procurement markets.[6] A case in point to which we will return is the audit industry in France, where every major company is required by law to procure the services of two separate audit firms. In the procurement setup, assuming that each supplier draws its cost independently from some distribution and that a merger between two suppliers means that the merged entity's cost is drawn from the distribution of the minimum of the two merging suppliers' costs, we show that some, but not all, markets are at risk for coordination. Because of the inefficiency involved in coordination, a procurement market need not even be at risk for coordination by all suppliers. For a given set of candidate coordinators, mergers by outsiders, that is, by firms that do not involve suppliers in the set of coordinators, are neutral insofar as they do not affect the coordinated effects index for the coordinators. Moreover, stronger outside competition (which can equivalently be caused by better cost distributions of the outsiders or by replicating a number of outside suppliers) reduces the risk for coordination by a given set of suppliers. Last but not least, if two suppliers become less symmetric in the sense that one supplier's cost distribution improves while the other one's worsens, while the distribution of their minimum cost remains the same, then the HHI and expected price that the buyer pays under competitive bidding increase. In contrast, the coordinated effects index decreases for a set of firms that includes these two suppliers when costs have a power-based distribution. This brings to light the possibility of a tension or tradeoff between unilateral and coordinated effects that is relevant, for example, in the context of remedies such as mandated divestitures.

Many of these insights from efficient procurement markets carry over to Cournot markets with constant marginal costs and linear demand. In particular, more outside competition reduces the risk for coordination by a given set of suppliers, and not all Cournot markets are at risk for coordination. However, in contrast to procurement markets, Cournot markets are always at risk for all-inclusive coordination. In both procurement and Cournot markets, coordination reduces expected buyer (or consumer) surplus and social surplus.

From the definition of a maverick, it follows that a merger between a candidate coordina-

[6] Government procurement accounts for a double-digit share of GDP in most developed economies (Kutlina-Dimitrova, 2018). U.S. business-to-business wholesale and distribution was $8 trillion in 2018 (Qasim Mohammad, "Amazon's Next Mountain: B2B Procurement," *Globe and Mail*, January 7, 2018).

tor and a maverick puts a market at risk for coordination if the acquisition of the maverick amounts to eliminating the maverick's productive assets. This is, for example, the case in a Cournot model with constant marginal costs when the maverick's marginal costs are at least as high as its acquirer's. Thus, in addition to developing a rigorous and disciplined approach for defining and analyzing mavericks and their effects, our framework and analysis suggest that a maverick-based merger evaluation can usefully be employed for Cournot markets, thereby providing support for the maverick-based approach, as called for by Baker (2002), in the context of Cournot markets. More generally, however, the acquisition of a firm does not necessarily eliminate the acquired firm's productive assets. For example, in a procurement market in which each supplier's cost is given by a draw from some distribution, it is natural to think of the merged entity as drawing its cost from the distribution of the minimum of the two costs.[7] In this case, a merger eliminates a bid (and, in a first-price auction, affects the bidding equilibrium) but leaves the number of cost draws the same. As we show, in this case, the acquisition of a maverick by a candidate coordinator need not put a market at risk for coordination.

There is a large legal and economics literature on coordinated effects and mavericks. Baker (2002, 2010) and Harrington (2013) provide overviews of the legal literature and Porter (2020) of the economics literature.[8] As mentioned, definitions and analyses of maverick firms have been provided by Kwoka (1989), Kolasky (2002), Ivaldi et al. (2007), Ivaldi and Lagos (2017), and de Roos and Smirnov (2019), among many others.

Formal analyses of coordinated effects in both the empirical and theoretical economics literature typically adhere to a repeated games framework, where the object of interest is the critical discount factor that permits collusion, with a lower critical discount factor having the interpretation that a market is more at risk for coordination. See, for example, Miller and Weinberg (2017), Miller et al. (2019), and Igami and Sugaya (2019) for empirical approaches and Compte et al. (2002), Vasconcelos (2005), Ivaldi et al. (2007), Bos and Harrington (2010), and Harrington and Wei (2012) for theoretical analyses.[9] Our approach shares with the repeated games approach the quantitative interpretation of a market being

[7]The same observation—that a merger does not eliminate the production capacity but only a bid—explains the different effects of bargaining power in the merger context of Loertscher and Marx (2019b) versus the setup of Bulow and Klemperer (1996), where removing a bidder means that both its bid *and* its productive capacity are eliminated.

[8]Coordinated effects arguments have played a central role in U.S. merger cases such as *Heinz/Beech-Nut, Anheuser-Busch InBev/Grupo Modelo,* and *H&R Block/TaxACT*. For European cases, see Amelio et al. (2009) on *ABF/GBI Business*, Motta (2000) on *Airtours/First Choice*, and Aigner et al. (2006) on *Sony/BMG* and *Impala* and on the evolution of coordinated effects' assessment in the EU.

[9]In an alternative approach, Kovacic et al. (2007a, 2009) and Gayle et al. (2011) view coordination as analogous to incremental mergers (i.e., perfect collusion) among post-merger firms and propose quantifying coordinated effects by using existing merger simulation tools to model coordination as incremental mergers.

more at risk when, in our setup, the coordination effects index is larger and, in the repeated games framework, the critical discount factor is smaller. A key distinguishing feature of our approach is that it naturally gives rise to a threshold—the sum of critical shares being less than one—above which a market is not at risk for coordination. No such threshold exists in the repeated games approach, implying that there, markets can only be more or less at risk. It is this threshold that also allows us to define mavericks, to test for whether a firm is a maverick, and to identify whether a market is at risk for coordination.

The remainder of the paper is organized as follows. In Section 2, we provide an overview of our approach without imposing specific assumptions about the price formation process. In Section 3, we analyze coordinated effects and mavericks for efficient procurement markets. Using data from the French audit industry, Section 3 also illustrates how one can test for risk for coordination and mavericks with data that are typically available pre-merger. Section 4 contains extensions to Cournot competition, buyer power, first-price auctions, and vertical integration. In Section 5, we discuss the relation to the HHI and possible tensions between unilateral and coordinated effects. Section 6 concludes the paper.

2 Overview

In this section, we present our framework with a considerable degree of generality by not imposing any assumptions on the underlying model, deferring to later sections more specific assumptions and analyses of their implications.

Throughout the paper, we model coordination as a selection scheme. In such a scheme, markets or customers are allocated among the colluding suppliers by taking turns or staying out of each other's territory, without the colluding suppliers ever transferring money among themselves.[10] The electrical contractors conspiracy that relied on the phases of the moon is one notorious example. Authorities recognize that selection schemes based on "geographic areas (e.g., one road contractor gets all the work in one county, another company all the work in the next), or by type of job or by time to give each member a chance to share in the spoils."[11] Likewise, according to the U.S. Department of Justice, four basic schemes are involved in most bid-rigging conspiracies: bid suppression, complementary bidding, bid rotation, and customer or market allocation (U.S. DOJ, 2015), all of which involve one bidder being designated to represent the coordinating firms.

We denote by $N \equiv \{1, ..., n\}$, with $n \geq 2$, the set of all suppliers (i.e., firms in an oligopoly), and we consider the possibility that the suppliers in a subset $K \subseteq N$ coordinate,

[10] See also Stigler (1964); McAfee and McMillan (1992); Aoyagi (2003).

[11] See the International Anti-Corruption Resource Center's *Guide to Combating Corruption & Fraud in Development Projects* available at https://guide.iacrc.org/potential-scheme-collusive-bidding/.

where K contains $k \geq 2$ suppliers. In other words, we allow for non-all-inclusive coordination without requiring it. The selection scheme is such that each supplier $i \in K$ is designated to be the active member with some probability s_i, in which case all other members of K are inactive. In a merger-review context, competition authorities may have reason to focus on specific subsets of suppliers based on historical conduct or other evidence. We illustrate the use of the coordinated effects index to identify sets of concern in the context of the application in Section 3.5.

Let Π_i denote supplier i's expected payoff when there is no coordination. Given coordination by suppliers in K, we denote the expected payoff of supplier $i \in K$ when it is the designated supplier from K by $\Pi_i(K)$.

The individual rationality of suppliers' participation in coordination depends on the assumption regarding what happens if a supplier declines to coordinate. Because coordination provides a public good to noncoordinating suppliers, the conservative approach, which we take, is to assume that the failure by any supplier in K to participate results in there being no coordination. To see this, note that if remaining suppliers in K continue to coordinate when a supplier i in K chooses not to participate in coordination, then supplier i's payoff is higher than if the coordination broke down completely, generating a higher critical share for supplier i. Given our assumption that coordination breaks down if not all suppliers in K participate, if a market is not at risk, then it is also not at risk under alternative assumptions of coalition formation.[12]

Participation by supplier i in coordination with suppliers in K is individually rational for supplier i if i is selected with probability s_i and if the probability with which supplier i is selected, multiplied by supplier i's coordination payoff, $\Pi_i(K)$, is greater than its payoff without coordination, Π_i. Thus, coordination is profitable for supplier i if it is selected with a sufficiently high probability, that is, coordination is profitable for supplier i if and only if it is selected with probability greater than a *critical share* $s_i(K)$ defined by

$$s_i(K) \equiv \frac{\Pi_i}{\Pi_i(K)}.$$

Of course, coordination is only feasible if it is profitable for all of the coordinating suppliers. This means that each coordinating supplier needs to be selected with probability greater than its critical share. Because the selection probabilities must sum to one, this is only possible if the critical shares sum to less than one, which gives us a natural candidate for a

[12] As illustrated in Section 3.5, it is straightforward to evaluate scenarios with different sets of coordinators.

coordinated effects index:

$$\mathcal{I}(K) \equiv 1 - \sum_{i \in K} s_i(K).$$

If the coordinated effects index is positive, that is, if $\mathcal{I}(K) > 0$, then selection probabilities exist for the coordinating suppliers such that each coordinating supplier finds it profitable to participate in the coordination. In that case, we say that the market is at risk for coordination by the suppliers in K. However, if the coordinated effects index is not positive, $\mathcal{I}(K) \leq 0$, then no such selection probabilities exist, and we say that the market is not at risk for coordination by the suppliers in K. Given a positive index, a further increase in the index allows greater scope for coordination in the sense that some inefficiencies in the coordination process can then be accommodated. In the extreme, as the index approaches 1, coordination can be sustained through coordination that selects each coordinating supplier with equal probability. This makes coordination easier to sustain with asymmetric firms insofar as, with asymmetric firms, the critical shares $s_i(K)$ vary across firms.

Because the test based on the coordinated effects index focuses on a necessary condition for coordination, it is biased in the direction of overestimating the gains from coordination. It thus provides a screen that allows one to dismiss concerns of coordinated effects as unlikely in some cases. Whenever the index is nonpositive, coordination as a selection scheme without communication or transfers is not profitable for the coordinating suppliers. If the index is positive, it may be profitable. Importantly, under parametric assumptions, data on the pre-merger market shares of candidate coordinating suppliers and one supplier's margin are sufficient to make the construction of the index a straightforward calculation, rendering it operational for practical purposes.

The interpretation of a larger, positive value of the coordinated effects index as indicating an increase in risk may also prove useful for practitioners. For example, the *U.S. Guidelines* (p. 26) state: "The Agencies regard coordinated interaction as more likely, the more the participants stand to gain from successful coordination." A larger positive value of the coordinated effects index implies that there is more leeway for coordination to remain profitable in the face of additional costs of coordination, including potential penalties for being caught coordinating. In contrast, when the index is close to zero, small positive costs or frictions could impede coordination.

We define a *maverick* with respect to a set of suppliers K to be a supplier whose presence prevents the pre-merger market from being at risk for coordination among suppliers in K, i.e., the market is not at risk for coordination by suppliers in K when the maverick is in the market, but is at risk when the maverick is not in the market. That is, writing $\mathcal{I}(K; N)$ to denote the coordinated effects index for coordination by the subset K of N suppliers, then supplier $m \in N\backslash K$ is a *maverick* if

$$\mathcal{I}(K;N) \leq 0 \text{ and } \mathcal{I}(K;N\backslash\{m\}) > 0.$$

This definition of a maverick allows the possibility that there is no maverick, as well as the possibility that more than one supplier in a market could be a maverick with respect to a particular set K of suppliers.[13]

The conditions for a market to be at risk for coordination or for the acquisition of a maverick to put a market at risk of coordination naturally depend on the specifics of the price formation process. In what follows, we first analyze efficient procurement markets, which are equivalent to efficient second-price auctions, and then turn our attention other price formation processes, including Cournot competition and first-price auctions.

3 Efficient procurement markets

In this section, we analyze coordinated effects in the context of efficient procurement markets. The price formation process in many markets is based on competitive procurement, including much of government procurement and business-to-business purchasing. Thus, a model of price formation through competitive bidding is relevant for many markets in which mergers, and hence concerns of coordinated effects, occur.

3.1 Setup

To model an efficient procurement market, we assume that a buyer with value v for a single unit purchases from n suppliers using a second-price procurement auction with reserve r. The supplier with the lowest bid that is less than or equal to r wins and is paid the lower of r and the second-lowest bid. We denote the coordinated effects index for a second-price procurement market by $\mathcal{I}^S(K)$.

Each supplier i independently draws its cost of producing a unit, which is its private information, from a distribution G_i with positive density g_i on $[\underline{c}, \overline{c}]$, where $\underline{c} < v$. We assume that suppliers follow their weakly dominant strategies of bidding truthfully. The reserve in the procurement is assumed to be $r \equiv \min\{v, \overline{c}\}$, which, together with truthful bidding, ensures that the procurement is efficient. In this model, symmetry among a set of suppliers means that the suppliers in that set draw their costs from the same distribution, i.e., $G_i = G$ for all i in the set.

[13] We provide additional discussion and examples in Section 3.3. The notion of multiple mavericks arises in practice, e.g., in the U.S. mobile communications market, prior to T-Mobile's acquisition of MetroPCS, both were considered "'mavericks' with a history of disrupting the industry" ("Sprint CEO Sees 'Enormous' Synergies in T-Mobile Merger," *Kansas City Star*, June 12, 2017).

To model mergers, we assume that a merged entity that combines suppliers i and j has a cost distribution that is the distribution of the minimum cost of the two pre-merger suppliers, i.e., the merged entity's cost distribution is $1 - (1 - G_i)(1 - G_j)$. It follows that the productivity of the merged entity is enhanced relative to that of either of the individual merging suppliers.

For some procurement results, it is useful to parameterize the suppliers' cost distributions as $G_i(c) = 1 - (1 - c)^{\alpha_i}$ with $\alpha_i > 0$ and support $[\underline{c}, \overline{c}] = [0, 1]$, which we refer to as the power-based parameterization. This parameterization allows us to talk about suppliers with larger values of α_i as being "larger" or "stronger." We refer to α_i as supplier i's strength parameter because, in a market with set N of competing suppliers, supplier i's market share is $\alpha_i / \sum_{j \in N} \alpha_j$. Letting $A \equiv \sum_{j \in N} \alpha_j$ and $A_{-X} \equiv \sum_{j \in N \setminus X} \alpha_j$, and we can write $\mathcal{I}^S(K)$ in terms of the distributional parameters as follows:

Lemma 1. *For the power-based parameterization with $v \geq \overline{c}$,*

$$\mathcal{I}^S(K) = 1 - \sum_{i \in K} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A_{-\{i\}})(1 + A)}.$$

Proof. See Appendix A.

3.2 Coordination is harmful but the test for at-risk markets has power

Coordination suppresses competition from one or more suppliers. With positive probability, the participation of the lowest-cost supplier is suppressed. Thus, coordination increases the expected cost conditional on trade and also reduces the probability of trade if the buyer's reserve binds with positive probability. As a result, coordination reduces expected social surplus. In addition, the elimination of bids as a result of coordination results in a higher expected price, which harms the buyer. Thus, we have the following result:

Proposition 1. *Coordination reduces expected buyer surplus and expected social surplus.*

Proposition 1 provides a foundation for competition authorities' concerns about coordination. In what follows, we show that the coordinated effects index has power as a test for which markets are at risk, or at a greater risk, for coordination. Also, the coordinated effects index allows one to identify mergers that would cause a market that is not at risk for coordination to become at risk. Intuitively, not all markets are at risk because, given the cost uncertainty across suppliers, randomly selecting one of them to be the designated bidder involves an element of inefficiency, which may outweigh the benefit to the coordinating suppliers from bid suppression. As we show, not even coordination by all suppliers necessarily puts a market at risk for coordination, i.e., $\mathcal{I}^S(N) \leq 0$ for some (but not all) cost distributions. Using the notation $c_{(j:n)}$ to denote the j-th lowest order statistic out of n independent draws from a common cost distribution, we have the following result:

Lemma 2. *Assume symmetry and $v \geq \bar{c}$. Then the market is at risk for all-inclusive coordination, $\mathcal{I}^S(N) > 0$, if and only if*

$$\mathbb{E}_{\mathbf{c}}[c_{(2:n)} - c_{(1:n)}] < \bar{c} - \mathbb{E}_c[c], \tag{1}$$

where (1) *is satisfied for some but not all cost distributions.*

Proof. See Appendix A.

As Lemma 2 shows, the market is *not* at risk for all-inclusive coordination if the suppliers' cost distribution has a "long left tail," where the distance between the upper bound of the support of the cost distribution and the expected cost is less than the expected distance between the first and second order statistics.[14] In this case, the price paid to the winning supplier is likely to be close to the reserve under competition, so the expected incremental payment under cooperation is small and outweighed by the loss associated with the possibility that the supplier is not selected to participate.

It follows that the coordinated effects index test has power in the sense that, based on the index, some, but not all, markets are at risk for coordination. Using "model specification" to mean the buyer's value and suppliers' cost distributions, we have:

Proposition 2. *There exist N, $K \subseteq N$, and model specifications such that $\mathcal{I}^S(K) > 0$, and other N, $K \subseteq N$, and model specifications such that $\mathcal{I}^S(K) \leq 0$.*

Having established the ability of the coordinated effects index to screen markets for their risk for coordination, we consider the application of the coordinated effects index in a merger context.

3.3 Mergers and mavericks

We now examine merger effects, including coordinated and unilateral effects—effects on expected buyer surplus assuming that there is no coordination—and examine the role of mavericks.

[14] The market is not at risk, for example, when $n = 2$ and the density $g(c)$ is such that $g(c) = 0.05$ for $c \in [0, 0.9]$ and $g(c) = 9.55$ for $c \in (.9, 1]$, which has a long left tail and high probability close to the upper bound of support. Specifically, $1 - \mathbb{E}_c[c] = 0.0725$ and $\mathbb{E}_{\mathbf{c}}\left[c_{(2:n)} - c_{(1:n)}\right] = 0.0740$. Hence, inequality (1) is not satisfied in this example.

In merger review, competition authorities commonly use a model of the price formation process, calibrated to pre-merger data, to predict the economic effects of a proposed merger. A merger raises concerns of coordinated effects if the merger enables or encourages post-merger coordinated interaction (*U.S. Guidelines*, p. 24). Thus, in the context of the coordinated effects index, a key scenario of concern occurs when the pre-merger market has $\mathcal{I}^S(K) \leq 0$, based on a calibration to pre-merger data, and, based on that same calibration, the post-merger market would have $\mathcal{I}^S(K) > 0$. In this case, the data are consistent with pre-merger competition and with the risk of post-merger coordination. If one obtains $\mathcal{I}^S(K) > 0$ for the pre-merger market, then concerns that the merger could exacerbate coordinated effects are justified when the post-merger market has an even larger positive value of $\mathcal{I}^S(K)$ because this implies that a larger set of possibilities for coordination become available following the merger.

To begin, we note that an implication of Lemma 2 is that even a "3-to-2" merger does not raise concerns of coordinated effects if the resulting duopoly is characterized by symmetric suppliers that draw their costs from a distribution satisfying (1). The implication of Lemma 2 that it is possible to have a 3-to-2 merger that does not raise concerns of coordinated effects, but that such circumstances are limited, is consistent with the view in practice that 3-to-2 defines a significant, but not insurmountable, hurdle for antitrust approval.[15]

In order to consider the effects of a merger among suppliers in K on incentives for coordination, consider a set K of three or more suppliers. The critical share for a supplier $i \in K$, $s_i(K)$, depends only on the distribution of the minimum cost of suppliers other than i in K and on the distribution of the minimum cost of suppliers outside K. Because a merger of two suppliers does not affect the distribution of the minimum cost of the two merging suppliers, it follows that a merger of two suppliers in K does not affect the critical shares of the nonmerging firms in K. Thus, the change in $\mathcal{I}^S(K)$ as a result of a merger depends on how the critical share of the merged entity compares to the sum of the critical shares of two merging suppliers in the pre-merger market. Letting $\hat{N}$ denote the set of post-merger suppliers and $\hat{K}$ denote the post-merger suppliers in K following the merger of suppliers k and ℓ in K, and letting μ denote the merged entity, we have

$$\mathcal{I}^S(\hat{K};\hat{N}) - \mathcal{I}^S(K;N) = s_k(K;N) + s_\ell(K;N) - s_\mu(\hat{K},\hat{N}). \tag{2}$$

The next lemma follows immediately.

[15] This is reflected in headlines such as "Is 4-3 the New 3-2? FTC Continues to Target Pharmaceutical Mergers" (Bruce Sokler and Helen Kim, MIntz Levin Antitrust Alert, 2014, https://www.mintz.com/newsletter/2014/Advisories/3893-0414-NAT-AFR/index.html) and analysis of antitrust enforcement trends (see, e.g., Kovacic et al., 2007b; Hawkins and King-Kafsack, 2014)).

Lemma 3. *A merger of suppliers in K increases the coordinated effects index if and only if the critical share of the merged entity is less than the sum of the pre-merger critical shares of the merging suppliers.*

To see the forces at work, consider a merger of suppliers k and ℓ, both of which are members of K, and assume for purposes of illustration that $\Pi_k(K;N) = \Pi_\ell(K;N)$. Under this assumption, we have

$$s_k(K;N) + s_\ell(K;N) = \frac{\Pi_k + \Pi_\ell}{\Pi_k(K;N)} \tag{3}$$

because a supplier's critical share is its expected payoff without coordination, divided by its expected payoff with coordination. A merger of suppliers k and ℓ is always profitable for those suppliers, i.e., $\Pi_k + \Pi_\ell < \Pi_\mu$ (Loertscher and Marx, 2019b, Prop. 6), so the numerator in the merged entity's critical share is larger than the numerator in (3). At the same time, the denominator in the merged entity's critical share is greater than the denominator in (3) because the merged entity draws its cost from a better distribution than does either supplier k or ℓ. Because either effect can dominate, the critical share of the merged entity can be greater than or less than the sum of the pre-merger critical shares of the merging suppliers, implying that the coordinated effects index can increase or decrease as a result of a merger.

In contrast to the case of a merger of suppliers in K, a merger of suppliers outside K does not affect $\mathcal{I}^S(K)$ because it only depends on suppliers outside K through the distribution of the minimum of their costs, which is not affected by a merger. Thus, we have the following result:

Proposition 3. *A merger of suppliers in K can, but need not, cause a market not at risk for coordination by suppliers in K to become at risk for coordination by the corresponding post-merger suppliers, and a merger of suppliers outside K does not affect the risk for coordination by suppliers in K.*

An implication of Proposition 3's result that a merger of outsiders does not affect the coordinated effects index is that merging parties cannot reduce the perceived risk of coordination associated with their merger by strategically varying the timing of their merger so that it falls either before or after a merger of outsiders.

Turning to the role of maverick firms, as we now show, the acquisition of a maverick can, but need not, put the market at risk. In a procurement market, the acquisition of a maverick is not the same as the elimination of the maverick's productive capability. Rather, it eliminates a bid in the procurement. Thus, if supplier $i \in K$ is the acquirer, then both Π_i and $\Pi_i(K)$ increase after the acquisition, so that the critical share $s_i(K)$ may well be larger after the acquisition than before it. For all other coordinating suppliers $j \in K \backslash \{i\}$, $\Pi_j(K)$ increases because the maverick has been eliminated as an outside supplier. In an efficient procurement, Π_j is not affected, so $s_j(K)$ decreases. Consequently, in a procurement market, the overall effect of the acquisition of a maverick with respect to K on the coordinated effects index depends on the details and can, as we show, go either way.

The following proposition provides conditions under which the decrease in the critical share of the nonmerging supplier dominates if and only if the maverick is acquired by the smaller of two coordinating suppliers.

Proposition 4. *For the power-based parameterization with $v \geq \bar{c}$ and $|K| = 2$, the acquisition of a maverick by the weakly smaller supplier in K puts the market at risk for coordination, whereas the acquisition by the larger supplier in K does not if the weaker supplier is sufficiently small.*

Proof. See Appendix A.

Proposition 4 implies that maverick-based merger review that focuses on blocking mergers that involve the acquisition of a maverick can be misleading for efficient procurement markets. Some other aspects of the perceived wisdom regarding mavericks are similarly not supported in our framework. In particular, one can easily construct examples in which the acquisition of a supplier that is not a maverick puts a market at risk for coordination. Thus, it is not the case that the acquisition of a maverick is necessary for coordinated effects. It is also not the case, as has been suggested, that the presence of a maverick prevents coordinated effects from mergers not involving the maverick.[16]

Similar to the neutrality result of Nocke and Whinston (2010) that shows that the order in which a competition authority addresses mergers does not matter for unilateral effects, we have a form of neutrality for coordinated effects because outside mergers do not affect coordination. If $\mathcal{I}^S(K) > 0$, a merger of outsiders cannot stop the market from being at risk nor create a maverick with respect to K because mergers among firms outside K do not affect $\mathcal{I}^S(K)$, which depends on outsiders only through the distribution of the minimum of their costs. Thus, a competition authority cannot reduce the risk of coordinated effects among one set of firms by first approving "balancing" mergers among outsiders. However, if $\mathcal{I}^S(K) < 0$, then a merger of outsiders could create a maverick, which would then be

[16] To see that this argument (e.g., by Baker, 2002, p. 180) does not apply for efficient procurement markets, consider a market with five firms drawing their cost types from the uniform distribution on $[0, 1]$. Then supplier 5 is a maverick with respect to coordination by suppliers in $\{1, 2, 3\}$, i.e., $\mathcal{I}^S(\{1, 2, 3\}; \{1, 2, 3, 4, 5\}) \leq 0$ and $\mathcal{I}^S(\{1, 2, 3\}; \{1, 2, 3, 4\}) > 0$, and following the merger of suppliers 1 and 2, the market is at risk for coordination by the merged entity and supplier 3, i.e., $\mathcal{I}^S(\{\mu, 3\}; \{\mu, 3, 4, 5\}) > 0$, where μ is the merged entity.

a potential acquisition target for suppliers in K. For example, suppose that there are two suppliers outside K and that $\mathcal{I}^S(K) < 0$. If neither of the outsiders is a maverick, but if they are jointly a maverick in the sense that if both outsiders were eliminated, then the market would be at risk for coordination by K, then a merger of those outsiders creates a maverick.

The merger of U.S. telecom firms Sprint and T-Mobile raised the question whether the merger of two mavericks might create a "super maverick": "Today, Verizon and AT&T have nearly 70 percent market share and 93 percent of the industry cash flow. Combining T-Mobile and Sprint will create a supercharged maverick, which will still be only Number Three. This newly invigorated third carrier will be better able to compete against the larger two."[17] In our model, a merger involving a maverick with respect to K and another supplier outside K does not affect $\mathcal{I}^S(K; N)$ and increases $\mathcal{I}^S(K; N\backslash\{\hat{m}\})$, where $\hat{m}$ denotes the merged entity formed by the maverick and the other outside supplier. As a result of such a merger, there continues to be a maverick with respect to K, and the difference between the coordinated effects index with and without the maverick is increased.[18] Thus, there is a sense in which the merger of two mavericks could indeed create a "super maverick."

3.4 Market and supplier characteristics

We now briefly show that the effects of market and supplier characteristics on the risk of coordination in our model are broadly consistent with the perceived wisdom and the prior literature. In particular, we discuss the effects of outside competition, supplier strength, and symmetry among the coordinating suppliers on the coordinated effects index. For ease of presentation, we keep this discussion in the main text brief and mostly informal, deferring the formal statements and their proofs to Appendix B.

Outside competition

A natural conjecture is that coordination by a given set of suppliers is more challenging when these suppliers face more outside competition. Although an increase in the number of (symmetric) outside suppliers decreases the payoffs of the suppliers in K both with and without coordination, one can show that, for symmetric suppliers, the decrease in payoffs

[17]Robert McDowell, "T-Mobile-Sprint merger would be win for David, not Goliath," Fox Business, April 29, 2019, available at https://www.foxbusiness.com/technology/t-mobile-sprint-merger-fcc-doj. See also Deutsche Telekom CEO Tim Hoettges stating that he was "intrigued by the idea of having a combination with Sprint and being the 'super-maverick' in the market" (Cam Bunton, "'Super-Maverick' T-Mobile and Sprint Merged Company 'Intrigued' Deutsche Telekom Chief," TmoNews, January 19, 2015, available at deutsch-telekom-chief-was-intrigued-by-the-idea-of-combining-with-sprint-to-create-super-maverick).

[18]This suggests that one could quantify the extent of the maverickness by the difference in the coordinated effects index with and without the maverick, i.e., one could measure the strength of a maverick m by $\mathcal{I}^S(K; N\backslash\{m\}) - \mathcal{I}^S(K; N)$. As an example, $n = 4$, $a_1 = 2$, $a_2 = 2$, $a_3 = 1$, $a_4 = 1$: 2 is a maverick for $K = \{1, 3\}$ and "more of a maverick" after 2 merges with 4.

under coordination is larger, and so critical shares increase and the market becomes less at risk for coordination. Specifically, assume that all suppliers are symmetric and consider a fixed set of coordinators. Then, the coordinated effects index is negative when the number of suppliers is sufficiently large, and, for the power-based parameterization, the index is decreasing in the number of suppliers (Proposition B.1).

The intuition is simple. As mentioned, coordination is the provision of a public good whose costs are borne by the insiders and whose benefits accrue to all active suppliers, which include the designated supplier among the insiders and all outsiders. For coordination to pay off, it must be the case that the insiders internalize enough of the benefits their coordination generates, which requires the outside competition to be limited.

Interestingly, the effect of the strength of outside competition on the incentives to coordinate for a given set of suppliers may make coordination contagious in the following sense. Suppose that, if the suppliers in some set K_1 do not coordinate, then $\mathcal{I}^S(K_2)$ is negative for some disjoint set of suppliers K_2. If the suppliers in K_1 coordinate, implying that the lowest-cost supplier in K_1 is not active with positive probability,[19] this is as if the outside competition for the suppliers in K_2 has weakened, making it possible that once the suppliers in K_1 coordinate, $\mathcal{I}^S(K_2)$ becomes positive. Of course, because the same logic applies to the suppliers in K_1, it is possible that each set of suppliers only finds it beneficial to coordinate if the other set coordinates as well.

Supplier strength

Next, we consider the effect of increases in the strength of the coordinating suppliers, where that increase in strength corresponds to a first-order stochastically dominated shift in the cost distribution. We focus on the power-based parameterization, which allows us to make clean statements about the strength of the suppliers.

We first assume that all suppliers in K are symmetric with a cost distribution by parameterized by $\alpha \in [0, \infty)$ and keep the set and cost distributions of the outsiders fixed. Increases in α correspond to increases in the productivity of the coordinating firms. Under these assumptions, efficient procurement markets are not at risk when the coordinators are sufficiently weak and are at risk when they are sufficiently strong (Proposition B.2). Intuitively, coordinating with a stronger supplier is more profitable because the bid suppression by that supplier is more likely to affect the outcome of the procurement.

With that in mind, we now allow suppliers to be asymmetric and examine the question of what characterizes the groups of suppliers that have the highest coordinated effects index. As we show in Appendix B, coordination in efficient procurement markets is characterized

[19] The contagion effect would not be present if coordination were efficient, as might be the case with explicit collusion involving the communication of private information and transfers.

by "positive assortative matching" insofar as the coordinated effects index for a set of k suppliers is greatest when the set contains the k strongest suppliers. Moreover, one can show that the coordinated effects for a given set of suppliers K increases as these suppliers becomes stronger while keeping fixed the distribution of the lowest cost draw in the market. Specifically, let $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$ be two parameterizations satisfying $\sum_{i \in N} \alpha_i = \sum_{i \in N} \beta_i$ and $\beta_i \geq \alpha_i$ for all $i \in K$ with a strict inequality for at least one i. Then, the coordinated effects index for the set K is higher under the parameterization $\boldsymbol{\beta}$ than under $\boldsymbol{\alpha}$ (Proposition B.3).

This implies that an efficient procurement market is most at risk for the type of coordination that is most problematic for buyers, namely coordination by the large suppliers. In addition, a market is more at risk for coordination by a group of the largest suppliers in the market, relative to an equal number of other suppliers. This is consistent with the prevailing view that a competition authority should be most concerned about coordination among the largest suppliers in a market.

Symmetry

Arguing that asymmetries may make it difficult for firms to agree to a common pricing policy and that incentive compatibility may be difficult to satisfy for low-cost firms that face relatively large gains from deviations and small costs from punishment, Ivaldi et al. (2007) provide intuition for the notion that collusion is easier for more symmetric firms.[20] We now show that a similar result applies in our context.

To consider the effects of the symmetry of suppliers in the market, we begin by defining a *neutral for rivals spread (NR spread)*. We say that a change in the cost distributions for suppliers i and j from (G_i, G_j) to (H_i, H_j) is an NR spread if for all $c \in [\underline{c}, \overline{c}]$,[21]

$$H_i(c) \quad \leq \quad G_i(c), G_j(c) \quad \leq \quad H_j(c), \tag{4}$$

with strict inequalities for costs in an open subset of $[\underline{c}, \min\{v, \overline{c}\}]$, and

$$(1 - G_i(c))(1 - G_j(c)) = (1 - H_i(c))(1 - H_j(c)). \tag{5}$$

The inequalities in (4) represent the spread, while (5) captures the neutrality for rivals because it means that the distribution of the minimum cost of suppliers i and j is the same

[20]For related arguments, see Compte et al. (2002), Kühn (2004), and Vasconcelos (2005). For the contrasting view that asymmetries can facilitate collusion in some settings, see Ganslandt et al. (2012).

[21]An NR spread applied to two suppliers in a market produces a market that is "more concentrated" according to Waehrer (2019). For example, if $G(c; \alpha) \equiv 1 - (1 - c)^\alpha$, we can construct an NR spread of $(G(c; \alpha_1), G(c; \alpha_2))$, where $\alpha_1, \alpha_2 > 0$, using $H_1(c) = G(c; \alpha_1 - \Delta)$ and $H_2(c) = G(c; \alpha_2 + \Delta)$ for $\Delta \in (0, \min\{\alpha_1, \alpha_2\}]$.

under (H_i, H_j) and (G_i, G_j).

For the power-based parameterization, an NR spread applied to two suppliers in K reduces the coordinated effects index (Proposition B.4). This highlights the impact of increased symmetry among potentially coordinating suppliers on the market risk for coordination. It is consistent with existing literature showing that coordination is easier to support among more symmetric suppliers. Intuitively, note that when two suppliers are made less symmetric, their expected second-lowest cost increases, increasing the expected price paid by the buyer and the suppliers' critical shares, and decreasing the coordinated effects index. In addition, with less symmetric suppliers, the inefficiency associated with randomly selecting the higher cost of the two suppliers also tends to increase the index.

3.5 Application

In this section we illustrate the applicability of the framework.

Use of market data for identification

One can use market data to identify the model by assuming that market shares, margins, and other observed data are generated as the average of outcomes over a large number of efficient procurements at which suppliers bid competitively given costs that are independently drawn for each procurement from a parameterized distribution.

Imposing the power-based parameterization and assuming that $v \geq \bar{c}$ (i.e., trade always occurs), the suppliers' cost distributions are pinned down by market shares and, for example, one supplier's margin. To see this, note that supplier i's market share σ_i satisfies $\sigma_i = \alpha_i/A$, and A can be related to supplier i's operating margin (Lerner Index), $\omega_i \equiv \Pi_i/R_i$, where R_i is supplier i's expected revenue, as follows: $A = 1/(1/\omega_i - 2 + \sigma_i)$, as long as supplier i's operating margin is less than $1/(2 - \sigma_i)$. (A sufficient condition is that the operating margin is less than 50%.) Or, noting that $\omega_i = 1/(1/A + 2 - \sigma_i)$, one could derive A from the (share weighted) average margin for the industry or for any subset of suppliers. If a positive share of procurements result in no trade, indicating that $v < \bar{c}$, then data on the share of procurements resulting in no trade allows v also to be identified.[22] In the absence of margin data or other identifying information, an alternative is to impose a restriction on A. For example, assuming that A is equal to the number of suppliers is equivalent to assuming that the suppliers' cost distributions are uniform on average.

In what follows, we assume that the data generating process is competitive bidding across a large number of procurements in which each bidder i's cost is an independent draw from

[22] In this case, $r = \min\{v, \bar{c}\} < \bar{c}$, $\Pi_i = \frac{1-(1-r)^{1+A-\alpha_i}}{1+A-\alpha_i} - \frac{1-(1-r)^{1+A}}{1+A}$, and $R_i = \frac{1-(1-r)^{1+A-\alpha_i}}{1+A-\alpha_i} - \frac{1-(1-r)^{A}}{A} + (1+\alpha_i)\frac{1-(1-r)^{A}(1+rA)}{A(1+A)}$.

a power-based distribution.

Application to the French audit industry

As described by Ivaldi et al. (2012), one industry in which coordinated effects and the potential for mavericks is a concern, and where price formation is, to a first-order approximation, characterized by efficient procurement, is the French audit industry.[23] Thus, the French audit industry provides an almost ideal application of the framework just presented. In addition, because large French firms are required to engage two auditors, the application allows us to illustrate the extension of the model to the case of multi-unit demand, which is detailed in Appendix D.1.

Ivaldi et al. (2012) raise the questions of whether the French audit industry is at risk for coordination among the Big 4 firms and whether the fifth-largest firm, Mazars, should be viewed as a maverick, by which they mean "a firm with a drastically different cost structure, which is thus unwilling to participate to a collusive action" (Ivaldi et al., 2012, p. 40). Ultimately, they conclude that Mazars is not a maverick based on their qualitative and quantitative analysis, including econometric results indicating that Mazars is not properly viewed as a competitor with comparable capabilities to a Big 4 firm. Given that, they then conclude that the market *is* at risk for coordination by the Big 4 based on the *Airtours* criteria of sufficient transparency, the possibility of retaliation, and the absence of either a disruptive rival (i.e., a maverick) or powerful buyers.[24]

We calibrate the model of second-price procurement to the data and then examine both whether the market is at risk for coordination by the Big 4 firms according to our framework and whether Mazars is a maverick according to our definition. In light of the characteristics of the French audit industry, including the feature that large companies must each hire two independent auditors (Ivaldi et al., 2012), we model each buyer as having value $v \geq \overline{c}$ for two-units, one from each of two different suppliers (and zero value for anything else), and as using an efficient procurement in its dominant strategy implementation.

To calibrate cost distributions, we use the data on 2006 revenue-based market shares for the top eight firms given in Ivaldi et al. (2012, Table 4.2) and assume power-parameterized cost distributions $G_i(c) = 1 - (1-c)^{\alpha_i}$ defined on $[0,1]$. It is then straightforward to write each supplier i's revenue share, $R_i / \sum_{j=1}^n R_j$, in terms of the parameters of the cost distributions (for details, see Appendix D.1). For identification, we assume that Ivaldi et al.'s "other" category consists of six symmetric suppliers (which ensures that they are among

[23] Ivaldi et al. (2012, pp. 79–80) describe the process of retaining an auditor as involving competitive bidding, which provides the foundation for a procurement model. In addition, they note that buyer power does not seem to be a strong force on the audit market (Ivaldi et al., 2012, p. 81), supporting the use of an efficient procurement model.

[24] CFI, 6 June 2002, case T-342/99, *Airtours v. Commission of the European Communities.*

the smallest suppliers) and that the average cost parameter is equal to one, which pins down the level of the parameters.

Using the calibrated cost distributions, we can then calculate the critical shares and $\mathcal{I}^S$(Big 4), which are shown in Table 1. We find that $\mathcal{I}^S$(Big 4) is positive, which implies that, consistent with the findings of Ivaldi et al. (2012), the market is at risk for coordination among the Big 4 and that Mazars is not a maverick (because Mazars' presence in the market does not prevent the market from being at risk for coordination).

Table 1: Calibration and analysis of coordination in the French audit industry

		Revenue-based shares	α_i	s_i(Big 4)	$\mathcal{I}^S$(Big 4)
Big 4	Ernst & Young	29.8%	3.8668	0.1289	0.5670
Big 4	Deloitte	21.4%	2.9934	0.1052	
Big 4	KPMG	22.2%	3.0820	0.1077	
Big 4	PWC	17.2%	2.5061	0.0913	
	Mazars	7.3%	1.1819		
	Grant Thornton	0.4%	0.0703		
	BDO	0.2%	0.0352		
	Constantin	0.3%	0.0528		
	6 others	0.2% each	0.0352		
	Total	100%	14		

To analyze the stability of coordination, we also calculate the coordinated effects indices for various groups of coordinating suppliers, as shown in Table 2. As the table shows, coordination by any three of the Big 4 is stable (the coordinated effects index for any three of the Big 4 is positive, but for any two is negative). The table also shows that the market is not at risk for pairwise coordination among any of the Big 4 firms (and the market is not at risk for pairwise coordination between Mazars and any one of the Big 4).

Overall our results are consistent with those of Ivaldi et al. (2012)—we find that the market is at risk for coordination by the Big 4 and that Mazars is not a maverick—but our analysis also points to the perhaps greater concern of coordination among subsets of three of the Big 4. Although the market is at risk for coordination by the Big 4, each individual member of the Big 4 would prefer not to coordinate if the alternative were for the remaining three firms to coordinate. But for any subset of three of the Big 4 firms, each firm is pivotal to the feasibility of coordination.

Table 2: $\mathcal{I}^S(K)$ for various sets K of coordinating suppliers. Firms are: 1. Ernst & Young, 2. Deloitte, 3. KPMG, 4. PWC, 5. Mazars.

K	$\mathcal{I}^S(K)$	K	$\mathcal{I}^S(K)$	K	$\mathcal{I}^S(K)$
{1,2,3,4}	0.5670	{1,2}	−0.0691	{1,2,3,4,5}	0.6115
{1,2,3}	0.2905	{1,3}	−0.0640	{1,2,5}	0.0103
{1,2,4}	0.2852	{1,4}	−0.1135	{1,3,5}	0.0172
{1,3,4}	0.2852	{2,3}	−0.1044	{1,4,5}	−0.0567
{2,3,4}	0.2917	{2,4}	−0.1710	{2,3,5}	−0.0357
		{3,4}	−0.1617	{2,4,5}	−0.1420
				{3,4,5}	−0.1271

4 Extensions

As foreshadowed by the overview in Section 2, our basic approach is equally applicable to alternative industrial organization models. To illustrate this, we now provide an extension to Cournot competition, and then reconsider procurement auctions first under the assumption that the buyer uses an optimal auction and then that the buyer uses a first-price auction.

4.1 Cournot

We now consider a Cournot model in which suppliers have constant marginal costs $(c_1, ..., c_n)$ that are common knowledge. The inverse demand function for aggregate quantity $Q \in [0, 1]$ is $P(Q) = 1 - Q$. We assume that costs are such that all suppliers are active in equilibrium without coordination.[25] For Cournot, a supplier is "stronger" if it has a lower cost. Being stronger is equivalent to having to a larger market share—in a Cournot market with n suppliers whose total cost is $C \equiv \sum_{j=1}^{n} c_j$, supplier i's market share is $(1+C-(n+1)c_i)/(n-C)$, which is decreasing in supplier i's cost.

To model coordination, we assume that $\Pi_i(K)$ is equal to supplier i's Cournot payoff when only supplier i and suppliers in $N\backslash K$ are present in the market. Below we discuss an alternative formulation in which we hold fixed the output of suppliers in $N\backslash K$ at their levels under Cournot competition involving all suppliers in N. This alternative amounts to assuming that coordination is not observed by outsiders, whereas the setup we study for now corresponds to assuming that outsiders are aware of coordination. We denote the coordinated effects index based on Cournot by $\mathcal{I}^C(K)$. Based on this index, the definition

[25] Farrell and Shapiro (1990) consider unilateral effects of mergers in a Cournot setup and provide conditions, which are satisfied in our Cournot setup with linear demand and constant marginal cost, under which a merger that involves no synergies must increase the price. For the relevance of the linear-demand, constant-marginal-cost Cournot setup in an empirical context, see, e.g., Igami and Sugaya (2019).

of a maverick carries over as before—a maverick is a supplier whose presence means that a market is not at risk for coordination, but whose removal would render the market at risk for coordination.

To model mergers, we assume that a merger results in the elimination of the merging supplier with the higher cost, or if the two merging suppliers have the same cost, then a merger simply eliminates one of them. As this suggests, a driving force for differences between merger effects in Cournot and procurement models is the assumption of constant marginal costs in the Cournot model. This means that the effect of a merger in the Cournot model is the elimination of the productive assets of the weakly less efficient merging supplier, which leaves the productivity of the merged entity the same as that of the weakly more efficient merging supplier.[26] In the procurement model, by contrast, the productivity of the merged entity is enhanced relative to that of either of the individual merging suppliers.

The gist of Proposition 1 extends to the Cournot setup insofar as coordination reduces consumer surplus and social surplus. The suppression of suppliers reduces the total quantity, which increases the price.

Turning to the coordinated effects index, letting $C_X \equiv \sum_{i \in X} c_i$, we have the following lemma:

Lemma 4. *For Cournot,* $\mathcal{I}^C(N) > 0$ *and*

$$\mathcal{I}^C(K) = 1 - \sum_{i \in K} \frac{(1-(n+1)c_i + C_N)^2}{(1-(n-k+1)c_i + C_{N \setminus K})^2} \frac{(n-k+2)^2}{(n+1)^2}. \tag{6}$$

Proof. See Appendix A for the proof of (6) and Appendix C for the proof that $\mathcal{I}^C(N) > 0$.

It follows from Lemma 4 that for Cournot, Proposition 2 also continues to hold: some, but not all, Cournot markets, are at risk for coordination. However, as shown in Lemma 4, Cournot markets are always at risk for all-inclusive coordination, which contrasts with the case of an efficient procurement market. With symmetric suppliers in K, regardless of symmetry among the costs of the outside suppliers,

$$\mathcal{I}^C(K) = 1 - k\left(\frac{n-k+2}{n+1}\right)^2, \tag{7}$$

[26] The role the assumption of constant marginal costs plays for this results is worth mentioning. As noted in McAfee and Williams (1992), in the Cournot model, a merger would no longer amount to the elimination of a supplier if, for example, supplier i's cost function for producing quantity q_i were q_i^2/κ_i, where κ_i is i's capacity. It that case, a merger of suppliers i and j would result in a supplier whose cost was only $q_i^2/(\kappa_i + \kappa_j)$. Related to this model of costs in a Cournot setup, see also Perry and Porter (1985); Farrell and Shapiro (1990); Whinston (2006).

which is positive if and only if $n < k - 1 + \sqrt{k}$. We illustrate the implications of this in Figure 1, which displays the combinations of n and k such that coordination is profitable in the symmetric Cournot model.

(a) Symmetric Cournot markets at risk

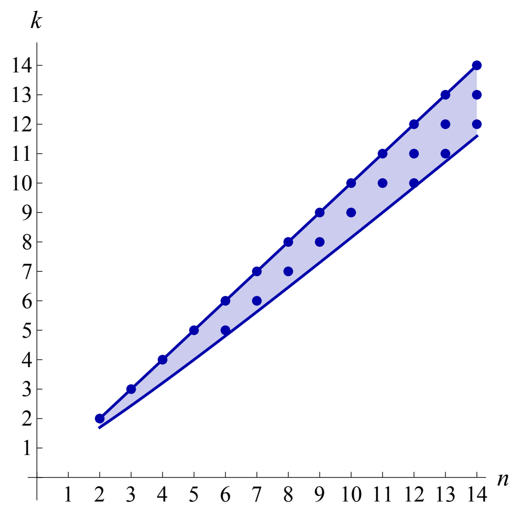

Figure 1: Values of n and k such that a linear demand Cournot market is at risk for coordination by a subset of k symmetric suppliers, i.e., $\mathcal{I}^C(K) > 0$.

In the Cournot model, coordination between two firms with identical constant marginal costs is equivalent to the two firms merging. In light of this and of the well-known Cournot merger paradox, according to which a merger without synergies only pays off if it is a merger to monopoly (Salant et al., 1983; Perry and Porter, 1985), we obtain the expected result that coordination by two firms only pays off if the coordination is all inclusive, that is, if there are only two firms. With the Cournot merger paradox in mind, consistent with the prior literature, partial coordination can be profitable when more than two firms coordinate. Specifically, for n greater than 5, partial coordination by k firms with $k < n$ can be profitable. For example, when $n = 6$, the market is at risk for coordination by conglomerations that consist of 5 of the 6 suppliers. Intuitively, the number of coordinating firms needs to be large enough for the insiders to coordination to benefit sufficiently from the public good their coordination provides, which accrues to the outsiders free of any cost. Although the number of suppliers that can remain outside and still allow coordination to be profitable increases with n, in the limit as n goes to infinity, the share of suppliers that must be included in coordination in order for a market to be at risk goes to 1.[27] This is consistent with the Cournot merger paradox because it only consider mergers between two firms at a time, whereas coordination may naturally involve multiple firms.

[27]The threshold value of k implied by Proposition 2 is $\hat{k}(n) \equiv \frac{1}{2}(3+2n-\sqrt{5+4n})$ and $\lim_{n\to\infty} \hat{k}(n)/n = 1$.

With symmetric suppliers, a rather specific set of circumstances is required for a merger to cause a Cournot market that is not at risk to become at risk. The merger, which reduces the number of suppliers from n to $n-1$, must leave the number of coordinators k unchanged.[28] This means that $k < n$ has to be the case and that the merger must be between a supplier in K and a supplier outside K, with the merged entity remaining part of K. Indeed, the merger must involve a maverick. It follows that the key concern in Cournot markets is the acquisition of a maverick by a supplier in K, as shown in the following proposition. In the statement of the proposition, we use the term "acquisition" of a supplier outside K by a supplier inside K to indicate that the number of suppliers in K is unchanged, with the merged-entity replacing the acquiring supplier and the acquired supplier being eliminated.

Proposition 5. *For a Cournot market, the acquisition of a maverick by a supplier in K puts the market at risk for coordination if the maverick's marginal cost is no less than that of the acquiring supplier. Moreover, with symmetric suppliers, the only merger that can cause a market that is not at risk for coordination by suppliers in K to become at risk for coordination by those suppliers is the acquisition of a maverick.*

Proof. See Appendix A.

Note that the assumption that the maverick firm has a higher marginal cost than the acquirer is consistent with the popular notion that mavericks are "small" firms. In that sense, Proposition 5 provides a foundation for a maverick-based merger review in Cournot markets. In order to have a maverick with respect to K, of course we must be talking about a potential set of coordinators that is not all inclusive, with the maverick and possibly other suppliers outside K. However, in order for the market to be at risk for coordination once the maverick is incorporated into one of the suppliers in K, it also must be that there are not too many other suppliers outside K. For symmetric suppliers, Figure 1 shows that there is a narrow band of (n, k) pairs such that a maverick exists. It must be that (n, k) lies outside the shaded area in the figure, but $(n-1, k)$ lies inside the shaded area. This would be the case, for example, with $(n, k) \in \{(3, 2), (4, 3), (5, 4)\}$, where the acquisition of the maverick eliminates all outside competition, or with, for example, $(n, k) \in \{(7, 5), (8, 6)\}$, where the acquisition of the maverick leaves one remaining outside supplier.

For symmetric Cournot, increasing the number of outside suppliers also eventually prevents a market from being at risk—using (7), $\lim_{n\to\infty} \mathcal{I}^C(K) = 1 - k < 0$. The intuition is the same as for the procurement model: Coordination is the provision of a public good whose costs are borne by the insiders and whose benefits accrue to all active suppliers, which

[28]For symmetric suppliers and $i \in K$, if $\mathcal{I}^C(K; N) \leq 0$, then $\mathcal{I}^C(K\backslash\{i\}; N\backslash\{i\}) < 0$.

include the designated supplier among the insiders and all outsiders. For coordination to pay off, it must be the case that the insiders internalize enough of the benefits their coordination generates, which requires the outside competition to be limited.

In contrast to the positive assortative matching obtained for the procurement model, Lemma 4 implies that for the Cournot model, supplier i's critical share is decreasing in its own cost but increasing in the cost of the other coordinating suppliers. As a result, for some parameters, the largest coordinated effects index for coordination between two suppliers (i.e., $k = 2$) can occur for the lowest-cost supplier coordinating with the highest-cost supplier.[29] Thus, in the Cournot model, we have negative assortative matching for coordinators in some cases, suggesting that in Cournot markets, competition authorities should be attentive to the possibility of coordination between combinations of small and large suppliers.

Proposition 5 leaves open the question of what happens to the risk for coordination when the maverick is more efficient than its acquirer. With a lower-cost maverick, it is straightforward to construct examples that go either way. By definition, the *elimination* of the maverick results in a positive index, but the reduction in the acquirer's cost—who now produces using the maverick's production technology and shuts down its own production facility—increases its payoffs both with and without coordination and so can increase the acquirer's critical share and decrease the index.[30]

Unobserved coordination

In a Cournot setting, the profitability of coordination is affected by whether outside suppliers optimally react to coordination or continue to use their pre-coordination strategies. (Dominant strategies eliminate this issue for efficient procurement markets.) Letting $\mathcal{I}_0^C(K)$ denote the coordinated effects index for a Cournot market in which outsiders do not adjust their quantities in light of coordination, we have $\mathcal{I}_0^C(K) \geq \mathcal{I}^C(K)$. Further, because we can relate payoffs from unobserved coordination to payoffs from observed all-inclusive coordination, but with a different demand intercept, Lemma 4 implies that for all K, $\mathcal{I}_0^C(K) > 0$. This implies that there can be no mavericks when coordination is not observed. There is also no harm from mergers for symmetric suppliers in K because under symmetry, $\mathcal{I}_0^C(K) = \left(\frac{k-1}{k+1}\right)^2$, which implies that a merger of suppliers in K reduces the coordinated effects index and other mergers have no effect.

More formally, given $i \in J \subseteq N$ and $a > 0$, let $\Pi_i(J, a)$ denote the Cournot profit of

[29] This occurs, for example, when $n = 3$ and $c_1 = 0$, $c_2 = 0.1$, and $c_3 = 0.2$, in which case $\mathcal{I}^C(\{1,3\}) > \max\{\mathcal{I}^C(\{1,2\}), \mathcal{I}^C(\{2,3\})\}$.

[30] To see that the acquisition of a maverick can result in a market that is not at risk, consider a setup with $c_1 = ... = c_{n-1} = 0.1$ and $c_n = 0$, where $K = \{1, ..., K\}$. If $n = 43$ and $k = 37$, then $\mathcal{I}^C(K; N) < 0$ and $\mathcal{I}^C(K; N \backslash \{n\}) > 0$, so supplier n is a maverick with respect to K. But when supplier 1's cost is reduced to zero, which is the maverick's cost, then $\mathcal{I}^C(K; N \backslash \{n\}) < 0$, so the post-acquisition market is not at risk.

supplier i when only the suppliers in J are active and when demand is $P = a - Q$. Given $i \in K \subseteq J \subseteq N$, and $a > 0$, let $\Pi_i(K; J, a)$ and $\mathcal{I}^C(K; J, a)$ be the profit of supplier i when it is the only supplier in K to be active and the index for coordination by suppliers in K, respectively, when only suppliers in J are active and demand is $P = a - Q$. Then given $i \in K \subseteq N$, $\Pi_i(N, 1) = \Pi_i(K, \hat{a})$, where

$$\hat{a} \equiv 1 - \sum_{j \in N \backslash K} q_j^C(\mathbf{c}) = \frac{1 + k - (n-k)C_N}{n+1} + C_{N \backslash K}.$$

In addition, letting $\hat{\Pi}_i(K; N, 1)$ the profit of supplier i when it is the only supplier in K to be active and suppliers in $N \backslash K$ choose their Cournot quantities associated with a market where all suppliers in N are active, then $\hat{\Pi}_i(K; N, 1) = \hat{\Pi}_i(K; K, \hat{a}) = \Pi_i(K; K, \hat{a})$, where the last equality holds because observability does not affect all-inclusive coordination. Thus, for $i \in K \subseteq N$,

$$\mathcal{I}_0^C(K) = 1 - \sum_{i \in K} \frac{\Pi_i(N, 1)}{\hat{\Pi}_i(K; N, 1)} = 1 - \sum_{i \in K} \frac{\Pi_i(K, \hat{a})}{\Pi_i(K; K, \hat{a})} = \mathcal{I}^C(K; K, \hat{a}) > 0,$$

where the final inequality uses the result of Lemma 4 that a Cournot market is at risk for all-inclusive coordination.

4.2 Powerful buyers

We now revisit procurement markets to consider the effects of buyer power. In the procurement context, it is natural to consider the possibility of powerful buyers that can employ potentially inefficient procurement procedures in order to maximize their payoffs. This contrasts with the Cournot model, where the assumption of price-taking buyers leaves no scope for buyer power.

Intuitively, one expects buyer power to reduce the risk for coordination because powerful buyers rely less on rivalry among bidders than buyers without power to obtain favorable prices. It is also in line with antitrust practice.[31] As mentioned in the introduction, there is a notion in the *U.S. Guidelines* (p. 27) that "the conduct or presence of large buyers" could undermine coordinated effects.[32] Yet, to our knowledge, there has been no formalization of

[31] The *U.S. Guidelines* (p. 27) state that the agencies "consider the possibility that powerful buyers may constrain the ability of the merging parties to raise prices. This can occur, for example, ... if the conduct or presence of large buyers undermines coordinated effects." The *EC Guidelines* discuss the possibility that "buyer power would act as a countervailing factor to an increase in market power resulting from the merger" (para. 11). The Australian *Merger Guidelines* view "countervailing power" as a competitive constraint that can limit merger harms (paras. 1.4, 5.3, 7.48).

[32] *U.S. Guidelines* (p. 27) also state: "In some cases, a large buyer may be able to strategically undermine coordinated conduct, at least as it pertains to that buyer's needs, by choosing to put up for bid a few large

this intuitive idea in the literature.

Of course, in our setup with single-unit demand, there is no natural notion of buyer size. We therefore adhere to the view that large buyers are *powerful* in the sense that, as in Loertscher and Marx (2019b), they use—the dominant strategy implementation of—the optimal mechanism, that is, the mechanism that maximizes the buyer's expected profit subject to suppliers' dominant strategy incentive compatibility and individual rationality constraints. The optimal mechanism contrasts with the efficient mechanism used by buyers without power, which we analyzed in Section 3.[33] Here, we assume that buyer power itself is not affected by a merger among suppliers. This is natural if buyer power derives from the size and/or sophistication of the buyer, as suggested by the *EC Guidelines* (para. 65), or from the ability to vertically integrate upstream or sponsor entry, as suggested by the *U.S. Guidelines* (p. 27), and if the merger does not increase the suppliers' ability to affect the price formation process.[34]

In the optimal procurement, the buyer ranks suppliers according to their virtual costs, defined as

$$\Gamma_i(c) \equiv c + \frac{G_i(c)}{g_i(c)}, \tag{8}$$

and applies a supplier-specific reserve price to the supplier with the lowest virtual cost. We impose the standard regularity assumption that Γ_i is increasing.[35] Because we allow the possibility that the densities are zero at $\underline{c}$ (and also possibly at $\overline{c}$), define $\Gamma_i(\underline{c}) = \lim_{c \to \underline{c}} \Gamma_i(c) = \underline{c}$. For $x > \Gamma_i(\overline{c})$, we define $\Gamma_i^{-1}(x) \equiv \overline{c}$. The reserve price that a powerful buyer applies to supplier i is $\hat{r}_i \equiv \Gamma_i^{-1}(v)$. (In the case of symmetric suppliers, we drop the subscript i on $\hat{r}$.) Because $\Gamma_i(c) > c$ for $c > \underline{c}$, it follows that $\hat{r}_i < v$.

As noted by Loertscher and Marx (2019b), buyer power consists of two components: the ability to *discriminate* between suppliers and the commitment to cancel a procurement even though it would be profitable, which may be called *monopsony power*. Whether the buyer optimally exerts one or both of these powers depends on the problem at hand. When all

contracts rather than many smaller ones, and by making its procurement decisions opaque to suppliers."

[33] Loertscher and Marx (2019b) build on Myerson (1981) and apply his mechanism design approach to merger analysis using the same notion of designers with and without power as Bulow and Klemperer (1996).

[34] The *EC Guidelines* raise the possibility that a merger could reduce buyer power "because a merger of two suppliers may reduce buyer power if it thereby removes a credible alternative" (*EC Guidelines*, para. 67); Loertscher and Marx (2019a) provide a formalization of countervailing powers along these lines. A nuance on the view that mergers decrease buyer power is provided by Loertscher and Marx (2019b), who observe that, with symmetric suppliers, a merger *increases* the buyer's incentive to become powerful.

[35] An intuitive interpretation of the virtual cost function and an understanding of the role of its monotonicity can be developed using standard monopsony pricing. Consider a buyer with value $v \leq \overline{c}$ who faces a single supplier i who draws his cost from the distribution G_i. The buyer's pricing problem is $\max_p (v - p) G_i(p)$, the first-order condition for which is $0 = g_i(p)(v - \Gamma_i(p))$. If Γ_i is increasing, the second-order condition is satisfied if the first-order condition is, i.e., the problem is quasi-concave.

suppliers are ex ante symmetric, that is, when $G_i = G$ for all $i \in N$, there is no point for the buyer to discriminate because ranking the suppliers according to their virtual costs is the same as ranking them according to their costs. Without ex ante symmetry, the buyer will optimally use its power to discriminate some of the time. The buyer will optimally refrain from ever using its monopsony power if and only if $v > \min_{i \in N}\{\Gamma_i(\overline{c})\}$.

The assumption that suppliers follow their weakly dominant strategies of reporting truthfully implies that if supplier i has the lowest virtual cost, then supplier i wins if $c_i \leq \hat{r}_i$ and is paid $\min_{j \neq i}\{\hat{r}_i, \Gamma_i^{-1}(\Gamma_j(c_j))\}$. Otherwise, there is no trade. Consequently, when the buyer is powerful,

$$\Pi_i = \mathbb{E}\left[\max\{0, \min_{j \in N \setminus \{i\}}\{\hat{r}_i, \Gamma_i^{-1}(\Gamma_j(c_j))\} - c_i\} \cdot \mathbf{1}_{\Gamma_i(c_i) \leq \min_{j \in N \setminus \{i\}} \Gamma_j(c_j)}\right]$$

and

$$\Pi_i(K) = \mathbb{E}\left[\max\{0, \min_{j \in N \setminus K}\{\hat{r}_i, \Gamma_i^{-1}(\Gamma_j(c_j))\} - c_i\} \cdot \mathbf{1}_{\Gamma_i(c_i) \leq \min_{j \in K} \Gamma_j(c_j)}\right].$$

The definitions of critical shares and of the coordinated effects index are then the same as in the case without buyer power.

As in the case of an efficient procurement, with buyer power, the buyer is harmed by coordination, and some but not all markets are at risk for coordination. Interestingly, with buyer power, coordination can increase social surplus for some type realizations. This occurs with ex ante heterogeneous suppliers when, absent coordination, the buyer does not purchase from the lowest-cost supplier because it discriminates between suppliers on the basis of their virtual costs and purchases from a supplier with a lower cost when there is coordination because the bid of the supplier it buys from absent coordination is suppressed.

With buyer power, we continue to have the result that markets with sufficient outside competition are not at risk for coordination. Assuming ex ante symmetric pre-merger suppliers, a decrease in the critical share of the merged entity relative to the sum of the pre-merger critical shares of the merging suppliers is sufficient, but no longer necessary for the merger to increase $\mathcal{I}^S(K)$. Nevertheless, it remains the case that a merger can, but need not, cause a market not at risk to become so.

To examine how $\mathcal{I}^S(K)$ is affected by buyer power in our framework, we first abstract from mergers and focus on ex ante symmetric suppliers. In this case, a powerful buyer never uses its power to discriminate, so the sole effect of buyer power is to reduce the reserve price from r to $\hat{r}$, which allows us to focus on the effects of a change in the reserve.

Proposition 6. *Assuming symmetric suppliers, $\mathcal{I}^S(K)$ is increasing in the buyer's reserve price, and thus decreases with buyer power.*

Proof. See Appendix A.

To the best of our knowledge, Proposition 6 is the first formal demonstration that, consistent with perceived wisdom, buyer power reduces concerns of coordination.

Let us now turn to a merger. With buyer power, a merger of two suppliers in K increases the noncoordinated payoff of the other suppliers in K as a result of the buyer's more aggressive discrimination against the merged entity. However, a merger does not affect the payoff of a nonmerging supplier under coordination. Thus, we have the following result:

Proposition 7. *With buyer power, a merger of two suppliers in K increases the critical shares of the non-merging suppliers in K.*

Proposition 7 implies that with buyer power, a merger of two suppliers in K reduces the incentive for the other suppliers in K to participate in coordination. In the face of buyer power, a merger of suppliers in K constrains the ability of those suppliers to coordinate by increasing the critical shares of the nonmerging suppliers.

Propositions 6 and 7 provide a foundation for the view that coordinated effects from a merger are less of a concern in the face of powerful buyers. In Appendix D.2, we demonstrate the tractability of the analysis of markets with buyer power through an application to the oilfield services market. In addition, in Appendix D.3, we discuss the extension to allow powerful buyers in the setup with two-unit demand that underlies the application to the French audit industry in Section 3.5.

4.3 First-price auctions

For first-price procurements, we focus on the unique Bayes Nash equilibrium (Lebrun, 1999) and denote the coordinated effects index by $\mathcal{I}_0^F(K)$ if coordination is not observed by outsiders and $\mathcal{I}_1^F(K)$ if coordination is observed by outsiders. In the first-price setting, the presence of strategic complements implies that $\mathcal{I}_1^F(K) \geq \mathcal{I}_0^F(K)$. It will be useful in what follows to note that the payoff equivalence theorem (see, e.g., Myerson, 1981; Krishna, 2002; Börgers, 2015) implies that under symmetry, suppliers' uncoordinated payoffs are the same under second-price and first-price procurements. If, in addition, coordination is observable, then the coordinated payoffs are also the same under second-price and first-price procurements because then, letting k be the number of coordinating suppliers, we simply have competition among $n - k + 1$ symmetric suppliers. Thus, for symmetric suppliers,

$$\mathcal{I}^S(K) = \mathcal{I}_1^F(K) \geq \mathcal{I}_0^F(K),$$

with a strict inequality if and only if $K \neq N$. This formalizes the commonly held notion that first-price procurements are less susceptible to coordination than second-price procurements. It shows that this is, indeed, the case for non-all-inclusive coordination among symmetric suppliers when coordination is not observable to the outside suppliers. However, it also shows that for symmetric suppliers, a first-price procurement provides no additional protection from coordination relative to a second-price procurement if the coordination is all-inclusive and/or observable.

As in the other models considered, in the first-price procurement model, the suppression of bids causes the designated supplier to increase its bid for every cost $c \in (\underline{c}, r)$, increasing the buyer's expected price, so Proposition 1 extends to this setup.

4.4 Vertical integration

Another question of concurrent interest concerns the effects of vertical integration on the risk for coordination.[36] To shed light on this question, we reconsider a procurement market and stipulate that there are $n \geq 3$ ex ante symmetric suppliers who draw their costs independently from the distribution G with density $g(c) > 0$ for $c \in [\underline{c}, \overline{c}]$. The exposition simplifies by assuming that G is such that for any $a \in [0,1]$, the functions $\Gamma_a(c) \equiv c + a\frac{G(c)}{g(c)}$ and $\Phi_a(c) \equiv c - a\frac{1-G(c)}{g(c)}$ are increasing in c. Let $v \geq \overline{c}$ be the buyer's willingness to pay without integration. This implies that prior to integration, the reserve price in the second-price auction is $\overline{c}$. Without loss of generality, assume that the buyer integrates with supplier 1. This implies that after integration, the buyer's willingness to pay for the service or product of the independent suppliers is c_1. By a straightforward extension of the impossibility theorem of Myerson and Satterthwaite (see, e.g., Gresik and Satterthwaite, 1989; Delacrétaz et al., 2019), this implies that post-integration ex post efficient trade is impossible without running a deficit.[37] It is then natural to focus, post-integration, on the second-best mechanism that maximizes ex ante expected social surplus subject to incentive compatibility, individual rationality, and budget constraints. As is well known (see, again, e.g. Gresik and Satterthwaite, 1989), this mechanism is characterized by an allocation rule that induces trade from the lowest-cost independent supplier i to the integrated firm if and only if $\Phi_{a^*}(c_1) \geq \Gamma_{a^*}(c_i)$, where $a^* \in (0,1)$.[38] For any $K \subset N \setminus \{1\}$, denote by $\mathcal{I}^S(K)$ and $\mathcal{I}^S_{VI}(K)$ coordinated effects index

[36] "FTC and DOJ Announce Draft Vertical Merger Guidelines for Public Comment," Press Release, U.S. Federal Trade Commission, January 10, 2020.

[37] See Loertscher and Marx (2019a) for a more detailed analysis of vertical integration and the efficiency of the price formation process.

[38] For our purposes, the derivation of a^* and its precise size, above and beyond $a^* > 0$, does not matter. It is the smallest number $a \in [0,1]$ such that an incentive compatible and individually rational mechanism that is based on the allocation rule that induces trade between the lowest-cost independent supplier i and the integrated firm if and only if $\Phi_a(c_1) \geq \Gamma_a(c_i)$ does not run a deficit.

before and after integration, respectively.

Intuitively, as far as the efficient allocation is concerned, vertical integration has no effect under the assumptions we stipulate in the sense that under ex post efficiency for any given cost realization it is the same supplier that produces. Hence, if the allocation were still efficient following integration, then vertical integration would have no impact on the risk for coordination. However, because vertical integration induces a Myerson-Satterthwaite problem, the allocation following integration is no longer efficient. Qualitatively, the second-best mechanism has the same effects as endowing the buyer with buyer power. To see this, notice that any allocation rule according to which trade occurs between the integrated firm and the independent supplier with the lowest cost if and only if $\Phi_a(c_1) \geq \Gamma_a(c_i)$ is implementable via a second-price auction in which the integrated buyer sets the reserve $p_a(c_1) \equiv \Gamma_a^{-1}(\Phi_a(c_1))$, which is decreasing in a and satisfies $p_0(c_1) = c_1$. Thus, for a given realization of c_1, vertical integration has the same effect as having the buyer set a more aggressive reserve. As we know from Proposition 6, buyer power reduces the risk for coordination among symmetric suppliers. Hence, from Proposition 6 follows Corollary 1:

Corollary 1. *Vertical integration decreases the risk for coordination by any subset of independent, symmetric suppliers. That is, for $v \geq \bar{c}$, $G_i = G$ for all $i \in N$ such that $\Phi_a(c)$ and $\Gamma_a(c)$ are increasing in c for any $a \in [0, 1]$ and $K \subset N \setminus \{1\}$, we have $\mathcal{I}^S(K) > \mathcal{I}^S_{VI}(K)$.*

To our knowledge, Corollary 1 is the first instance of a formal connection between buyer power and vertical integration. Although merger guidelines refer to vertical integration as a *source* of buyer power,[39] the independent private values framework that embeds our setup does not distinguish between what determines the buyer's, or more generally, the designer's, value. Either the designer has the power to choose the profit-maximizing mechanism, or it does not, irrespective of its value. Nevertheless, as the above shows, in the context of coordinated behavior when the suppliers are ex ante symmetric, vertical integration has qualitatively the same effects as buyer power.

5 Tension between unilateral and coordinated effects

Our clear-cut definition and measure of a market being at risk of coordination also brings to light a possible tension between unilateral and coordinated effects. To see this, reconsider the efficient procurement market. Because an NR spread results in a spread of the affected suppliers' market shares, without changing their total market share, an NR spread increases the Herfindahl-Hirschman Index (HHI), which is a widely used measure of merger effects that is defined as the sum of the squared market shares.

[39]For example, the *U.S. Guidelines* (p. 27) describe the "ability and incentive to vertically integrate upstream" as a possible source of buyer power.

As shown in Section 3.4, for the power-based parameterization, an NR spread applied to two firms within K decreases $\mathcal{I}^S(K)$. In this sense, an NR spread is good medicine because it decreases the risk of coordination. At the same time, an NR spread implies that $H_iH_j < G_iG_j$,[40] that is, the distribution of the higher of the two cost draws after the spread first-order stochastically dominates the distribution of the higher of the two cost draws before the spread. This is illustrated in Figure 2. It implies that the buyer's expected price increases, and the more so the larger is the spread. Thus, an NR spread is bad medicine if the firms bid competitively because it harms the buyer.

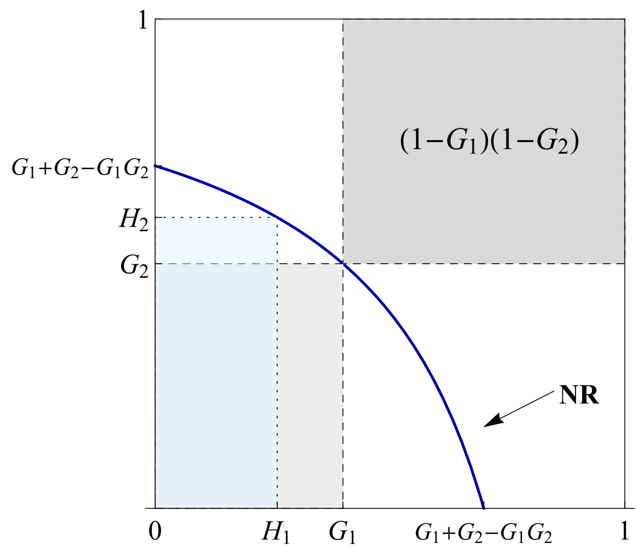

Figure 2: Illustration of a symmetry-reducing NR change from (G_1, G_2) to (H_1, H_2). As illustrated by the shaded rectangles below the NR curve, for any symmetry-reducing NR change, $H_1H_2 < G_1G_2$.

We summarize these results in the following proposition, where given an NR spread from (G_1, G_2) to (H_1, H_2), we say that we have a *larger* NR spread if we have a change from (G_1, G_2) to $(\hat{H}_1, \hat{H}_2)$, where $(\hat{H}_1, \hat{H}_2)$ is an NR spread of (H_1, H_2):

Proposition 8. *An NR spread has the unilateral effect of increasing the HHI and the buyer's expected price, with a larger NR spread resulting in both a larger increase in the HHI and a larger increase in the expected price.*

Proof. See Appendix A.

Proposition 8 resonates with the idea that the presence of a supplier with a "dominant

[40]Indeed, the result that $H_iH_j < G_iG_j$ is implied by any symmetry reducing change, where a change in distributions for suppliers 1 and 2 from (F_1, F_2) to (H_1, H_2) is *symmetry reducing* if, for all $c \in [\underline{c}, \overline{c}]$, we have $\max_i\{H_i(c)\} \geq \max_i\{F_i(c)\}$ and $\min_i\{H_i(c)\} \leq \min_i\{F_i(c)\}$, with strict inequalities for costs in an open subset of $[\underline{c}, \min\{v, \overline{c}\}]$.

position" harms buyers and that greater dominance results in greater harm.[41] It is noteworthy that the buyer harm from a decrease in symmetry identified in Proposition 8 arises irrespective of the sizes of the suppliers under consideration relative to their rivals—any NR spread increases the expected price.

Interestingly, while Proposition 8 shows that an NR spread increases the HHI, the discussion in Section 3.4 (and associated formalization in Proposition B.4) shows that for the power-based parameterization, an NR spread applied to suppliers in K reduces $\mathcal{I}^S(K)$. In contrast to the positive and reaffirming news from Proposition 8 for the use of the HHI as in indicator of unilateral effects, the fact that a NR spread applied to two suppliers can decrease the coordinated effects index implies that a larger change in the HHI need not be indicative of an increase in risk for coordination as measured by the coordinated effects index. That is, the HHI is not a reliable indicator of coordinated effects, except inversely so for efficient procurement markets. This raises concerns given competition authorities' historical reliance on the HHI as an indicator of coordinated effects.[42]

The contrast between unilateral effects and the risk for coordinated effects extends to dynamic considerations. For example, Nocke and Whinston (2013) show that in a dynamic setting, a competition authority might have an incentive to block a merger with a large supplier in order to induce a merger with smaller supplier that is less harmful in terms of unilateral effects. In contrast, our results show that when considering the risk of coordinated effects, a competition authority might have an incentive to block a large firm from acquiring a smaller firm in order to induce it to instead acquire a larger firm, with a resulting lower coordinated effects index.

The inherent conflict between unilateral and coordinated effects is particularly salient in the context of merger remedies. For example, a competition authority considering requiring a divestiture as a condition for merger approval faces the dilemma that a divestiture that results in relatively more symmetric post-merger suppliers reduces concerns of unilateral effects, but increases concerns of coordinated effects, and conversely for a divestiture that results in relatively less symmetric post-merger suppliers.[43] Letting a "merger plus divestiture" be a transaction that takes two suppliers and reorganizes them to create two different suppliers:

Corollary 2. *A merger plus divestiture involving suppliers in K that results in an NR spread increases the HHI, but, for the power-based parameterization, it decreases $\mathcal{I}^S(K)$.*

[41] Regarding the possibility of a "significant impediment to effective competition," the *EC Guidelines* (para. 2) states, "The creation or the strengthening of a dominant position is a primary form of such competitive harm." See also Compte et al. (2002).

[42] See, for example, the U.S. Department of Justice & Federal Trade Commission's "Commentary on the 1992 Horizontal Merger Guidelines," 2006 (http://www.ftc.gov/os/2006/03/CommentaryontheHorizontalMergerGuidelinesMarch2006.pdf).

[43] See Loertscher and Marx (2019c) for further analysis of mergers plus divestitures.

While our measure of risk for coordination focuses on the individual rationality of coordination, Igami and Sugaya (2019) show that when one measures coordinated effects based on the incentive compatibility constraint, it is again possible for the HHI and the incentive to collude to go in opposite directions.

For the case of Cournot competition, it is straightforward to construct examples in which an NR spread moves the HHI and $\mathcal{I}^C(K)$ in opposite directions. For example, let $n = 3$, $c_1 = 0$, $c_2 = 0.01$, and $c_3 = 0.02$, with the spread being to reduce c_2 to $c_2' = 0$ and increase c_3 to $c_3' = 0.03$. Because this is an NR spread, the HHI increases, but the effect on the coordinated effects index depends on the set of coordinating suppliers: $\mathcal{I}^C(K)$ decreases if $K = \{1, 2, 3\}$ and increases if $K = \{2, 3\}$. This raises the interesting point that, although suppliers contemplating a merger might have an incentive to manipulate their market shares (for example by shifting sales from one merging party to the other) in hopes of improving their chances of surviving a merger review, a manipulation that reduces concerns of unilateral effects based on the HHI could increase concerns of coordinated effects based on the coordinated effects index.

6 Conclusion

Modelling coordination as a selection scheme among coordinating suppliers, where only one of the coordinating suppliers is designated to participate in the market, we provide a framework that allows us to define and measure when a market is at risk for coordination and to analyze the potential coordinated effects of mergers, including effects related to maverick firms.

Notions of coordinated effects and mavericks have significant traction in merger analysis and litigation. A concern is that a merger can put a market at risk for coordination, to the detriment of consumers and society. In analyzing coordinated effects, competition authorities have embraced mavericks as a useful construct. Also viewed as relevant for coordinated effects are the role of outside competition, supplier strength, supplier symmetry, and market wide trends. In this paper, we provide a framework that pulls together these components and that allows one to see where prior thinking was sound and where it was muddled, in some instances because it was, without being explicit about it, switching between different models of the price formation process. For example, merger guidelines promote the idea that buyer power lessens concerns of coordinated effects, and our framework concurs, but with the qualification or clarification that this is grounded in procurement-based thinking. At the same time, there is a view that mavericks play a critical role, but, as we show, that is grounded in Cournot-based thinking. Thus, a key contribution of our paper is to separate the overall framework for analyzing coordinated effects from the model of the price formation process. The framework is general. The price formation process is specific to a market.

A Appendix: Proofs

Proof of Lemma 1. In the power-based parameterization, assuming $v \geq \bar{c} = 1$, it is straightforward to show, letting $A_{-X} \equiv \sum_{j \in N \setminus X} \alpha_j$ and $A \equiv A_N$, that for $i \in K$,

$$\Pi_i = \frac{\alpha_i}{(1 + A_{-\{i\}})(1 + A)},$$

$$\Pi_i(K) = \frac{\alpha_i}{(1 + \alpha_i + A_{-K})(1 + A_{-K})},$$

and, thus,

$$s_i(K) = \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A_{-\{i\}})(1 + A)}.$$

The expression for $\mathcal{I}^S(K)$ then follows. ■

Proof of Lemma 2. Under symmetry and $v \geq \bar{c}$, the definition of $\Pi_i(N)$ implies that

$$\Pi_i(N) = \mathbb{E}_c \left[\max\{0, \bar{c} - c\} \right],$$

and the definition of Π_i (and the payoff equivalence theorem) implies that

$$\Pi_i = \frac{1}{n} \mathbb{E}_{\mathbf{c}} \left[\max\{0, c_{(2:n)} - c_{(1:n)}\} \right].$$

Using symmetry, the market is at risk for all-inclusive coordination if and only if coordination based on symmetric selection probabilities increases the expected surplus for all suppliers, i.e., for all $i \in N$, $\frac{1}{n}\Pi_i(N) > \Pi_i$, which holds if and only if

$$\mathbb{E}_c \left[\max\{0, r - c\} \right] > \mathbb{E}_{\mathbf{c}} \left[\max\{0, c_{(2:n)} - c_{(1:n)}\} \right].$$

This inequality is satisfied, for example, for the uniform distribution on $[0, 1]$ and $n = 2$. In this case, $\bar{c} - \mathbb{E}\left[c_i\right] = 1/2$ and $\mathbb{E}\left[c_{(2:n)} - c_{(1:n)}\right] = 1/3$. It is not satisfied, for example, when $n = 2$ and the density $g(c)$ is such that $g(c) = 0.05$ for $c \in [0, 0.9]$ and $g(c) = 9.55$ for $c \in (.9, 1]$, which has a long left tail and high probability close to the upper bound of support. Specifically, $1 - \mathbb{E}[c] = 0.0725$ and $\mathbb{E}\left[c_{(2:n)} - c_{(1:n)}\right] = 0.0740$. ■

Proof of Proposition 4. Using Lemma 1, in the power-based parameterization,

$$s_i(K; N) = \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A_{-\{i\}})(1 + A)},$$

where we augment the arguments of the critical share to include the set of all suppliers. Let $N = \{1, ..., n\}$ for some $n \in \{3, 4, ...\}$. Let $K = \{1, 2\}$ and assume that supplier $m \in \{3, ..., n\}$ is a maverick with respect to K. Define $X \equiv \sum_{i \in N \backslash \{1,2,m\}} \alpha_i$. By the definition of a maverick, $1 - \sum_{i \in K} s_i(K; N\backslash\{m\}) > 0$, which we can be written as

$$\sum_{i \in \{1,2\}} \frac{(1 + \alpha_i + X)(1 + X)}{(1 + \alpha_1 + \alpha_2 - \alpha_i + X)(1 + \alpha_1 + \alpha_2 + X)} - 1 < 0. \tag{9}$$

Following the merger of suppliers 1 and m, the coordinated effects index for suppliers in $\hat{K} \equiv \{\mu_{1,m}, 2\}$, where $\mu_{1,m}$ denotes the merged entity, is

$$\mathcal{I}^S(\hat{K}) = 1 - \frac{(1+\alpha_1+X+\alpha_m)(1+X)}{(1+\alpha_2+X)(1+\alpha_1+\alpha_2+X+\alpha_m)} - \frac{(1+\alpha_2+X)(1+X)}{(1+\alpha_1+X+\alpha_m)(1+\alpha_1+\alpha_2+X+\alpha_m)}.$$

It follows that

$$\lim_{\alpha_2 \to 0} \mathcal{I}^S(\hat{K}) = 1 - 1 - \frac{(1 + X)^2}{(1 + \alpha_1 + X + \alpha_m)^2} < 0,$$

which proves the second part of the proposition.

Using the above expression for $\mathcal{I}^S(\hat{K})$ and adding the expression on the left side of (9), which is negative, we have

$$\begin{aligned} \mathcal{I}^S(\hat{K}) &> -\frac{(1+\alpha_1+X+\alpha_m)(1+X)}{(1+\alpha_2+X)(1+\alpha_1+\alpha_2+X+\alpha_m)} - \frac{(1+\alpha_2+X)(1+X)}{(1+\alpha_1+X+\alpha_m)(1+\alpha_1+\alpha_2+X+\alpha_m)} \\ &\quad + \frac{(1+\alpha_1+X)(1+X)}{(1+\alpha_2+X)(1+\alpha_1+\alpha_2+X)} + \frac{(1+\alpha_2+X)(1+X)}{(1+\alpha_1+X)(1+\alpha_1+\alpha_2+X)} \\ &\equiv (1 + X) f(\alpha_1, \alpha_2, \alpha_m, X), \end{aligned}$$

where we factor out $(1+X)$ and define $f(\alpha_1, \alpha_2, \alpha_m, X)$ to be equal to the remainder. Thus, $\mathcal{I}^S(\hat{K}) > 0$ if $f(\alpha_1, \alpha_2, \alpha_m, X) > 0$. Collecting the terms in $f(\alpha_1, \alpha_2, \alpha_m, X)$ over the common denominator of

$$(1 + \alpha_1 + X)(1 + \alpha_2 + X)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X),$$

it follows that $\mathcal{I}^S(\hat{K}) > 0$ if the associated numerator,

$$\begin{aligned} \hat{f}(\alpha_1, \alpha_2, \alpha_m, X) &\equiv -(1 + \alpha_1 + X)(1 + \alpha_1 + X + \alpha_m)^2(1 + \alpha_1 + \alpha_2 + X) \\ &\quad +(1 + \alpha_1 + X + \alpha_m)(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_1 + X)^2 \\ &\quad -(1 + \alpha_2 + X)^2(1 + \alpha_1 + X)(1 + \alpha_1 + \alpha_2 + X) \\ &\quad +(1 + \alpha_1 + \alpha_2 + X + \alpha_m)(1 + \alpha_2 + X)^2(1 + \alpha_1 + X + \alpha_m), \end{aligned}$$

is positive. Differentiating with respect to X, it is straightforward to show that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, X)$ is convex in X and increasing at $X = 0$, which implies that for all $X \geq 0$, $\hat{f}(\alpha_1, \alpha_2, \alpha_m, X) \geq \hat{f}(\alpha_1, \alpha_2, \alpha_m, 0)$. Thus, it is sufficient to show that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, 0) > 0$. Straightforward calculations show that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, 0)$ is convex in α_2 and is positive and increasing in α_2 at $\alpha_2 = \alpha_1$, which implies that $\hat{f}(\alpha_1, \alpha_2, \alpha_m, 0) > 0$ for all $\alpha_2 \geq \alpha_1$, i.e., as long as the acquiring supplier has the weakly smaller distributional parameter, completing the proposition. ■

Proof of Lemma 4. Here we establish (6). (See Appendix C for the proof that $\mathcal{I}^C(N) > 0$.) In the asymmetric, linear demand Cournot model, $q_i = \frac{1+C_N-(n+1)c_i}{n+1}$ and $Q = \frac{n-C_N}{n+1}$, where $C_X \equiv \sum_{i \in X} c_i$. It follows that

$$\Pi_i = \frac{(1-(n+1)c_i + C_N)^2}{(n+1)^2}$$

and

$$\Pi_i(K) = \frac{(1-(n-k+2)c_i + C_{(N \setminus K) \cup \{i\}})^2}{(n-k+2)^2},$$

which implies that

$$s_i(K) = \frac{(1-(n+1)c_i + C_N)^2}{(1-(n-k+2)c_i + C_{(N \setminus K) \cup \{i\}})^2} \frac{(n-k+2)^2}{(n+1)^2}.$$

The expression in (6) then follows. ■

Proof of Proposition 5. The first part follows by the definition of a maverick and the assumption that a merger in a Cournot market eliminates the higher-cost supplier. The result for symmetric suppliers is an implication of (7). ■

Proof of Proposition 6. With buyer power and symmetric suppliers, letting $L_X(c)$ denote the distribution of the lowest cost among suppliers in X, i.e., $L_X(c) = 1 - (1 - G(c))^{|X|}$, for all $i \in K$,

$$s_i(K) = \frac{\int_{\underline{c}}^{\hat{r}} (1 - L_{N \setminus \{i\}}(c)) G(c) dc}{\int_{\underline{c}}^{\hat{r}} \left(1 - L_{N \setminus K}(c)\right) G(c) dc}. \tag{10}$$

If $v \geq \Gamma(\overline{c})$, then $\hat{r} = \overline{c}$, and so the critical shares are not affected by buyer power. Focusing on the case with $\hat{r} < \overline{c}$ and differentiating the expression in (10) with respect to $\hat{r}$, we get an expression with sign equal to the sign of

$$
\begin{aligned}
& (1-L_{N\setminus\{i\}}(\hat r))\int_{\underline{c}}^{\hat r}\left(1-L_{N\setminus K}(c)\right)G(c)c-\left(1-L_{N\setminus K}(\hat r)\right)\int_{\underline{c}}^{\hat r}(1-L_{N\setminus\{i\}}(c))G(c)dc \\
= \; & (1-G(\hat r))^{n-1}\int_{\underline{c}}^{\hat r}(1-G(c))^{n-k}G(c)dc-(1-G(\hat r))^{n-k}\int_{\underline{c}}^{\hat r}(1-G(c))^{n-1}G(c)dc \\
= \; & (1-G(\hat r))^{2n-1-k}\int_{\underline{c}}^{\hat r}\left\{\left(\frac{1-G(c)}{1-G(\hat r)}\right)^{n-k}-\left(\frac{1-G(c)}{1-G(\hat r)}\right)^{n-1}\right\}G(c)dc.
\end{aligned}
$$

Because $\frac{1-G(c)}{1-G(\hat r)}>1$ for $c<\hat r$ and because $k\geq 2$, which implies $n-k<n-1$, it follows that the expression above is negative. Thus, critical shares are weakly decreasing in buyer power, and strictly so for $v<\Gamma(\overline{c})$, which completes the proof. ■

Proof of Proposition 8. Suppose we have an NR spread for suppliers 1 and 2, causing their distributions to change from (G_1,G_2) to (H_1,H_2) satisfying (4) and (5) for $i=1$, $j=2$, and all $c\in[\underline{c},\overline{c}]$, with (4) satisfied with strict inequalities for costs in an open subset of $[\underline{c},\min\{v,\overline{c}\}]$. Letting p_1 and p_2 be the probabilities of trade for suppliers 1 and 2, respectively, prior to the NR spread and $\hat p_1$ and $\hat p_2$ be their probabilities after the NR spread, then $\hat p_1<\min\{p_1,p_2\}\leq\max\{p_1,p_2\}<\hat p_2$. Because an NR spread affects neither the overall probability of trade nor the probability of trade of suppliers other than 1 and 2, we have $p_1+p_2=\hat p_1+\hat p_2$. Letting $\Delta\equiv\min\{p_1,p_2\}-\hat p_1>0$, the change in HHI as a result of the NR spread is

$$
\begin{aligned}
\hat p_1^2+\hat p_2^2-p_1^2-p_2^2 &= (\min\{p_1,p_2\}-\Delta)^2+(p_1+p_2-(\min\{p_1,p_2\}-\Delta))^2-p_1^2-p_2^2 \\
&= 2\Delta\left(\max\{p_1,p_2\}-\min\{p_1,p_2\}\right),
\end{aligned}
$$

which is positive and increasing in Δ.

An NR spread affects the price that the buyer pays only in the event that one of suppliers 1 and 2 has the lowest cost and the other one has the second-lowest cost. Because the distribution of their lowest cost is by construction not affected by the NR spread, all that is left to do is to compare the distribution of their second-lowest draw, which is $G_1(c)G_2(c)$ without the spread and $H_1(c)H_2(c)$ after the spread.

Take as given a $c\in(\underline{c},\overline{c})$ in the open subset of $[\underline{c},\min\{v,\overline{c}\}]$ such that (4) is satisfied with strict inequalities. Let $A=1-H_1(c)$, $B=1-H_2(c)$, $C=1-F_1(c)$, and $D=1-F_2(c)$. Then we have (i) $AB=CD$ and (ii) $1-A\geq 1-C, 1-D\geq 1-B$, with $A,B,C,D\in(0,1)$.

Consider the problem

$$\max_{(A,B)\in[0,1]^2}(1-A)(1-B) \quad \text{s.t.} \quad AB = CD.$$

Substituting the constraint $A = CD/B$ yields the univariate maximization problem

$$\max_{B\in[0,1]} \frac{B-CD}{B}(1-B),$$

whose first and second derivatives are $-1+\frac{CD}{B^2}$ and $-\frac{2CD}{B^3} < 0$, respectively. Thus, the problem is strictly concave and maximized at $A = B = \sqrt{CD}$. This implies that for any A and B satisfying (i) and (ii) with $B \neq \sqrt{CD}$, we have $(1-A)(1-B) < (1-\sqrt{CD})^2$, which we can write as (dropping the argument c):

$$H_1H_2 \;<\; \left(1-\sqrt{(1-G_1)(1-G_2)}\right)^2 = G_1G_2 - G_1 - G_2 - 2\sqrt{(1-G_1)(1-G_2)} < G_1G_2.$$

This establishes that the distribution of the second-lowest cost after the NR spread first-order stochastically dominates the distribution of the second-lowest cost prior to the spread, for all $c \in [\underline{c}, \overline{c}]$, $H_1H_2 \leq G_1G_2$, with a strict inequality for costs in an open subset of $[\underline{c}, \min\{v, \overline{c}\}]$. Because the second-lowest cost determines the buyer's price, this implies that the buyer's expected price is higher under (H_1, H_2) than under (G_1, G_2).

Further, a larger NR spread, i.e., a change from (G_1, G_2) to $(\hat{H}_1, \hat{H}_2)$, where $(\hat{H}_1, \hat{H}_2)$ is an NR spread of (H_1, H_2), implies a larger Δ, and so a larger increase in the HHI, and also implies an increase in the buyer's expected price relative to (H_1, H_2). ■

References

Aigner, G., O. Budzinski, and C. Arndt (2006): "The Analysis of Coordinated Effects in EU Merger Control: Where Do We Stand After Sony/BMG and Impala?" *European Competition Journal*, 2, 311–336.

Amelio, A., P. Asbo, M. de la Mano, R. Maximiano, and V. Porubsky (2009): "ABF/GBI Business: Coordinated Effects Baked Again," Competition Policy Newsletter, European Commission.

Aoyagi, M. (2003): "Bid Rotation and Collusion in Repeated Auctions," *Journal of Economic Theory*, 112, 79–105.

Baker, J. B. (2002): "Mavericks, Mergers, and Exclusion: Proving Coordinated Competitive Effects Under the Antitrust Laws," *New York University Law Review*, 77, 135–203.

——— (2010): "Market Concentration in the Antitrust Analysis of Horizontal Mergers," in *Antitrust Law & Economics*, ed. by K. N. Hylton, Edward Elgar Publishing, vol. 4, 234–260, corrected in a working paper available at http://ssrn.com/abstract=1092248.

Börgers, T. (2015): *An Introduction to the Theory of Mechanism Design*, Oxford University Press.

Bos, I. and J. E. Harrington, Jr. (2010): "Endogenous Cartel Formation with Heterogeneous Firms," *RAND Journal of Economics*, 41, 92–117.

Bromfield, J. and M. Olczak (2018): "The Role of the Maverick Firm Concept in European Commission Merger Decisions," *Journal of Copmpetition Law & Economics*, 1–14.

Bulow, J. and P. Klemperer (1996): "Auctions versus Negotiations," *American Economic Review*, 86, 180–194.

Compte, O., F. Jenny, and P. Rey (2002): "Capacity Constraints, Mergers and Collusion," *European Economic Review*, 46, 1–29.

de Roos, N. and V. Smirnov (2019): "Collusion, Price Dispersion, and Fringe Competition," Working Paper, Sydney University.

Delacrétaz, D., S. Loertscher, L. Marx, and T. Wilkening (2019): "Two-Sided Allocation Problems, Decomposability, and the Impossibility of Efficient Trade," *Journal of Economic Theory*, 416–454.

European Commission (2004): "Guidelines on the Assessment of Horizontal Mergers Under the Council Regulation on the Control of Concentrations Between Undertakings," Official Journal of the European Union 2004/C 31/03.

Farrell, J. and C. Shapiro (1990): "Horizontal Mergers: An Equilibrium Analysis," *American Economic Review*, 80, 107–126.

Fellner, W. (1950): "Collusion and Its Limits Under Oligopoly," *American Economic Review*, 40, 54–62.

Ganslandt, M., L. Persson, and H. Vasconcelos (2012): "Endogenous Mergers and Collusion in Asymmetric Market Structures," *Economica*, 79, 766–791.

Gayle, W.-R., R. C. Marshall, L. M. Marx, and J.-F. Richard (2011): "Coordinated Effects in the 2010 Horizontal Merger Guidelines," *Review of Industrial Organization*, 39, 39–56.

Gresik, T. and M. Satterthwaite (1989): "The Rate at which a Simple Market Converges to Efficiency as the Number of Traders Increases: An Asymptotic Result for Optimal Trading Mechanisms," *Journal of Economic Theory*, 48, 304–332.

Harrington, J. E. (2013): "Evaluating Mergers for Coordinated Effects and the Role of "Parallel Accommodating Conduct"," *Antitrust Law Journal*, 78, 651–668.

Harrington, J. E. and Z. Wei (2012): "Signaling and Tacit Collusion in an Infinitely Repeated Prisoners' Dilemma," *Mathematical Social Sciences*, 64, 277–289.

Hawkins, N. and J. King-Kafsack (2014): "Federal Merger Enforcement Actions: A Look at Recent Trends," *Practical Law The Journal*, March 2014, 34–44.

Igami, M. and T. Sugaya (2019): "Measuring the Incentive to Collude: The Vitamin Cartels, 1990–1999," Working Paper, Yale University.

IVALDI, M., B. JULLIEN, P. REY, P. SEABRIGHT, AND J. TIROLE (2007): "The Economics of Tacit Collusion: Implications for Merger Control," in *The Political Economy of Antitrust*, ed. by V. Ghosal and J. Stennek, Elsevier, 217–239.

IVALDI, M. AND V. LAGOS (2017): "Assessment of Post-Merger Coordinated Effects: Characterization by Simulations," *International Journal of Industrial Organization*, 53, 267–305.

IVALDI, M., S. MITRAILLE, AND C. MULLER (2012): "Testing for the Presence of a Maverick in the French Audit Industry," in *Recent Advances in the Analysis of Competition Policy and Regulation*, ed. by J. E. Harrington, Jr. and Y. Katsoulacos, Cheltenham, UK: Edward Elgar Publishing, 75–94.

KOLASKY, W. J. (2002): "Coordinated Effects In Merger Review: From Dead Frenchmen To Beautiful Minds And Mavericks," Remarks by Deputy Assistant Attorney General of the Antitrust Division at the U.S. DOJ William J. Kolasky at the ABA Section of Antitrust Law Spring Meeting, Washington, D.C.

KOVACIC, W. E., R. C. MARSHALL, L. M. MARX, AND M. E. RAIFF (2007a): "Lessons for Competition Policy from the Vitamins Cartel," in *The Political Economy of Antitrust*, ed. by V. Ghosal and J. Stennek, Elsevier, 149–176.

KOVACIC, W. E., R. C. MARSHALL, L. M. MARX, AND S. P. SCHULENBERG (2007b): "Coordinated Effects in Merger Review: Quantifying the Payoffs from Collusion," in *International Antitrust Law & Policy: Fordham Competition Law 2006*, ed. by B. E. Hawk, Juris Publishing, 271–285.

——— (2009): "Quantitative Analysis of Coordinated Effects," *Antitrust Law Journal*, 76, 397–430.

KRISHNA, V. (2002): *Auction Theory*, Elsevier Science, Academic Press.

KÜHN, K.-U. (2004): "The Coordinated Effects of Mergers in Differentiated Products Market," Law & Economics Working Papers Archive: 2003–2009, University of Michigan Law School.

KUTLINA-DIMITROVA, Z. (2018): "Government Procurement: Data, Trends and Protectionist Tendencies," DG TRADE Chief Economist Notes 2018-3, Directorate General for Trade, European Commission.

KWOKA, J. E. (1989): "The Private Profitability of Horizontal Mergers with Non-Cournot and Maverick Behavior," *International Journal of Industrial Organization*, 7, 403–411.

LEBRUN, B. (1999): "First Price Auctions in the Asymmetric N Bidder Case," *International Economic Review*, 40, 125–142.

LOERTSCHER, S. AND L. M. MARX (2019a): "Countervailing Power," Working Paper, University of Melbourne.

——— (2019b): "Merger Review for Markets with Buyer Power," *Journal of Political Economy*, 127, 2967–3017.

——— (2019c): "Mix-and-Match Divestitures and Merger Harm," *Japanese Economic Review*, 70, 346–366.

Marshall, R. C. and L. M. Marx (2012): *The Economics of Collusion: Cartels and Bidding Rings*, MIT Press.

McAfee, R. P. and J. McMillan (1992): “Bidding Rings,” *American Economic Review*, 82, 579–599.

McAfee, R. P. and M. A. Williams (1992): “Horizontal Mergers and Antitrust Policy,” *Journal of Industrial Economics*, 40, 181–187365–369.

Miller, N. H., G. Sheu, and M. C. Weinberg (2019): “Oligopolistic Price Leadership and Mergers: An Empirical Model of the Beer Industry,” Working Paper, Georgetown University.

Miller, N. H. and M. Weinberg (2017): “Understanding the Price Effects of the Miller-Coors Joint Venture,” *Econometrica*, 85, 1763–1791.

Motta, M. (2000): “EC Merger Policy, and the Airtours Case,” *European Competition Law Review*, 21, 199–207.

Myerson, R. and M. Satterthwaite (1983): “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory*, 29, 265–281.

Myerson, R. B. (1981): “Optimal Auction Design,” *Mathematics of Operations Research*, 6, 58–73.

Nocke, V. and M. D. Whinston (2010): “Dynamic Merger Review,” *Journal of Political Economy*, 118, 1200–1251.

——— (2013): “Merger Policy with Merger Choice,” *American Economic Review*, 103, 1006–1033.

Perry, M. K. and R. H. Porter (1985): “Oligopoly and the Incentive for Horizontal Merger,” *American Economic Review*, 75, 219–227.

Porter, R. H. (2020): “Mergers and Coordinated Effects,” *International Journal of Industrial Organization*.

Salant, S. W., S. Switzer, and R. J. Reynolds (1983): “Losses From Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium,” *Quarterly Journal of Economics*, 98, 185–199.

Stigler, G. J. (1964): “A Theory of Oligopoly,” *Journal of Political Economy*, 72, 44–61.

U.S. Department of Justice (2015): “Preventing and Detecting Bid Rigging, Price Fixing, and Market Allocation in Post-Disaster Rebuilding Projects,” https://www.justice.gov/sites/default/files/atr/legacy/2013/01/31/disaster_primer.pdf.

U.S. Department of Justice and Federal Trade Commission (2010): “Horizontal Merger Guidelines,” https://www.justice.gov/atr/horizontal-merger-guidelines-0.

Vasconcelos, H. (2005): “Tacit Collusion, Cost Asymmetries, and Mergers,” *RAND Journal of Economics*, 36, 39–62.

Waehrer, K. (2019): “Pass-Through of Cost Reductions in Open Auction Markets with Application to Merger Effects,” SSRN Working Paper No. 3334607.

Whinston, M. D. (2006): “Antitrust Policy Toward Horizontal Mergers,” in *Handbook of Industrial Organization*, ed. by M. Armstrong and R. Porter, Elsevier, vol. 3, 2369–2440.

B Online Appendix: Formal statements of results and proofs for Section 3.4

This appendix contains the formal statements of propositions and the associated proofs for the material in Section 3.4.

Outside competition

Proposition B.1. *Assuming symmetric suppliers and a fixed set of coordinators K, (i) the market is not at risk if n is sufficiently large, i.e., there exists $\underline{n} > k$ such that for all $n > \underline{n}$, $\mathcal{I}^S(K) < 0$, and (ii) for the power-based distribution, $\mathcal{I}^S(K)$ decreases in n.*

Proof of Proposition B.1. In the limit as n grows large, with probability 1, both the lowest and second-lowest order statistics are less than the reserve r. Thus, we can focus on the case in which the reserve does not bind, in which case $\Pi_i = \frac{1}{n}\mathbb{E}\left[c_{(2:n)} - c_{(1:n)}\right]$ and $\Pi_i(K) = \frac{1}{n-k+1}\mathbb{E}\left[c_{(2:n-k+1)} - c_{(1:n-k+1)}\right]$. It is then sufficient to show that there exists $\underline{n}$ such that for all $n > \underline{n}$,

$$\frac{n-k+1}{n}\frac{\mathbb{E}\left[c_{(2:n)} - c_{(1:n)}\right]}{\mathbb{E}\left[c_{(2:n-k+1)} - c_{(1:n-k+1)}\right]} > \frac{1}{k}. \tag{11}$$

As shown by Loertscher and Marx (2020, Lemma 1),

$$j\mathbb{E}[c_{(j+1:n)} - c_{(j:n)}] = \mathbb{E}\left[\frac{G(c_{(j:n)})}{g(c_{(j:n)})}\right],$$

which, using $\mathbb{E}\left[\frac{G(c_{(1:\ell)})}{g(c_{(1:\ell)})}\right] = \int_{\underline{c}}^{\overline{c}} \ell G(x)(1-G(x))^{\ell-1}dx$, allows us to write (11) as

$$\frac{\int_{\underline{c}}^{\overline{c}} G(x)(1-G(x))^{n-1}dx}{\int_{\underline{c}}^{\overline{c}} G(x)(1-G(x))^{n-k}dx} - \frac{1}{k} > 0,$$

which we can write as

$$\frac{\int_{\underline{c}}^{\overline{c}}(1-G(x))^{n-k}H(x)dx}{k\int_{\underline{c}}^{\overline{c}} G(x)(1-G(x))^{n-k}dx} > 0, \tag{12}$$

where

$$H(x) \equiv kG(x)(1-G(x))^{k-1} - G(x).$$

Note that $H(\underline{c}) = 0$, $H'(\underline{c}) > 0$, and $H(x) < 1$ for all $x \in [\underline{c}, \overline{c}]$. Thus, there exists $c^* \in (\underline{c}, \overline{c})$ such that for all $x \in (\underline{c}, c^*)$,

$$H(x) \in (0,1). \tag{13}$$

In addition, note that for all $x \in [\underline{c}, \overline{c}]$,

$$H(x) \geq -1, \tag{14}$$

with equality only at $x = \overline{c}$.

Because the denominator on the left side of (12) is positive for all n, it is sufficient to show that there exists $\underline{n}$ such that the numerator is positive for all $n > \underline{n}$. Letting $\hat{c} \in (\underline{c}, c^*)$, we have

$$\begin{aligned}
& \int_{\underline{c}}^{\overline{c}} (1 - G(x))^{n-k} H(x) dx \\
= \; & \int_{\underline{c}}^{c^*} (1 - G(x))^{n-k} H(x) dx + \int_{c^*}^{\overline{c}} (1 - G(x))^{n-k} H(x) dx \\
> \; & \int_{\underline{c}}^{\hat{c}} (1 - G(x))^{n-k} H(x) dx - \int_{c^*}^{\overline{c}} (1 - G(x))^{n-k} dx \\
> \; & (1 - G(\hat{c}))^{n-k} \int_{\underline{c}}^{\hat{c}} H(x) dx - (1 - G(c^*))^{n-k} (\overline{c} - c^*) \\
= \; & (1 - G(\hat{c}))^{n-k} \left[\int_{\underline{c}}^{\hat{c}} H(x) dx - \left(\frac{1 - G(c^*)}{1 - G(\hat{c})} \right)^{n-k} (\overline{c} - c^*) \right],
\end{aligned}$$

where the first inequality, which replaces the upper bound of integration in the first integral with $\hat{c} \in (\underline{c}, c^*)$ and replaces $H(x)$ in the second integral with -1, uses (13) and (14); the second inequality uses the properties of G as a cdf; and the final equality rearranges. Setting the expression in square brackets in the last line equal to zero and solving for n, we define

$$\underline{n} \equiv \ln \left(\frac{\int_{\underline{c}}^{\hat{c}} H(x) dx}{\overline{c} - c^*} \right) / \ln \left(\frac{1 - G(c^*)}{1 - G(\hat{c})} \right) + k > k,$$

where, using (13) and the properties of G as a cdf, the two logarithms are both negative, which gives the result that $\underline{n} > k$. It follows that for all $n > \underline{n}$, $\int_{\underline{c}}^{\overline{c}} (1 - G(x))^{n-k} H(x) dx > 0$, and so (12) holds for all $n > \underline{n}$, which completes the proof of the first part of the proposition.

Turning to the second part of the proposition, using Lemma 1, under symmetry and the power-based distribution,

$$I^S(K) = 1 - k \frac{(1 + (n - k + 1)\alpha)(1 + (n - k)\alpha)}{(1 + (n - 1)\alpha)(1 + n\alpha)}.$$

Differentiating with respect to n, we have

$$\frac{\partial I^S(K)}{\partial n} = -\frac{a^2 (k - 1) k \left[2 - 2a(k - 2n) + a^2 (k - 2kn + 2n^2)\right]}{(1 + a(n - 1))^2 (1 + an)^2},$$

which has sign equal to the sign of

$$-2 - 2a(2n - k) - a^2(k - 2kn + 2n^2)$$

which is increasing in k and at $k = n$ is equal to $-2 - 2an - a^2 n$, which is negative. Thus, $I^S(K)$ is decreasing in n for all $k \in \{2, ..., n\}$. ■

Supplier strength

Proposition B.2. *For the power-based parameterization with $v \geq \bar{c}$ and symmetric suppliers in K, as the strength of suppliers in K decreases, eventually the market is not at risk, that is,* $\lim_{\alpha \to 0} \mathcal{I}^S(K) < 0$, *and as the strength of suppliers in K increases, eventually the market is at risk, that is,* $\lim_{\alpha \to \infty} \mathcal{I}^S(K) > 0$.

Proof. The proof follows straightforwardly from Lemma 1. ■

Proposition B.3. *For the power-based parameterization with $v \geq \bar{c}$, $\mathcal{I}^S(K)$ is largest, conditional on $|K| = k$ if K includes the k strongest suppliers. Further, increasing the strength of the suppliers in K, while holding fixed the distribution of the lowest cost draw in the market, causes $\mathcal{I}^S(K)$ to increase. That is, given parameters $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)$ satisfying $\sum_{i \in N} \alpha_i = \sum_{i \in N} \beta_i$ and $\beta_i \geq \alpha_i$ for all $i \in K$ with a strict inequality for at least one i, $\mathcal{I}^S(K; \boldsymbol{\beta}) > \mathcal{I}^S(K; \boldsymbol{\alpha})$.*

Proof of Proposition B.3. We begin by proving the first statement of the proposition. If $n = k$, then the result holds trivially, so assume that $n > k$. Without loss of generality, assume that $K = \{1, ..., k\}$. Using Lemma 1, we have

$$\sum_{i \in K} s_i(K) = \sum_{i \in K} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_i)(1 + A)}.$$

Because we assume that $1 \in K$, we can rewrite this as

$$\sum_{i \in K} s_i(K) = \sum_{i \in K \setminus \{1\}} \frac{(1 + \alpha_i + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_i)(1 + A)} + \frac{(1 + \alpha_1 + A_{-K})(1 + A_{-K})}{(1 + A - \alpha_1)(1 + A)}.$$

Suppose we deduct $\varepsilon \in (0, \alpha_n)$ from α_n and add ε to α_1. Then the sum of all suppliers' strength parameters, A, remains unchanged, and we have

$$\begin{aligned} &\mathcal{I}^S(K; \alpha_1 + \varepsilon, \alpha_2, ..., \alpha_{n-1}, \alpha_n - \varepsilon) \\ = \;& 1 - \left[\sum_{i \in K \setminus \{1\}} \frac{(1+\alpha_i+A_{-K}-\varepsilon)(1+A_{-K}-\varepsilon)}{(1+A-\alpha_i)(1+A)} + \frac{(1+\alpha_1+A_{-K})(1+A_{-K}-\varepsilon)}{(1+A-\alpha_1-\varepsilon)(1+A)} \right]. \end{aligned}$$

Differentiating with respect to ε, we have

$$-\frac{1}{1+A}\left[\sum\nolimits_{i\in K\backslash\{1\}}\frac{-(1+A_{-K}-\varepsilon)-(1+\alpha_i+A_{-K}-\varepsilon)}{1+A-\alpha_i}-\frac{(1+\alpha_1+A_{-K})(A-\alpha_1-A_{-K})}{(1+A-\alpha_1-\varepsilon)^2}\right],$$

which is positive. Thus, for all $\varepsilon \in (0, \alpha_n)$,

$$\mathcal{I}^S(K; \alpha_1+\varepsilon, \alpha_2, ..., \alpha_{n-1}, \alpha_n-\varepsilon) > \mathcal{I}^S(K; \alpha_1, \alpha_2, ..., \alpha_{n-1}, \alpha_n). \tag{15}$$

If follows that $\mathcal{I}^S(K)$ increases when a member of K is replaced with a member of $N\backslash K$ with a larger distributional parameter and that $\mathcal{I}^S(K)$ is maximized when its members are the set K of suppliers with the largest distributional parameters. This completes the proof of the first sentence of the proposition.

Turning to the second sentence of the proposition, let $\boldsymbol{\beta}$ be as given in the statement of the proposition. Because $\mathcal{I}^S(K)$ relies on the strength parameters of suppliers in $N\backslash K$ only through the sum of those parameters, we have

$$\mathcal{I}^S(K; \alpha_1, ..., \alpha_k, \alpha_{k+1}, ..., \alpha_n) = \mathcal{I}^S(K; \alpha_1, ..., \alpha_k, \beta_{k+1}, ...\beta_{n-1}, A_{-K} - \sum_{i=k+1}^{n-1}\beta_i),$$

where $A_{-K} - \sum_{i=k+1}^{n-1}\beta_i > 0$ by the assumptions on $\boldsymbol{\beta}$. But then, letting $\varepsilon_i \equiv \beta_i - \alpha_i \geq 0$ for $i \in K$, we have

$$\begin{aligned}
& \mathcal{I}^S(K; \alpha_1, ..., \alpha_k, \beta_{k+1}, ...\beta_{n-1}, A_{-K} - \sum_{i=k+1}^{n-1}\beta_i) \\
< \; & \mathcal{I}^S(K; \alpha_1+\varepsilon_1, ..., \alpha_k+\varepsilon_k, \beta_{k+1}, ...\beta_{n-1}, A_{-K} - \sum_{i=k+1}^{n-1}\beta_i - \sum_{i=1}^{k}\varepsilon_i) \\
= \; & \mathcal{I}^S(K; \beta_1, ..., \beta_k, \beta_{k+1}, ...\beta_{n-1}, \beta_n),
\end{aligned}$$

where the inequality uses the repeated application of (15) and the assumption that ε_i is strictly positive for at least one $i \in K$, and where the final inequality uses the definition of ε_i and the assumption that $\sum_{i\in N}\beta_i = A$. This completes the proof. ■

Symmetry

Proposition B.4. *For the power-based parameterization, an NR spread applied to two suppliers in K reduces $\mathcal{I}^S(K)$.*

Proof of Proposition B.4. We show that for the power-based parameterization, an NR spread applied to two suppliers in K reduces $\mathcal{I}^S(K)$. We denote the two suppliers experiencing the

NR spread as 1 and 2, with $\{1,2\} \subseteq K$. Let F_1 and F_2 be their distributions prior to the spread, with parameters f_1 and f_2, and let H_1 and H_2 be their distributions after the spread, with parameters h_1 and h_2. Note that we abuse notation by using, for $i \in \{1,2\}$, f_i and h_i to denote the parameters of the distributions F_i and H_i, rather than their pdfs. Denote the parameters of suppliers other than 1 and 2 by α_j for $j \in \{3,...,n\}$.

An NR spread in the power-based parameterization implies the existence of $a > 0$ such that

$$h_1 > f_1 \geq a \geq f_2 > h_2$$

and

$$h_1 + h_2 = 2a = f_1 + f_2.$$

Thus, $h_2 = 2a - h_1$ and $f_2 = 2a - f_1$. Let $s_i(K)$ be critical share of supplier $i \in K$ under (F_1, F_2), and let $\hat{s}_i(K)$ be the critical share for supplier $i \in K$ under (H_1, H_2). The shift from (F_1, F_2) to (H_1, H_2) decreases the $\mathcal{I}^S(K)$ if and only if it increases the sum of the critical shares of suppliers 1 and 2. In what follows, we show that $\hat{s}_1(K)+\hat{s}_2(K)-s_1(K)-s_2(K) > 0$.

In the power-based parameterization, letting $A \equiv 2a + \sum_{i\in\{3,...,n\}} \alpha_i$,

$$s_i(K) = \frac{(1+f_i+A_{-K})(1+A_{-K})}{(1+A-f_i)(1+A)} \quad \text{and} \quad \hat{s}_i(K) = \frac{(1+h_i+A_{-K})(1+A_{-K})}{(1+A-h_i)(1+A)}.$$

Consequently, the sign of $\hat{s}_1(K) + \hat{s}_2(K) - s_1(K) - s_2(K)$ is equal to the sign of

$$\frac{(1+h_1+A_{-K})}{(1+A-h_1)} + \frac{(1+h_2+A_{-K})}{(1+A-h_2)} - \frac{(1+f_1+A_{-K})}{(1+A-f_1)} - \frac{(1+f_2+A_{-K})}{(1+A-f_2)}.$$

Substituting $2a - h_1$ for h_2 and $2a - f_1$ for f_2 and collecting terms, we get

$$\frac{2(f_1-h_1-2a)(h_1-f_1)(1+a+X)(2+2a+X)}{(1+2a-f_1+X)(1+f_1+X)(1+2a-h_1+X)(1+h_1+X)},$$

where $X \equiv A_{-\{1,2\}} + A_{-K}$, which is positive. ■

C Online Appendix: Longer proofs

Proof of Lemma 4. Here we show that $\mathcal{I}^C(N) > 0$. (See Appendix A for the proof of (6).) The Cournot and monopoly quantities are given by:

$$q_i^C(\mathbf{c}) = \frac{1-(n+1)c_i + C_N}{n+1} \quad \text{and} \quad q_i^M(c_i) = \frac{1-c_i}{2}.$$

Because supplier i's Cournot and monopoly profits are the squares of the corresponding quantities,

$$\mathcal{I}^C(N) = 1 - \sum_{i\in N}\left(\frac{q_i^C(\mathbf{c})}{q_i^M(c_i)}\right)^2.$$

Consider the problem of minimizing $\mathcal{I}^C(N)$ with respect to $\mathbf{c}$, subject to $q_i^C(\mathbf{c}) \geq 0$ for all $i \in N$, i.e., temporarily, we allow quantities to be zero. Using (6), given $\mathbf{c}_{-i} \in [0,1)^{n-1}$, $\lim_{c_i\to 1}\mathcal{I}^C(N) = -\infty$. It follows that when $\mathcal{I}^C(N)$ is minimized with respect to $\mathbf{c}$, subject to $q_i^C(\mathbf{c}) \geq 0$ for all $i \in N$, at least one constraint must be binding, i.e., must be satisfied with equality.

Let $\mathbf{c}$ be such that $\mathcal{I}^C(N)$ is minimized subject to $q_i^C(\mathbf{c}) \geq 0$ for all $i \in N$. Let $Z \subset N$ be the set of suppliers such that $q_i^C(\mathbf{c}) > 0$ and let $z \equiv |Z|$. By the arguments above $z \in \{1, ..., n\}$.

Case 1: $z = 1$. Then $\mathcal{I}^C(N) = 0$.

Case 2: $z \geq 2$ and for all $i \in Z$, $c_i = 0$. Then for all $j \in N\backslash Z$, $q_j^C(\mathbf{c}) = 0$, which implies that $c_j = \frac{1}{z+1}$, so $C_N = \frac{n-z}{z+1}$. Using (6),

$$\mathcal{I}^C(N) = 1 - \sum_{i\in Z}\left(1+\frac{n-z}{z+1}\right)^2\frac{4}{(n+1)^2} = 1 - \frac{4z}{(z+1)^2} = \frac{(z-1)^2}{(z+1)^2} > 0,$$

which contradicts the supposition of being at the minimum.

Case 3: $z \geq 2$ and there exists $\ell \in Z$ such that $c_\ell > 0$ and for all $i \in Z\backslash\{\ell\}$, $c_i = 0$. Then for all $j \in N\backslash Z$, $q_j^C(\mathbf{c}) = 0$, which implies that $c_j = \frac{1+c_\ell}{1+z}$ and $C_N = \frac{(n-z)(1+c_\ell)}{1+z} + c_\ell = \frac{n-z+(n+1)c_\ell}{1+z}$. Further, in order to have $q_\ell(\mathbf{c}) > 0$, it must be that $1-(n+1)c_\ell + C_N > 0$, which we can write as $c_\ell < \frac{1}{z}$. Using (6),

$$\begin{aligned}\mathcal{I}^C(N) &= 1-(z-1)\left(1+\frac{n-z+(n+1)c_\ell}{1+z}\right)^2\frac{4}{(n+1)^2} \\ &\quad -\frac{\left(1-(n+1)c_\ell+\frac{n-z+(n+1)c_\ell}{1+z}\right)^2}{(1-c_\ell)^2}\frac{4}{(n+1)^2} \\ &= \frac{-(1+c_\ell)(z-1)(1-z+3(z-1)c_\ell-4c_\ell^2+4c_\ell^3)}{(1-c_\ell)^2(1+z)^2},\end{aligned}$$

which is positive for all $c_\ell < \frac{1}{z}$ and $z \in \{2, 3, ..., n\}$,[44] contradicting the supposition of being at the minimum.

Case 4: $z \geq 2$ and there exist $j, \ell \in Z$ with $j \neq \ell$ such that $c_j, c_\ell > 0$. Without loss of generality, suppose that $c_j \leq c_\ell$. Let $\Delta \in (0, \min\{c_j, q_\ell^C(\mathbf{c})\})$ and consider a change in costs that decreases c_j by Δ and increases c_ℓ by Δ. (As we discuss in Section 3.4, this is a neutral-for-rivals spread.) Let $\hat{\mathbf{c}}$ be the new vector of costs, i.e., $\hat{c}_j = c_j - \Delta$, $\hat{c}_\ell = c_\ell + \Delta$, and for all $i \in N\backslash\{j, \ell\}$, $\hat{c}_i = c_i$. Given our choice of Δ, $q_j^C(\hat{\mathbf{c}}) > 0$ and $q_\ell^C(\hat{\mathbf{c}}) > 0$. The change in supplier j's critical share is

$$\begin{aligned}\frac{q_j^C(\mathbf{c})}{q_j^M(c_j)} - \frac{q_j^C(\hat{\mathbf{c}})}{q_j^M(\hat{c}_j)} &= \frac{2}{n+1}\left(\frac{1-(n+1)c_j+C_N}{1-c_j} - \frac{1-(n+1)(c_j-\Delta)+C_N}{1-(c_j-\Delta)}\right) \\ &= \frac{2\Delta}{n+1}\left(\frac{C_N-n}{(1-c_j)(1-c_j+\Delta)}\right) \\ &< 0,\end{aligned}$$

and, analogously, the change in supplier ℓ's critical share is

$$\frac{q_\ell^C(\mathbf{c})}{q_\ell^M(c_\ell)} - \frac{q_\ell^C(\hat{\mathbf{c}})}{q_\ell^M(\hat{c}_\ell)} = \frac{-2\Delta}{n+1}\left(\frac{C_N-n}{(1-c_\ell)(1-c_\ell-\Delta)}\right) > 0.$$

[44] To see this, note that the expression has sign equal to the sign of $1+z-3(z-1)c_\ell+4c_\ell^2-4c_\ell^3$, which at $c_\ell = \frac{1}{z}$ is equal to

$$-4+4z+3z^2-4z^3+z^4,$$

which is zero for $z = 2$ and positive for $z > 2$. In addition, the expression has derivative with respect to c_ℓ of

$$\frac{8c_\ell(z-1)(2-z-2c_\ell^2+c_\ell^3)}{(1-c_\ell)^3(1+z)^2},$$

which has sign equal to the sign of $2-z-c_\ell^2(2-c_\ell)$, which is negative for $z \geq 2$ and $c_\ell \in (0,1)$. Thus, the expression is positive for all $z \geq 2$ and $c_\ell \in (0, 1/z)$.

Because $\Delta > 0$ and $c_j \le c_\ell$, it follows that

$$\left| \frac{q_1^C(\mathbf{c})}{q_1^M(c_1)} - \frac{q_1^C(\hat{\mathbf{c}})}{q_1^M(\hat{c}_1)} \right| < \left| \frac{q_2^C(\mathbf{c})}{q_2^M(c_2)} - \frac{q_2^C(\hat{\mathbf{c}})}{q_2^M(\hat{c}_2)} \right|,$$

which implies that the decrease in supplier j's critical share is less than the increase in supplier ℓ's critical share, and so overall $\mathcal{I}^C(N)$ decreases as a result of the change. Because all constraints continue to be satisfied, this contradicts the supposition of being at the minimum.

Thus, we conclude that when $\mathcal{I}^C(N)$ is minimized subject to $q_i^C(\mathbf{c}) \ge 0$ for all $i \in N$, the quantity constraint is slack for one and only one supplier and $\mathcal{I}^C(N) = 0$. Because the minimum is not achieved when all constraints are satisfied strictly, we conclude that under our assumption that all suppliers in N have positive Cournot quantities, $\mathcal{I}^C(N) > 0$. ■

D Online Appendix: Extensions

D.1 Procurement setup with multi-unit suppliers and buyer power

Our definition of and test for coordinated effects generalize straightforwardly to an efficient procurement setup in which the buyer has multi-unit demand and suppliers have multi-unit capacities. (Without multi-unit demand, multi-unit capacities play no substantial role.) To be specific, we can allow the buyer to be characterized by a commonly known marginal value vector $\mathbf{v} = (v_1, ..., v_Q)$, where Q is the buyer's maximal demand, with $v_i \ge v_{i+1}$ for $i \in \{1, \dots, Q-1\}$ and for each supplier j to be characterized by a capacity κ_j and a vector of marginal costs $\mathbf{c}^j = (c_1^j, \dots, c_{\kappa_j}^j)$ satisfying $c_i^j \le c_{i+1}^j$ for $i \in \{1, \dots, \kappa_j - 1\}$, where κ_j is (now) an integer. Assume that each supplier j's capacity κ_j is common knowledge, but that each supplier's marginal cost is its own private information. Assume also that for all j, $\mathbf{c}^j$ is distributed according to the commonly known, continuous distribution $G_j(\mathbf{c}^j)$ with support $[\underline{c}, \overline{c}]^{\kappa_j}$.

A simple and particularly convenient specification for the multivariate distribution $G_j(\mathbf{c}^j)$ is to assume that j's cost draw is the realization of κ_j independent, univariate random variables c drawn from the distribution $G_j(c)$. This implies that $G_j(\mathbf{c}^j)$ is given by the distribution if the κ_j-th order statistic from G_j. For example, the distribution of c_1^j is $G_{j,[1]}(c) = 1 - (1 - G_j(c))^{\kappa_j}$. Consequently, we refer to this as the *order statistics model.* This model also makes clear the sense in which the power-based parameterization captures a supplier's strength.

Following a merger between suppliers h and j, the merged entity's capacity is $\kappa_h + \kappa_j$. In the order statistics model, assuming pre-merger symmetry between j and h, so that

$G_j = G_h = G$, the distribution of the minimum cost c_1^{hj} of the merged firm is $G_{hj,[1]}(c) = 1 - (1 - G(c))^{\kappa_h + \kappa_j}$.

The payoff (or revenue) equivalence theorems for multi-dimensional type spaces of Williams (1999) and Krishna and Maenner (2001) imply that the generalized second-price auction with reserve prices for the m-th unit given by $\min\{v_m, \overline{c}\}$ is without loss of generality insofar as this is the profit-maximizing mechanism for the buyer subject to efficiency and individual rationality and incentive compatibility constraints for the suppliers. Consequently, the profit of every supplier j is pinned down by $\mathbf{v}$ and the distributions $(G_i(\mathbf{c}^i))_{i \in N}$ when all suppliers play their dominant strategies of reporting their types $\mathbf{c}^i$ truthfully.

Likewise, the expected profit $\Pi_i(K)$ when the suppliers $j \in K$ participate in a bidder selection scheme when $i \in K$ is the designated bidder is pinned down by $G_i(\mathbf{c}^i)$ for $i \in K$ and $(G_h(\mathbf{c}^h))_{h \in N \backslash K}$. Consequently, $s_i(K) = \Pi_i / \Pi_i(K)$ as in the single-unit case, and the $\mathcal{I}^S(K)$ can be defined in the same way and with the same interpretation as before.

Two-unit demand

With two-unit demand, $v \geq \overline{c}$, supplier i's expected revenue is the expected value of the second-lowest cost among the other $n-1$ suppliers, conditional on that cost being greater than supplier i's cost, and multiplied by the probability that supplier i's cost is one of the two lowest, which we can write as:

$$R_i = \int_{\underline{c}}^{\overline{c}} \int_{c}^{\overline{c}} y dH_i(y) dG_i(c),$$

where H_i is the distribution of the second-lowest cost among suppliers other than i. Similarly, supplier i's expected profit is

$$\Pi_i = \int_{\underline{c}}^{\overline{c}} \int_{c}^{\overline{c}} (y - c) dH_i(y) dG_i(c).$$

Letting H_K be the distribution of the second-lowest cost among suppliers not in set K, then for $i \in K$, supplier i's expected profit when suppliers in K coordinate and supplier i is selected to be the only member of K to bid is

$$\Pi_i(K) = \int_{\underline{c}}^{\overline{c}} \int_{c}^{\overline{c}} (y - c) dH_K(y) dG_i(c).$$

Now consider the power-based parameterization. Letting $A \equiv \sum_{k \in N} \alpha_k$ and $A_{-X} \equiv \sum_{k \in N \backslash X} \alpha_k$, we can write R_i, Π_i, and $\Pi_i(K)$ in terms of the parameters of the cost distributions as shown in the following lemma.

Lemma D.1. *Assuming $G_i(c) = 1 - (1-c)^{\alpha_i}$ and $n \geq 3$, if the buyer has two-unit demand with $v \geq 1$, then supplier i trades with probability $q_i = \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}A} \right)$, has expected revenue*

$$R_i = \alpha_i \sum_{\ell \neq i} \left(\frac{\alpha_i + A^2_{-\{\ell\}}}{\left(1 + A_{-\{i,\ell\}}\right) A_{-\{\ell\}}(1 + A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}\left(1 + \alpha_i\right)}{A_{-\{i\}}(1 + A_{-\{i\}})A\left(1 + A\right)} \right),$$

and has expected profit $\Pi_i = R_i - C_i$, where C_i is supplier i's expected cost, given by

$$C_i = \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}(1 + A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}A(1 + A)} \right).$$

In addition, for $i \in K$, q_i, R_i and C_i can be adjusted for the case of coordination by suppliers in K by summing over $\ell \in N \backslash K$ and letting α_j be zero for all $j \in K \backslash \{i\}$.

Proof. Using the parameterization $G_i(c) = 1-(1-c)^{\alpha_i}$, for $n \geq 3$, the cdf of the second-lowest among the $n-1$ suppliers other than supplier i is

$$\begin{aligned} H_i(c) \ &\equiv \ 1 - \left(\prod_{j \neq i}(1 - G_j(c)) + \sum_{\ell \neq i} G_\ell(c) \times_{j \in N \backslash \{i,\ell\}} (1 - G_j(c)) \right) \\ &= \ 1 - \left((1-c)^{A_{-\{i\}}} + \sum_{\ell \neq i}(1 - (1-c)^{\alpha_\ell})(1-c)^{A_{-\{i,\ell\}}} \right), \end{aligned}$$

and the associated pdf is

$$h_i(c) \ = \ \sum_{\ell \neq i}(1 - (1-c)^{\alpha_\ell})A_{-\{i,\ell\}}(1-c)^{A_{-\{i,\ell\}}-1}.$$

The probability of trade for supplier i is

$$\begin{aligned} q_i \ &= \ \int_0^1 \int_c^1 \left(\sum_{\ell \neq i}(1 - (1-y)^{\alpha_\ell})A_{-\{i,\ell\}}(1-y)^{A_{-\{i,\ell\}}-1} \right) \alpha_i(1-c)^{\alpha_i - 1} dy dc \\ &= \ \int_0^1 \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left((1-y)^{A_{-\{i,\ell\}}-1} - (1-y)^{A_{-\{i\}}-1} \right) dy \right) \alpha_i(1-c)^{\alpha_i - 1} dc \\ &= \ \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}A} \right). \end{aligned}$$

So the market share of supplier i is $q_i/2$ (because the sum of all suppliers' probabilities of trade is 2 in the case of two-unit demand and no buyer power and $v \geq 1$).

Supplier i's expected revenue is

$$\begin{aligned} R_i &= \int_0^1 \int_c^1 y \left(\sum_{\ell \neq i} (1-(1-y)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-y)^{A_{-\{i,\ell\}}-1} \right) \alpha_i (1-c)^{\alpha_i - 1} dy dc \\ &= \int_0^1 \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left(y(1-y)^{A_{-\{i,\ell\}}-1} - y(1-y)^{A_{-\{i\}}-1} \right) dy \right) \alpha_i (1-c)^{\alpha_i-1} dc \\ &= \alpha_i \sum_{\ell \neq i} \left(\frac{1}{1+A_{-\{i,\ell\}}} \left(1 - \frac{A_{-\{i,\ell\}}}{A_{-\{\ell\}}(1+A_{-\{\ell\}})} \right) - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}}(1+A_{-\{i\}})A} \left(1 - \frac{A_{-\{i\}}}{1+A} \right) \right). \end{aligned}$$

Supplier i's expected cost is

$$\begin{aligned} C_i &= \int_0^1 \int_c^1 c \left(\sum_{\ell \neq i} (1-(1-y)^{\alpha_\ell}) A_{-\{i,\ell\}} (1-y)^{A_{-\{i,\ell\}}-1} \right) \alpha_i (1-c)^{\alpha_i - 1} dy dc \\ &= \int_0^1 c \left(\sum_{\ell \neq i} A_{-\{i,\ell\}} \int_c^1 \left((1-y)^{A_{-\{i,\ell\}}-1} - (1-y)^{A_{-\{i\}}-1} \right) dy \right) \alpha_i (1-c)^{\alpha_i-1} dc \\ &= \alpha_i \sum_{\ell \neq i} \left(\frac{1}{A_{-\{\ell\}}(1+A_{-\{\ell\}})} - \frac{A_{-\{i,\ell\}}}{A_{-\{i\}} A (1+A)} \right). \end{aligned}$$

The remaining results follow by substitution and rearrangement. ■

D.2 Applications with buyer power

In the U.S. DOJ's analysis of the proposed merger of oilfield services providers Halliburton and Baker Hughes, the agency identified the $400 million market of offshore cementing services as a relevant antitrust market.[45] According to the DOJ Complaint, the pre-merger market had essentially three suppliers: Halliburton, Baker Hughes, and Schlumberger.[46] Further, the information in the DOJ's complaint indicates that Halliburton and Baker Hughes had pre-merger market shares of 32% and 24% and that Schlumberger had a pre-merger market share of 43%, with the three suppliers accounting for 99% of the market.[47]

In this application, it seems reasonable to assume (as the merging parties argued) that the buyers, which include BP, Shell, and Exxon-Mobil, have buyer power. These buyers

[45] U.S. v. Halliburton Co. and Baker Hughes Inc., Complaint, Case 1:16-cv-00233-UNA, filed 6 April 2016 (DOJ Complaint).

[46] "In a strategic planning session, Halliburton's cementing executives recognized that this market is already a 'pure oligopoly' among the Big Three" (DOJ Complaint, p. 18).

[47] This can be deduced from the information provided in the DOJ Complaint that Schlumberger's market share was approximately 43%, the combined market share of Halliburton and Baker Hughes was approximately 56%, the pre-merger HHI was approximately 3500, and the post-merger HHI was approximately 5000. Although we can identify the shares of Halliburton and Baker Hughes as approximately 32% and 24%, it is not clear which supplier has the 32% share and which has the 24% share.

are large, sophisticated firms that purchased through competitive procurements. Thus, we calibrate distributions and calculate the coordinated effects index under the assumption of buyer power, but we also contrast the results with the case of no buyer power.

To facilitate the analysis of the case with buyer power, we use the parameterization $G_i(c) = c^{\alpha_i}$ (which implies linear virtual type functions), and we assume that v is sufficiently large that $v \geq \Gamma_i(\overline{c})$ for all i. As an identifying assumption, we assume that $\sum_{i=1}^{4} \alpha_i = 4$. Letting supplier 1 be Schlumberger and letting supplier 2 have market share 34% and supplier 3 have market share 24%, our calibration delivers $\alpha_1 = 0.0760$, $\alpha_2 = 0.0999$, $\alpha_3 = 0.1274$, and $\alpha_4 = 3.6967$. The calculation of the associated coordinated effects index for different sets of coordinators is shown in Table D.1.

Pre-merger		Post-merger	
K	$\mathcal{I}^S(K)$	K	$\mathcal{I}^S(K)$
$\{1,2,3\}$	0.7617	$\{1,\mu_{2,3}\}$	0.6338
$\{1,2\}$	0.4625		
$\{1,3\}$	0.4198		
$\{2,3\}$	0.3780		

Table D.1: Results for the oilfield services market of offshore cementing. Supplier 1 is Schlumberger, with pre-merger share 43%. Suppliers 2 and 3 are Halliburton and Baker Hughes (in unknown order), with pre-merger shares 34% and 24%. We denote the merger of Halliburton and Baker Hughes by $\mu_{2,3}$.

Holding fixed the distributions, without buyer power, we would have instead $\mathcal{I}^S(\{1,2,3\}) = 0.9510$ and $\mathcal{I}^S(\{1,\mu_{2,3}\}) = 0.8960$, which are larger than their corresponding values with buyer power, illustrating that the market would be at even greater risk, before and after the merger, if the buyers were not powerful.

As this shows, the market is at risk despite the presence of powerful buyers. And, holding fixed cost distributions, the market would be at greater risk if buyers were not powerful.

The market is also at risk for coordination by any pair of the suppliers in the Big 3.

D.3 Generalization to multi-unit demand with buyer power

With buyer power, the main obstacle to the generalization to multi-unit supply and demand is that the optimal mechanism is not known when agents have multi-dimensional types. Even if one assumed single-unit suppliers in the pre-merger market, a merger would naturally lead to a multi-unit supplier.

However, all is not lost because there are circumstances in which even multi-unit buyers restrict themselves to buying at most one unit from each individual supplier. This may be due to (non-modelled) preferences for diversification, protection against further hold-up, or imposed by law (as in one of the applications in Section 3.5). Under these circumstances, all that matters for the buyer's optimal mechanism are the distributions of each seller j's lowest cost c_1^j, that is, $G_{j,[1]}(c_1^j)$, which is a one-dimensional variable. Hence, the standard mechanism design tools and results apply.

Let us briefly elaborate. The profit-maximizing mechanism for the buyer subject to incentive compatibility and individual rationality constraints given $n > Q$ is characterized as follows: For notational simplicity, let $c_j \equiv c_1^j$ and $G_j(c_j) \equiv G_{j,[1]}(c_1^j)$ with support $[\underline{c}, \overline{c}]$ and density $g_j(c_j)$ for all $j \in N$. Moreover, to simplify the analysis, assume as before that, for all $j \in N$, the virtual cost function $\Gamma_j(c_j)$ defined by

$$\Gamma_j(c_j) \equiv c_j + \frac{G_j(c_j)}{g_j(c_j)}$$

is increasing in c_j. Then, for a given realization $\mathbf{c} = (c_1, \ldots, c_n)$ and for given $\mathbf{v}$, the profit-maximizing mechanism for the buyer has the allocation rule of purchasing $m \in \{0, \ldots, Q\}$ units from the m suppliers with the lowest *virtual* costs, where, if $m < Q$, m is such that the m-th lowest virtual cost is less than v_m and the $m+1$-st lowest virtual cost exceeds v_{m+1}.

In the dominant strategy implementation of this mechanism, suppliers who do not produce receive (and make) no payments. Each supplier who trades is paid a threshold payment, that is, the highest cost that it could have reported without changing the fact that it trades. This pins down Π_i and $\Pi_i(K)$, and thereby $s_i(K)$ and $\mathcal{I}^S(K)$, just as in the single-unit case. For example, in the special case in which all suppliers are ex ante symmetric with $G_j = G$ for all j and thus $\Gamma_j = \Gamma$ for all j, the optimal mechanism can be implemented as via a second-price auction, in which the reserve price for the l-th unit is $\Gamma^{-1}(v_l)$. If the quantity traded is m, the m successful suppliers are paid $\min\{\Gamma^{-1}(v_m), c_{[m+1]}\}$, where $c_{[m+1]}$ denotes the $m+1$-st lowest cost.

References for Online Appendix

Krishna, V. and E. Maenner (2001): "Convex Potentials with an Application to Mechanism Design," *Econometrica*, 69, 1113–1119.

Loertscher, S. and L. M. Marx (2020): "Asymptotically Optimal Prior-Free Clock Auctions," *Journal of Economic Theory*.

Williams, S. R. (1999): "A Characterization of Efficient, Bayesian Incentive Compatible Mechanisms," *Economic Theory*, 14, 155–180.