

Countervailing Power*

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Abstract

Countervailing power arguments hypothesize that mergers on one side of the market increase social surplus by offsetting power on the other. Despite its popular appeal, countervailing power has met with skepticism and been difficult to formalize. We provide a model with a price formation process whose efficiency depends on agents' bargaining weights. Keeping these fixed, mergers never increase and sometimes decrease social surplus. However, mergers that level the playing field by equalizing bargaining weights can increase social surplus. Moreover, our model with endogenous efficiency of price formation sheds new light on vertical integration, investment incentives, bargaining breakdown, and bargaining externalities.

Keywords: price formation, bargaining power, productive power, vertical integration, investment incentives

JEL Classification: D44, D82, L41

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1 Introduction

The concept of *countervailing power* features prominently in antitrust debates. While the idea that increasing market power on one side of the market to countervail existing market power on the other side has appeal, the concept of countervailing power has been controversial since its beginning, partly because formalizing it has proved challenging. According to the New Palgrave Dictionary, the reason is that “it is difficult to model bilateral monopoly or oligopoly, and there exists no single canonical model.”¹ Obviously, for countervailing power to increase social surplus requires a model with an explicit price formation process in which bargaining powers affect not only the division of social surplus, but also the size of social surplus.²

In this paper, we provide a model that has precisely these features. We consider a procurement problem with one buyer and one or more sellers in which private information pertains to both sides of the market. We assume that the buyer’s value and the sellers’ costs are independent draws from continuous distributions with compact supports. The distributions are common knowledge, but the realized value and costs are the buyer’s and the sellers’ private information. We model the price formation process as an incentive compatible, individually rational mechanism that maximizes the weighted sum of buyer surplus, seller surplus, and social surplus, subject to a no-deficit constraint. The weights in the objective represent the relative bargaining powers of the buyer and the sellers. These weights affect the division of the social surplus when ex post efficient trade is possible, and whether or not ex post efficient trade is possible. Typically, ex post inefficiency is impossible both when the buyer has all the bargaining power and when the sellers have all the bargaining power. Social surplus is maximized with equal bargaining weights. Thus, the setting offers scope for countervailing power.

With this in hand, we derive the following results and insights. First, while a horizontal merger between suppliers that does not affect the bargaining weights never improves social surplus and always makes achieving the first-best more difficult, a horizontal merger that “levels the playing field” by equalizing bargaining weights can improve social surplus. Indeed, such a merger can make the first-best possible when, prior to the merger and the change in bargaining weights, the first-best was not achievable because the price formation process was too strongly tilted towards the buyer. Thus, our framework allows for the possibility of countervailing power.

To develop a sense for how countervailing power is obtained, it is useful to distinguish

¹Snyder (2008, p. 1188).

²As Carlton and Israel (2011, p. 128) put it: “For changes in bargaining outcomes due to a buyer merger to create a true efficiency, it must be that, post-merger, the parties are better able to arrive at an optimal non-linear price schedule, perhaps due to lower transactions costs, which moves output closer to the competitive level.”

between an agent's *productive power* (or *strength*) and its *bargaining power*. An agent's productive power is its value or cost, or the distribution from which its value or cost is drawn. Productively stronger buyers have, or tend to have, higher values, and productively stronger sellers have, or tend to have, lower costs. In contrast, an agent's bargaining power captures its ability to bias the price formation process in its favor. While, empirically, productive power and bargaining power may be correlated, conceptually, they are distinct and independent. As a case in point, both business and leisure air travellers are price-takers and hence have the same amount of bargaining power, that is, none. However, business passengers are productively stronger, which is why they are charged higher prices. We show that changes in bargaining power are necessary, without being sufficient, for there to be countervailing power.

Second, vertical integration between the buyer and a supplier can create a bilateral trade problem à la Myerson and Satterthwaite (1983) in which the first-best is impossible when it was possible prior to integration. This occurs, for example, when vertical integration leaves only one independent seller in the market and when the buyer's lowest possible value before integration exceeds the suppliers' highest possible cost. In situations like these, vertical integration is thus socially harmful. It is so in ways and for reasons that are absent when the efficiency of the price formation process is exogenously fixed. Of course, vertical integration also eliminates a bilateral trade problem, namely that within the newly created entity. Therefore, the social surplus effects of vertical integration can go either way. Importantly, we show that under appropriate assumptions, the likely effects of vertical integration can be estimated using pre-integration data. Although our analysis does not imply that vertical integration is universally bad, it does show that a presumption that vertical integration improves social surplus is not warranted.

Third, our incomplete information approach also sheds new light on incentives to invest. These incentives are at the center stage of current debates in antitrust and have been at the heart of the theory of the firm for more than thirty years. We assume that investments improve distributions in the sense of first-order stochastic dominance shifts without affecting the supports and that, as in the theory of the firm, investments are not contractible, which implies that the price formation process does not vary with investments. In this setup, we show that the equilibrium investments are efficient if and only if the price formation process is efficient.³ This is the opposite of the result obtained in the theory of the firm, where efficient price formation (for example, Nash bargaining) with *complete* information induces hold up and thereby inefficient investments. Thus, the privacy of information in incomplete information models protects agents against hold up. To understand the intuition, recall that with incomplete information and incentive

³As we discuss, a similar result holds for investment by the suppliers in quality.

compatible prices that ensure an efficient allocation, every agent has to be paid his or her social marginal product.⁴ But this is exactly the condition that has to be satisfied for investments to be efficient. Hence, with incomplete information, investments are efficient if and only if the price formation process induces the ex post efficient allocation. Beyond highlighting another fundamental difference to complete information models, this analysis allows us to connect market structure, which affects the efficiency of the price formation process, with the efficiency of investment.

In extensions, we discuss the implications for bargaining breakdown of having a price formation process whose efficiency properties are endogenous, we extend the setup to allow for multi-object demand by the buyer and for supplier-specific preferences, and we show that bargaining externalities naturally arise in that extended setup.

The remainder of the paper is structured as follows. Section 2 introduces the setup. In Section 3, we define the price formation process. Section 4 derives the results for countervailing power, vertical integration, and investment. In Section 5, we use our setup to analyze bargaining breakdown and extend the setup to allow for buyers with multi-object demand and seller-specific preferences. Section 6 discusses related literature, and Section 7 concludes the paper. The formal mechanism design results and longer proofs are relegated to appendices.

2 Setup

We consider a procurement setup with M sellers indexed by $i \in \mathcal{M} \equiv \{1, \dots, M\}$, each with the capacity to produce one unit of the good, and one buyer, indexed by B , with demand for one unit. We let $\mathcal{N} = \mathcal{M} \cup \{B\}$ and $n \equiv M + 1$ denote the set and total number of all agents, respectively.

The buyer draws its value v from a distribution F with support $[\underline{v}, \bar{v}]$ and density $f(v)$ that is positive for all $v \in [\underline{v}, \bar{v}]$. Seller i draws its cost c_i independently from distribution G_i with support $[\underline{c}, \bar{c}]$ and density $g(c)$ that is positive for all $c \in (\underline{c}, \bar{c}]$. We assume that F and G_1, \dots, G_M are independent and common knowledge, while the realized value v and the realized costs c_1, \dots, c_M are the private information of the buyer and individual sellers, respectively. To save on notation, we ignore ties among the agents' types.

The buyer and the sellers have quasilinear preferences. The payoff of seller i with cost c_i when producing the good with probability q_i and receiving the payment p is $p - c_i q_i$. The buyer's payoff when receiving the object from seller i with probability q_i and making the payment p is $\sum_{i \in \mathcal{M}} v q_i - p$. Under (ex post) efficiency (and ignoring ties), trade occurs between the buyer and seller i if and only if $v - c_i > \max_{j \neq i} \{0, v - c_j\}$. The problem

⁴See, for example, Green and Laffont (1977) and Holmström (1979).

is trivial if $\bar{v} \leq \underline{c}$ because then it is never ex post efficient to have trade with any seller. Therefore, from now on, we assume that $\bar{v} > \underline{c}$.

For $M = 1$, our setup encompasses the classical Myerson-Satterthwaite setting (Myerson and Satterthwaite, 1983). In contrast to their assumptions, we allow for the possibility that $\underline{v} \geq \bar{c}$ without requiring it. We refer to the case with $\underline{v} \geq \bar{c}$ as the case of *nonoverlapping supports* and the case with $\underline{v} < \bar{c}$ as the case with *overlapping supports*. As Myerson and Satterthwaite (1983) showed, ex post efficient trade is impossible if and only if supports are overlapping. Thus, with one seller and nonoverlapping supports, we obtain ex post efficient trade even under incomplete information.

Allowing for nonoverlapping supports permits us to contrast the results for the price formation process with incomplete information to those under complete information. Even though price formation is efficient both in our setting with one seller and nonoverlapping supports and in settings with complete information, the predictions of the models are starkly different when trade and price formation are preceded by noncontractible investments. In our setting, the protection that privacy of information provides against hold-up extends to the setup with nonoverlapping supports. Consequently, the equilibrium predictions of the models are different.

We denote the buyer's virtual value function by $\Phi(v) \equiv v - \frac{1-F(v)}{f(v)}$ and seller i 's virtual cost function by $\Gamma_i(c) \equiv c + \frac{G_i(c)}{g_i(c)}$.⁵ We assume that the virtual value and virtual cost functions are increasing.⁶ For $a \in [0, 1]$, we define the a -weighted virtual value function by $\Phi_a(v) \equiv v - a\frac{1-F(v)}{f(v)}$ and the a -weighted virtual cost function for seller i by $\Gamma_{i,a}(c) \equiv c + a\frac{G_i(c)}{g_i(c)}$. Observe that monotonicity of $\Phi(v)$ and $\Gamma_i(c)$ implies that $\Phi_a(v)$ and $\Gamma_{i,a}(c)$ are also monotone.

We endow the agents with bargaining weights, where the bargaining weight of the buyer is denoted by $b \in [0, 1]$ and the bargaining weight of seller i is denoted by $s_i \in [0, 1]$, with $b + \sum_{i \in \mathcal{M}} s_i = 1$ so that the vector of bargaining weights is an element of the M -dimensional simplex.

It is worth highlighting two important features of this setup. First, we allow both the buyer's value and the sellers' costs to be random variables whose realizations are the agents' private information. This makes the setup symmetric with respect to the privacy of information, with the important consequence that ex post efficiency need not be possible.⁷ Second, we allow for arbitrary bargaining weights (b, \mathbf{s}) , and we allow these

⁵If $f(\bar{v}) = 0$, define $\Phi(\bar{v})$ to be the limit of $\Phi(v)$ as v approaches \bar{v} from below, and if $g(\underline{c}) = 0$, define $\Gamma(\underline{c})$ to be the limit of $\Gamma(c)$ as c approaches \underline{c} from above.

⁶The assumption of increasing virtual type functions can be relaxed through the use of "ironing."

⁷To avoid the resulting informed-principal problem when the buyer chooses the mechanism, we model the mechanism design problem as one where a third party without private information—such as a broker or "the market"—organizes the exchange. Although our setup has properties that are sufficient for the informed-principal problem to have no material consequences (see Mylovanov and Tröger, 2014), it seems

to change in the wake of a merger. As the following analysis shows, changes in the agents’ bargaining powers are necessary for there to be countervailing power.

Although we focus on a procurement setting with one buyer and one or more sellers, all of our results extend with the appropriate adjustments to a sales auction with one seller and one or more buyers. In addition, as we show in Section 5, we can extend the setup to allow for multi-unit demand and seller-specific preferences.

3 Price formation process

As does essentially any economic model of a market, our model relies on assumptions about how prices and allocations are determined. Using mechanism design concepts, any *price formation process* is a mechanism that maps agents’ types into prices and probabilities of trading.⁸ For our purposes, we model the price formation process as a direct mechanism (\mathbf{q}, \mathbf{m}) that maps the buyer’s and sellers’ types to quantities (or probabilities of trade) for the sellers, $\mathbf{q} : [\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}]^M \rightarrow [0, 1]^M$, and transfers for the buyer and sellers, $\mathbf{m} : [\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}]^M \rightarrow \mathbb{R}^n$. This *price formation mechanism* is required to satisfy incentive compatibility, individual rationality, and no deficit. A direct mechanism is *incentive compatible* if it is in the best interest of every agent to report its type truthfully to the mechanism.⁹ A mechanism is *individually rational* if each agent, for every possible type, is weakly better off participating in the mechanism than walking away. Normalizing the payoffs of not trading and of walking away—that is, the value of the outside option—to zero, a mechanism is individually rational if each agent always has a payoff of at least zero.¹⁰ A direct mechanism has *no deficit* if the expected payment from the buyer is equal to the sum of the expected payments to the sellers. For more formal definitions, see Appendix A.1.

to wise circumvent the associated technicalities. Of course, by choosing the bargaining weight $b = 1$, we still obtain the buyer-optimal mechanism, just as we would if we assumed that the buyer organizes the exchange and has all the bargaining power.

⁸This is true for complete information models as it is for incomplete information models. However, for complete information models, the dependence on agents’ types is often degenerate insofar as each agent has only one (known) type.

⁹For private values settings, one typically distinguishes between Bayesian incentive compatibility, which stipulates that reporting truthfully is a Bayes Nash equilibrium, and dominant strategy incentive compatibility, which requires that reporting truthfully be a dominant strategy equilibrium. In our setting, nothing is gained or lost by using one notion of incentive compatibility rather than the other, which is why we do not distinguish between the two.

¹⁰Again, in our setup, nothing is gained or lost by using interim versus ex post individual rationality, so as with incentive compatibility, we do not distinguish between the two. To see this, note that an incentive compatible, interim individually rational mechanism has an equivalent dominant strategy implementation (Gershkov et al., 2013), which satisfies ex post individual rationality.

Fix a mechanism (\mathbf{q}, \mathbf{m}) and type realizations (v, \mathbf{c}) . Then, the buyer's surplus is

$$U_B(v, \mathbf{c}) \equiv v \sum_{i \in \mathcal{M}} q_i(v, \mathbf{c}) - m_B(v, \mathbf{c}),$$

while the surplus of seller i is given by

$$U_i(v, \mathbf{c}) \equiv m_i(v, \mathbf{c}) - c_i q_i(v, \mathbf{c}).$$

The budget surplus generated by the mechanism is

$$R(v, \mathbf{c}) \equiv m_B(v, \mathbf{c}) - \sum_{i \in \mathcal{M}} m_i(v, \mathbf{c}), \quad (1)$$

while the welfare (or social surplus) generated by the mechanism is

$$W(v, \mathbf{c}) \equiv \sum_{i \in \mathcal{M}} (v - c_i) q_i(v, \mathbf{c}) = R(v, \mathbf{c}) + U_B(v, \mathbf{c}) + \sum_{i \in \mathcal{M}} U_i(v, \mathbf{c}).$$

A mechanism is a *first-best* mechanism if it maximizes $\mathbb{E}_{v, \mathbf{c}}[W(v, \mathbf{c})]$ subject to incentive compatibility and individual rationality, and it is a *second-best* mechanism if it maximizes that objective with the additional constraint of no deficit. The first-best and second-best quantities or outcomes are then the quantities or outcomes that arise in the first-best and second-best mechanisms, respectively.

To incorporate bargaining weights for the buyer and sellers, we define *weighted welfare* as follows:

$$W_{b, \mathbf{s}}(v, \mathbf{c}) \equiv R(v, \mathbf{c}) + \left(\min\{n \cdot b, 1\} U_B(v, \mathbf{c}) + \sum_{i \in \mathcal{M}} \min\{n \cdot s_i, 1\} U_i(v, \mathbf{c}) \right),$$

where, relative to unweighted welfare, weighted welfare allocates weight n (the total number of agents) over buyer and seller surplus with weight b on the buyer's surplus and weight s_i on seller i 's surplus, with the constraint that the weight on any agent's surplus cannot exceed one.

It follows that when the buyer and sellers have equal bargaining weights, the associated weighted welfare is equal to unweighted welfare:

$$W_{\frac{1}{n}, \dots, \frac{1}{n}}(v, \mathbf{c}) = W(v, \mathbf{c}).$$

Although the price formation mechanism defined below takes bargaining weights into account, we evaluate market outcomes in the usual way according to the expected value

of unweighted welfare, $\mathbb{E}_{v,\mathbf{c}}[W(v, \mathbf{c})]$.

Given bargaining weights (b, \mathbf{s}) , we define the *price formation mechanism* as the mechanism that maximizes expected weighted welfare, $\mathbb{E}_{v,\mathbf{c}}[W_{b,\mathbf{s}}(v, \mathbf{c})]$, subject to incentive compatibility, individual rationality, and no deficit. Using arguments that were first developed in the working paper version of Gresik and Satterthwaite (1989) and that were first used in published form in Myerson and Satterthwaite (1983), this price formation mechanism is the mechanism that maximizes

$$\mathbb{E}_{v,\mathbf{c}}[\alpha R(v, \mathbf{c}) + (1 - \alpha)W_{b,\mathbf{s}}(v, \mathbf{c})],$$

subject to incentive compatibility and individual rationality, and that has the smallest value of $\alpha \in [0, 1]$ such that the no deficit condition is satisfied. We denote this smallest α by $\alpha^*(b, \mathbf{s})$.¹¹ Ignoring ties, which have probability zero, the allocation rule for this mechanism is as characterized in the following lemma:

Lemma 1. *The price formation mechanism has the allocation rule*

$$q_{i,\alpha}(v, \mathbf{c}; b, \mathbf{s}) \equiv \begin{cases} 1 & \text{if } \Phi_{\max\{\alpha, 1-bn(1-\alpha)\}}(v) > \Gamma_{i,\max\{\alpha, 1-s_in(1-\alpha)\}}(c_i) \\ & \text{and } \Gamma_{i,\max\{\alpha, 1-s_in(1-\alpha)\}}(c_i) = \min_{j \in \mathcal{M}} \Gamma_{j,\max\{\alpha, 1-s_jn(1-\alpha)\}}(c_j), \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

with $\alpha = \alpha^*(b, \mathbf{s})$.

Proof. See Appendix B.

An immediate implication of Lemma 1 is that the probability of trade, and hence social surplus, are decreasing in $\alpha^*(b, \mathbf{s})$.

In addition, as shown in the following lemma, under symmetric bargaining weights $b = s_i = 1/n$ for all $i \in \mathcal{M}$, the price formation mechanism delivers the second-best quantities, which are defined by trade with seller i when $\Phi_\alpha(v) > \Gamma_{i,\alpha}(c_i) = \min_{j \in \mathcal{M}} \Gamma_{j,\alpha}(c_j)$, with the smallest $\alpha \in [0, 1]$ such that the no-deficit constraint is satisfied.

Lemma 2. *With equal bargaining weights, the price formation mechanism generates the second-best quantities.*

¹¹While we do not pursue this here, our approach generalizes directly to the requirement that the mechanism needs to generate a budget surplus of $K \in \mathbb{R}$, which is not more than the maximum budget surplus that any incentive compatible, individually rational mechanism can generate. The second-best mechanism that generates K , but otherwise maximizes the same objective, is then characterized by a mechanism with an allocation rule $q_{i,\alpha_K(b,\mathbf{s})}(v, \mathbf{c}; b, \mathbf{s})$ as defined in Lemma 1, where $\alpha_K(b, \mathbf{s})$ is an increasing function of K . Interpreted in this way, we have $\alpha^*(b, \mathbf{s}) = \alpha_0(b, \mathbf{s})$.

Proof. See Appendix B.

Further, for certain setups with extreme bargaining weights or equal bargaining weights, the characterization of α^* is straightforward:

Lemma 3. *For $[\underline{c}, \bar{c}] = [\underline{v}, \bar{v}]$, we have $\alpha^*(1, 0, \dots, 0) = \frac{M}{M+1}$ and, assuming symmetric sellers, i.e., $G_i = G$ for all $i \in \mathcal{M}$, we have $\alpha^*(0, \frac{1}{M}, \dots, \frac{1}{M}) = \frac{1}{M+1}$.*

Proof. See Appendix B.

We are left to augment the allocation rule of Lemma 1 with a consistent payment rule. By the *payoff equivalence theorem* (see, e.g., Myerson, 1981; Krishna, 2002; Börgers, 2015),¹² the interim expected payoff of an agent, $U_B(v)$ for the buyer or $U_{S_i}(c_i)$ for seller i , is pinned down by the allocation rule and incentive compatibility up to a constant that is equal to the interim expected payoff of the worst-off type for that agent. Incentive compatibility implies further that \underline{v} and \bar{c} are the *worst-off types* of the buyer and sellers, respectively.¹³

Thus, to complete the definition of the price formation mechanism, all that remains to be done is to define these constants. By standard mechanism design arguments, the expected budget surplus for the mechanism with the allocation rule in Lemma 1, not including the constants reflecting payments to worst-off types, can be written in terms of the allocation rule and virtual types as follows:

$$\Pi_\alpha \equiv \sum_{i \in \mathcal{M}} \mathbb{E}_{v, \mathbf{c}} [(\Phi(v) - \Gamma_i(c_i)) \cdot q_{i, \alpha}(v, \mathbf{c}; b, \mathbf{s})]. \quad (3)$$

Of course, if $\alpha^*(b, \mathbf{s}) > 0$, then the no deficit constraint binds and it must be that $\Pi_{\alpha^*(b, \mathbf{s})} = 0$, in which case the question of how to allocate the budget surplus is moot. In contrast, when $\alpha^*(b, \mathbf{s}) = 0$, $\Pi_0 > 0$ is possible. In this case, we assume that Π_0 is allocated among the buyer and sellers according to their bargaining weights. Specifically, letting $U_B(v) \equiv \mathbb{E}_{\mathbf{c}}[U_B(v, \mathbf{c})]$ and $U_{S_i}(c_i) \equiv \mathbb{E}_{v, \mathbf{c}_{-i}}[U_{S_i}(v, \mathbf{c})]$, we assume that the price formation mechanism with bargaining weights (b, \mathbf{s}) has,

$$U_B(\underline{v}) = b\Pi_{\alpha^*(b, \mathbf{s})} \quad \text{and, for } i \in \mathcal{M}, \quad U_i(\bar{c}) = s_i\Pi_{\alpha^*(b, \mathbf{s})}. \quad (4)$$

As an illustration, consider a bilateral trade problem, i.e., assume $M = 1$. As men-

¹²Sometimes, the payoff equivalence theorem is also referred to as the revenue equivalence theorem. However, revenue equivalence is an implication of the payoff equivalence theorem, so the alternative label is somewhat loose.

¹³That is, for any mechanism satisfying incentive compatibility, $\underline{v} \in \arg \min_{v \in [\underline{v}, \bar{v}]} U_B(v)$ and $\bar{c} \in \arg \min_{c \in [\underline{c}, \bar{c}]} U_i(c)$.

tioned, efficient trade is impossible if and only if the supports overlap, i.e., $\underline{v} < \bar{c}$. Because by Lemma 2 equal bargaining weights yield the second-best outcome, this implies that $\alpha^*(1/2, 1/2) > 0$ holds if and only if the supports overlap. With nonoverlapping supports, i.e., $\underline{v} \geq \bar{c}$, the incentive compatibility and individual rationality constraints can be satisfied by charging the buyer \underline{v} and paying the seller \bar{c} , generating a surplus of $\Pi_0 = \underline{v} - \bar{c} \geq 0$. With $b = s_1 = 1/2$, this surplus is shared evenly between the buyer and the seller.

The outcome of the price formation process is then given by the expected buyer and seller payoffs from the price formation mechanism.

Proposition 1. *Given bargaining weights (b, \mathbf{s}) , the price formation process generates expected payoffs $u_B = U_B(\underline{v}) + \mathbb{E}_v \left[\sum_{i \in \mathcal{M}} \int_{\underline{v}}^v \mathbb{E}_{\mathbf{c}} [q_{i, \alpha^*(b, \mathbf{s})}(x, \mathbf{c}; b, \mathbf{s})] dx \right]$ and, for $i \in \mathcal{M}$, $u_i = U_i(\bar{c}) + \mathbb{E}_{c_i} \left[\int_c^{\bar{c}} \mathbb{E}_{v, \mathbf{c}_{-i}} [q_{i, \alpha^*(b, \mathbf{s})}(v, x, \mathbf{c}_{-i}; b, \mathbf{s})] dx \right]$, with $U_B(\underline{v})$ and $U_i(\bar{c})$ given by (4).*

Proof. See Appendix B.

To illustrate further, reconsider the bilateral trade problem of Myerson and Satterthwaite and assume, for concreteness, that $\underline{v} = \underline{c} = 0$ and $\bar{v} = \bar{c} = 1$.¹⁴ When $b = 1$, the price formation process is the buyer-optimal mechanism, which consists of the buyer of type v making the take-it-or-leave-it offer $\Gamma_1^{-1}(v)$ to the seller. For example, when G_1 is the uniform distribution, the buyer offers $v/2$, yielding $u_B(1) = 1/12$ and $u_1(1) = 1/24$, where the arguments reflect the value of b . Conversely, for $b = 0$ and $s_1 = 1$, we have the seller-optimal mechanism. In this mechanism, the seller with cost c makes the take-it-or-leave-it offer $\Phi^{-1}(c)$. For F uniform, this is $(c + 1)/2$, yielding $u_1(0) = 1/12$ and $u_B(0) = 1/24$.

With interior bargaining weights, that is, $b \in (0, 1)$, one can, of course, use randomized take-it-or-leave-it offers with the buyer (resp. seller) making the offer with probability b (resp. $1 - b$). Thus, any convex combination between $(u_B(1), u_1(1))$ and $(u_B(0), u_1(0))$ is achievable, as Figure 1(a) illustrates.

Importantly, however, one can in general do better than using randomized take-it-or-leave-it offers by allowing the allocation rule to vary with the bargaining weights beyond just being a linear combination of the extremes.¹⁵ Indeed, as is evident from Lemma 1, this is what happens in our price formation mechanism whose allocation rule depends on the bargaining weights.

Interestingly, this is also the case for the k -double auction of Chatterjee and Samuelson (1983). To see this, recall that given $k \in [0, 1]$, the buyer and seller in a k -double auction

¹⁴Of course, the assumption of identical supports imposes some restrictions. Given this assumption, setting $\underline{v} = 0$ and $\bar{v} = 1$ is then an innocuous normalization.

¹⁵It is perhaps worth noting that this reasoning is essentially the same as that invoked by Samuelson (1949) to demonstrate that with constant returns to scale, the production possibility frontier is concave.

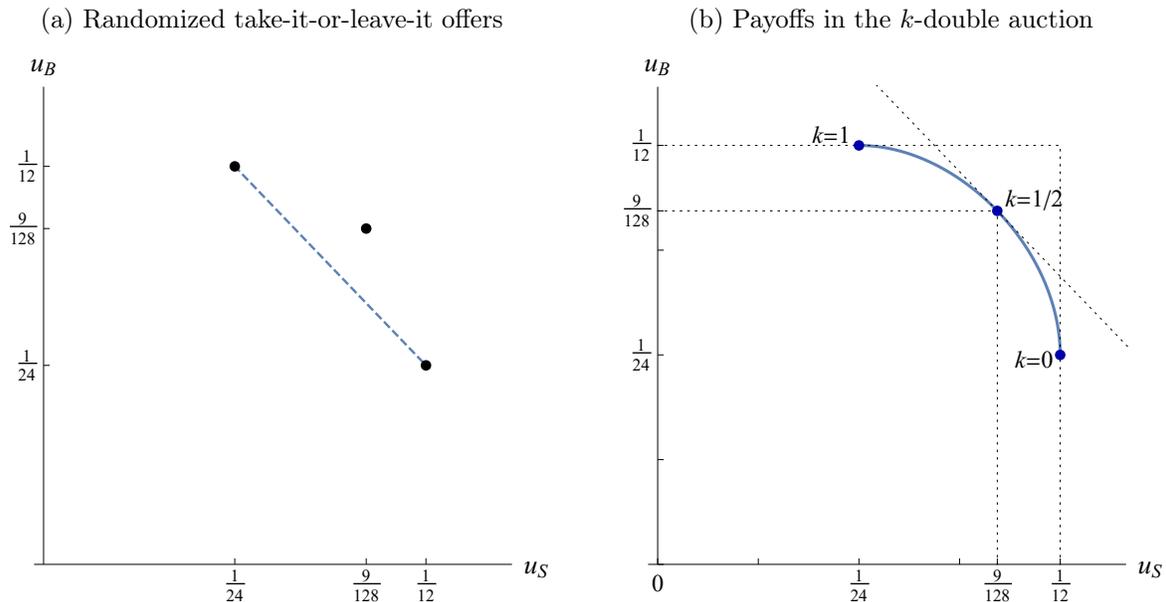


Figure 1: Panel (a): Buyer and seller payoffs for randomized take-it-or-leave-it offers and for the k -double auction with $k = 1/2$. Panel (b): Payoffs in the k -double auction for all $k \in [0, 1]$. Both panels assume F and G_1 are uniform on $[0, 1]$ and $\mathcal{M} = \{1\}$.

simultaneously submit bids p_B and p_S , and trade occurs at the price $p = kp_B + (1 - k)p_S$ if and only if $p_B \geq p_S$. By construction, the k -double auction never incurs a deficit. If F and G_1 are uniform on $[0, 1]$, then the linear Bayes Nash equilibrium of the k -double auction reduces to take-it-or-leave-it offers for $k \in \{0, 1\}$,¹⁶ but for $k = 1/2$, the payoffs are $9/128$ for each agent, yielding social surplus of $9/64$. This is larger than the social surplus of $1/8$ under take-it-or-leave-it offers.¹⁷ The comparison between the payoffs from the k -double auction with $k = 1/2$ and take-it-or-leave-it offers is shown in Figure 1(a), while Figure 1(b) shows the full frontier of equilibrium payoffs for the k -double auction as k varies from zero to one. In the next section, we discuss the connection with our price formation mechanism.

¹⁶In the linear Bayes Nash equilibrium, the buyer of type v bids $p_B(v) = (1 - k)k/(2(1 + k)) + v/(1 + k)$ and the seller with cost c bids $p_S(c) = (1 - k)/2 + c/(2 - k)$. Thus, for $k = 1$, $p_B(v) = v/2$ and $p_S(c) = c$, and for $k = 0$, $p_B(v) = v$ and $p_S(c) = (c + 1)/2$.

¹⁷Myerson and Satterthwaite (1983) were the first to observe that for $k = 1/2$ and F and G_1 uniform, the k -double auction yields the second-best outcome.

4 Results

We now illustrate the usefulness of the price formation mechanism introduced in the previous section, whose efficiency properties are endogenous, by applying it to three questions that are pertinent in industrial organization and beyond. Specifically, we show that our approach offers the possibility of a countervailing power defense for mergers, that social surplus decreasing (and anticompetitive) vertical integration is a robust phenomenon in our setting, and that there is a tight connection between the efficiency of the price formation process and the incentives to invest.

4.1 Countervailing power

The question of whether a merger can be defended on the grounds that it endows merging parties with *countervailing power* that somehow “levels the playing field” features prominently in concurrent antitrust debates and cases. For example, in a merger context, the Australian government’s 1999 (now superseded) guidelines stated: “If pre-merger prices are distorted from competitive levels by market power on the opposite side of the market, a merger may actually move prices closer to competitive levels and increase market efficiency. For example, a merger of buyers in a market may create countervailing power which can push prices down closer to competitive levels” (ACCC, 1999, para. 5.131). Ho and Lee (2017) find evidence of countervailing power, estimating that mergers among insurers increase the insurers’ bargaining power in negotiations with hospitals. Based on an analysis of hundreds of mergers, Bhattacharyya and Nain (2011, p. 99) find outcomes that are “consistent with the creation of buyer power through downstream consolidation to countervail upstream market power.”

Despite the relevance of the issue, a major obstacle to analyzing the effects of countervailing power in existing modelling approaches is that these take the efficiency of the price formation process as given. This is true for all oligopoly models, in which agents on one side of the market (typically buyers) are assumed to be price-takers. It also applies to the randomized take-it-or-leave-it offers model that give rise to the straight line in Figure 1(a). In contrast, as illustrated in Figure 1(a,b), if the equalization of bargaining weights changes the price formation mechanism from, for example, one based on take-it-or-leave-it offers to one based on a k -double auction with $k = 1/2$, then a change in bargaining weights has an impact on social surplus.

The price formation mechanism that we study captures the effects of bargaining weights on social surplus because the efficiency of the mechanism varies with bargaining weights.¹⁸ As noted by Williams (1987), for the case of $M = 1$ and uniform distribu-

¹⁸For experimental results consistent with the our price formation mechanism, see Valley et al. (2002,

tions, the outcomes in the price formation mechanism that we study are the same as in the linear Bayes Nash equilibrium of the k -double auction of Chatterjee and Samuelson (1983) when the bargaining weights are $(b, s_1) = (k, 1 - k)$. Because an equalization of bargaining weights moves the outcome towards the second-best outcome and away from less efficient outcomes, there is social surplus increasing countervailing power.

In general, when F and G_1 are not uniform or $M > 1$, then for $k \in (0, 1)$, the k -double auction does not yield same payoffs as the price formation mechanism with bargaining weights (Williams, 1987). However, as we discuss in Appendix A.4, for general distributions and any number of symmetric sellers, the price formation mechanism with bargaining weights can be implemented by a fee-setting mechanism in which trade occurs via a broker (Loertscher and Niedermayer, 2019).

For the setting with general distributions and $M \geq 1$, we can now define the *Williams frontier*. Assuming that each seller i has equal bargaining weight $(1 - b)/M$, we can write u_B and u_i as a function of only the buyer's bargaining weight b , and then, letting $u_S(b) \equiv \sum_{i \in \mathcal{M}} u_i(b)$, the *Williams frontier* is the set of payoff pairs given by:

$$\mathcal{F} \equiv \{(u_S(b), u_B(b)) \mid b \in [0, 1]\},$$

with associated mapping $\omega : [u_S(1), u_S(0)] \rightarrow [u_B(0), u_B(1)]$ defined by

$$\omega(u) = \max\{y \mid (u, y) \in \mathcal{F}\}.$$

As discussed above, for the special case of $M = 1$ and F and G_1 uniform on $[0, 1]$, the Williams frontier coincides with the payoff frontier for the k -double auction, which is depicted in Figure 1(b).

As we now show, the Williams frontier is concave.

Proposition 2. *The Williams frontier ω is concave.*

Proof. See Appendix B.

As foreshadowed in footnote 15, the concavity of the Williams frontier follows a logic that is reminiscent of Paul Samuelson's argument that the production possibility frontier is concave under constant returns to scale. Given bargaining weights b and $s_i = (1 - b)/M$ for all $i \in \mathcal{M}$, the price formation mechanism could randomize over the mechanism that is optimal for the buyer and the mechanism that is (jointly) optimal for sellers. Hence, any linear combination between $(u_S(1), u_B(0))$ and $(u_S(0), u_B(1))$ can be achieved. This corresponds to the thought experiment of Samuelson (1949, pp. 184–185) whereby equal

Fig. 3.A). See Larsen (2018) on the first-best and second-best frontiers for wholesale used cars.

proportions of all inputs are shifted from one sector to another, with which, in his words, a “neophyte bureaucrat might be satisfied.” However, in general, one can do better by reoptimizing.

The concavity of the Williams frontier implies that a change in the bargaining weights that moves them closer to symmetry can increase social surplus. When the buyer’s bargaining weight is b and the sellers have equal bargaining weights $\frac{1-b}{M}$, then we have for all $b \in [0, 1]$,

$$u_B(b) + u_S(b) \leq u_B(1/n) + u_S(1/n).$$

Further, if a merger leads to an equalization of the bargaining power in the post-merger market, then the merger (combined with countervailing power) can increase efficiency. As an example, Figure 2(a) shows a case in which a merger reduces social surplus if the buyer has all the bargaining power both before and after the merger, but a merger increases social surplus if the buyer’s bargaining weight falls from 1 before the merger to 1/2 after the merger. Indeed, Figure 2(b) provides an example in which countervailing power restores full efficiency to the post-merger market—specifically, if the buyer has all the bargaining weight prior to the merger, then the pre-merger outcome is not fully efficient, but with equal bargaining weights in the post-merger market, the outcome is fully efficient.

Given M suppliers with symmetric bargaining weights, we define a “merger with countervailing power” to mean a supplier merger that reduces the buyer’s bargaining weight from a value greater than $1/M$ to a value closer to $1/M$, and a “merger with no countervailing power” to mean a supplier merger that does not affect the buyer’s bargaining weight. Then we have the following result:

Corollary 1. *A merger with countervailing power is no more harmful to welfare than the same merger with no countervailing power and is, in some settings, welfare increasing.*

4.2 Vertical integration

We now analyze vertical integration between a buyer and a supplier. We assume that the integrated entity can efficiently solve its internal agency problem. This assumption is standard and can be rationalized on the grounds that integration slackens the individual rationality constraints within the integrated entity.

The price formation mechanism following vertical integration between the buyer and supplier i is as before, but with the vertically integrated firm acting as a buyer with value $w = \min\{v, c_i\}$, whose distribution is $1 - (1 - F(w))(1 - G_i(w))$, and attempting to procure from one of the nonintegrated suppliers. If there is no trade between the vertically integrated firm and the nonintegrated suppliers, then the integrated firm has payoff equal to $\max\{0, v - c_i\}$ due to internal sourcing, and the nonintegrated sellers have payoffs of

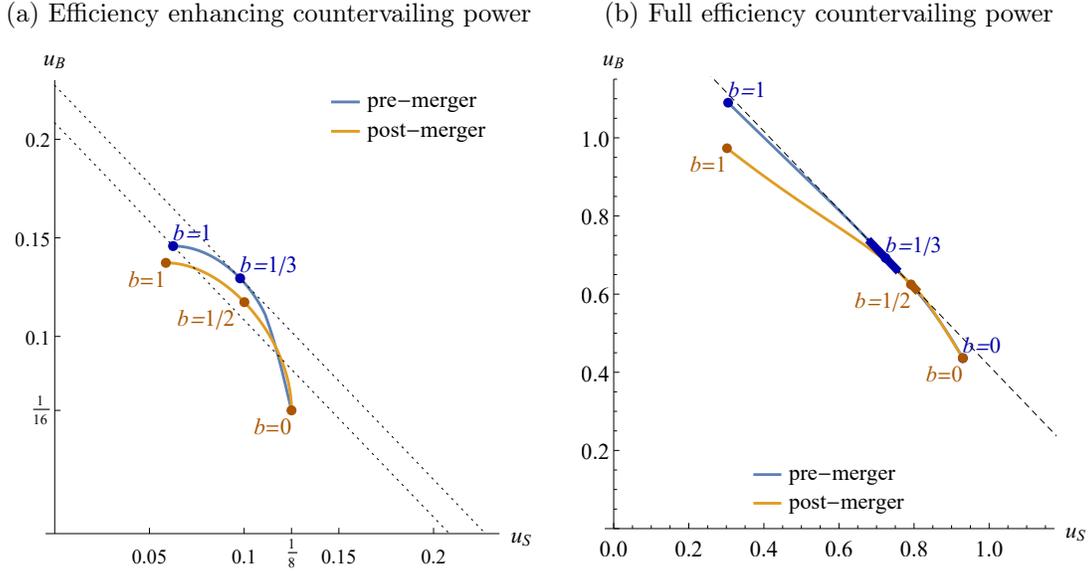


Figure 2: Williams frontiers for 2 pre-merger sellers (blue) and 1 post-merger seller (orange), where the post-merger seller’s cost is the minimum of two independent cost draws from the pre-merger distributions. Panel (a): Assumes that buyer’s and pre-merger sellers’ types are drawn from the uniform distribution on $[0, 1]$. Panel (b): Assumes that the pre-merger sellers draw their costs from the uniform distribution on $[0, 1]$ and the buyer draws its value from the uniform distribution on $[1.25, 2.25]$. The pre-merger market is fully efficient for $b \in [0.27, 0.41]$, and the post-merger market is fully efficient for $b \in [0.42, 0.50]$, as indicated by thick portions of the blue and orange lines, respectively, that coincide with the dashed line.

zero.

Consider first a bilateral trade setting with overlapping supports before integration (i.e., $M = 1$ and $\underline{v} < \bar{c}$). Because ex post efficient trade is impossible when the buyer and seller are independent entities, it follows immediately from our assumption that the integrated entity can resolve the internal agency problem that vertical integration can increase social surplus. It does so by essentially eliminating a Myerson-Satterthwaite problem. We state this as follows:

Proposition 3. *With one seller and overlapping supports, vertical integration increases social surplus.*

By Proposition 3, vertical integration can eliminate a Myerson-Satterthwaite problem. However, it can also create one, as we show now.

The assumption of nonoverlapping supports implies that prior to vertical integration, ex post efficient trade is possible. Hence, vertical integration cannot possibly increase social surplus. This leaves the question of whether vertical integration could be neutral. The following proposition shows that the answer is negative.

Proposition 4. *With two or more sellers and nonoverlapping supports, vertical integration decreases social surplus.*

Proof. After integration between the buyer and seller i , the buyer’s willingness to pay is the cost realization of its supplier, that is, $w = c_i$, whose support is $[\underline{c}, \bar{c}]$. Thus, we have a generalized Myerson-Satterthwaite problem (generalized insofar as there is one buyer but $M - 1 \geq 1$ sellers). For this setting, impossibility of efficient trade obtains (see, e.g., Delacrétaz et al., 2019). ■

Proposition 4 provides a clear-cut case in which vertical integration is harmful from the perspective of society. This result, as well as the result in Proposition 3, is robust in that it does not depend on specific assumptions about distributions or beliefs of agents. Indeed, because there is always a dominant strategy implementation of the price formation mechanism, beliefs play no role. Moreover, we obtain social surplus decreasing vertical integration without imposing *any* restrictions on the contracting space and without invoking exertion of market power by any player (above and beyond requiring individual rationality and incentive compatibility constraints to be satisfied). These are noticeable differences relative to the post-Chicago school literature on vertical contracting and integration, whose predictions rely on assumptions about beliefs, feasible contracts, and/or market power.¹⁹

At the heart of both Proposition 3 and Proposition 4 is the fact that the efficiency of the price formation process is endogenous in our setting. (Of course, our results do rely, inevitably, on support assumptions.) The elimination of a Myerson-Satterthwaite problem through vertical integration is the incomplete-information analogue to the classic double mark-up problem. In contrast to the classical literature, however, there is now a new effect, namely that the market with the remaining suppliers becomes less efficient. Further, it is possible for this latter effect to dominate so that the market as a whole is made less efficient as a result of vertical integration.²⁰

4.3 Investment

Investment incentives feature prominently, and at times controversially, in concurrent policy debates.²¹ They have been at center stage in the theory of the firm since the seminal

¹⁹For an overview, see Riordan (2008). On the sensitivity of complete information vertical contracting results to assumptions of “symmetric,” “passive,” and “wary” beliefs see, e.g., McAfee and Schwartz (1994).

²⁰This occurs, for example, with $M = 2$ and symmetric bargaining weights if F is uniform on $[0, 1]$ and for $i \in \{1, 2\}$, $G_i(c) = c^{1/10}$, also with support $[0, 1]$. Then vertical integration causes social surplus to decrease from 0.4827 to 0.4815.

²¹For example, related to the 2017 Dow-DuPont merger, the U.S. DOJ’s “Competitive Impact Statement” identifies reduced innovation as a key concern (<https://www.justice.gov/atr/case->

works of Grossman and Hart (1986) and Hart and Moore (1990) (G-H-M hereafter). To account for the possibility of investment by the buyer and the sellers, we now extend our model, modelling an agent's investment as an action taken prior to the realization of private information that improves the agent's type distribution.

We show that the results under incomplete information differ starkly from those obtained in the G-H-M literature. This literature stipulates complete information and efficient bargaining and, as a consequence, obtains hold-up and inefficient investment. In contrast, in our setting, incomplete information protects agents from hold-up, and investments are efficient if and only if price formation is efficient.

Intuitively, if the price formation mechanism implements the first-best allocation, each agent is paid its marginal contribution to social welfare. By the usual Vickrey-Clarke-Groves logic, this makes truthfully reporting one's type a dominant strategy for every agent and aligns each agent's objective with the planner's at the allocation stage. Anticipating that this will be the case once types are realized, each agent's incentives are also aligned with the planner's at the investment stage because each agent's and the planner's reward from investment are the same.

For the purposes of this analysis, we assume that bargaining weights are symmetric, i.e., $b = s_1 = \dots = s_M$. In this case, $\alpha^* = 0$ implies that the price formation mechanism is efficient, i.e., trade occurs if and only if $v \geq \min_{i \in \mathcal{M}} c_i$.

To incorporate the possibility of investment by the agents to improve their type distributions, we suppose that each agent i can improve (or more generally change) its type distribution by investing e_i at cost $\Psi_i(e_i)$. The resulting type distributions are denoted by $F(v, e_B)$ and by $G_i(c, e_i)$ for $i \in \mathcal{M}$, with densities $f(v, e_B)$ and $g_i(c, e_i)$, respectively. We assume that the supports of the distributions are fixed and not affected by investment, and we assume that the investment cost functions and distributions are sufficiently well behaved that optimality is characterized by first-order conditions. Consistent with G-H-M, we assume that investments are not contractible. Thus, the price formation mechanism depends only on equilibrium investment levels and not on realized investment levels. One implication of this is that the payments to the worst-off types, $U_B(\underline{v})$ and $U_i(\bar{c})$, are not affected by actual investments. We suppose that the buyer and seller first simultaneously make their investments and then price formation takes place.

The social planner's problem is to solve

$$\max_{\mathbf{e}} \mathbb{E}_{v, \mathbf{c}} \left[\left(v - \min_{i \in \mathcal{M}} c_i \right) \cdot \mathbf{1}_{v > \min_{i \in \mathcal{M}} c_i} \right] - \sum_{i \in \mathcal{N}} \Psi_i(e_i). \quad (5)$$

We denote by $\bar{\mathbf{e}} = (\bar{e}_i)_{i \in \mathcal{N}}$ a solution to (5).

Let $\mathbf{e}_S \equiv (e_i)_{i \in \mathcal{M}}$ and $\mathbf{e}_{S \setminus \{i\}} \equiv (e_j)_{j \in \mathcal{M} \setminus \{i\}}$ be vectors of sellers' investments and denote by

$$L(x, \mathbf{e}_S) \equiv 1 - \prod_{i \in \mathcal{M}} (1 - G_i(x, e_i)) \quad \text{and} \quad L_{-i}(x, \mathbf{e}_{S \setminus \{i\}}) \equiv 1 - \prod_{j \in \mathcal{M} \setminus \{i\}} (1 - G_j(x, e_j))$$

the distributions of the minimum of all the sellers' cost draws and of the minimum of all of supplier i 's rivals' cost draws, respectively. Using integration by parts, the social planner's problem in (5) can be rewritten as:²²

$$\max_{\mathbf{e}} \int_{\underline{c}}^{\bar{v}} L(x, \mathbf{e}_S) (1 - F(x, e_B)) dx - \sum_{i \in \mathcal{N}} \Psi_i(e_i).$$

Under our assumptions, the social planner's optimal investments $\bar{\mathbf{e}}$ are thus characterized by

$$- \int_{\underline{v}}^{\bar{v}} L(x, \bar{\mathbf{e}}_S) \frac{\partial F(x, e_B)}{\partial e_B} \Big|_{e_B = \bar{e}_B} dx = \Psi'_B(\bar{e}_B), \quad (6)$$

and for each $i \in \mathcal{M}$

$$\int_{\underline{c}}^{\bar{c}} \frac{\partial G_i(x, e_i)}{\partial e_i} \Big|_{e_i = \bar{e}_i} (1 - F(x, \bar{e}_B)) (1 - L_{-i}(x, \bar{\mathbf{e}}_{S \setminus \{i\}})) dx = \Psi'_i(\bar{e}_i). \quad (7)$$

Although we do not require these specific assumptions, simple and natural conditions for the first-order conditions to be satisfied are that the investments induce first-order stochastic dominance shifts in the sense that $\frac{\partial F(x, \bar{e}_B)}{\partial e_B} < 0$ and $\frac{\partial G_i(x, \bar{e}_i)}{\partial e_i} > 0$ and that each Ψ'_i is nondecreasing.

We now show that investments $\bar{\mathbf{e}}$ are a Nash equilibrium of a simultaneous investment game played by the agents when $\bar{\mathbf{e}}$ is such that $\alpha^* = 0$. (To make the dependence of α^* on investments explicit, we write $\alpha_{\bar{\mathbf{e}}}^*$.)

Assuming that $\alpha_{\bar{\mathbf{e}}}^* = 0$, the price formation mechanism based on $\alpha_{\bar{\mathbf{e}}}^*$ induces trade if and only if $v > \min_{i \in \mathcal{M}} c_i$, and the buyer is paid its threshold type. This is the lowest type in $[\underline{v}, \bar{v}]$ that the buyer could report and still trade, which is $\max\{\underline{v}, \min_{i \in \mathcal{M}} c_i\}$. Thus,

²²For the case of nonoverlapping supports, this can be written as

$$\int_{\underline{v}}^{\bar{v}} (1 - F(x, e_B)) dx + \int_{\underline{c}}^{\bar{c}} L(x, \mathbf{e}_S) dx + \underline{v} - \bar{c} - \sum_{i \in \mathcal{N}} \Psi_i(e_i).$$

This implies that with nonoverlapping supports the problems of optimizing the buyer's investment and the sellers' investments are separable.

the buyer's problem is to choose e_B to maximize

$$\max_{e_B} \mathbb{E}_{v,c} \left[\left(v - \max\{v, \min_{i \in \mathcal{M}} c_i\} \right) \cdot \mathbf{1}_{v > \max\{\underline{v}, \min_{i \in \mathcal{M}} c_i\}} \right] - \Psi_B(e_B) + U_B(\underline{v}). \quad (8)$$

Analogously, seller i 's threshold type is $\min_{j \in \mathcal{M} \setminus \{i\}} \{v, c_j, \bar{c}\}$,²³ and so the problem for seller i is to choose e_i to maximize

$$\max_{e_i} \mathbb{E}_{v,c} \left[\left(\min_{j \in \mathcal{M} \setminus \{i\}} \{c_j, v, \bar{c}\} - c_i \right) \cdot \mathbf{1}_{c_i < \min_{j \in \mathcal{M} \setminus \{i\}} \{c_j, v\}} \right] - \Psi_i(e_i) + U_i(\bar{c}). \quad (9)$$

Integrating by parts, the problems in (8) and (9) can be written as

$$\max_{e_B} \int_{\underline{v}}^{\bar{v}} L(x, \mathbf{e}_S)(1 - F(x, e_B)) dx - \Psi_B(e_B) + U_B(\underline{v}),$$

and

$$\max_{e_i} \int_{\underline{c}}^{\bar{c}} G_i(x, e_i)(1 - F(x, e_B))(1 - L_{-i}(x, \bar{\mathbf{e}}_{S \setminus \{i\}})) dx - \Psi_i(e_i) + U_i(\bar{c}),$$

whose first order conditions are the same as (6) and (7). Moreover, because the conditions for a Nash equilibrium are less restrictive than those for a social optimum (as there are no cross-partials from i to j to worry about in Nash equilibrium), the fact that (6) and (7) characterize a social optimum implies that they also characterize a Nash equilibrium. In other words, $\bar{\mathbf{e}}$ is a Nash equilibrium outcome when $\alpha_{\bar{\mathbf{e}}}^* = 0$.

In contrast, as we show in the proof of Proposition 5, when $\alpha_{\bar{\mathbf{e}}}^* > 0$, $\bar{\mathbf{e}}$ is not a Nash equilibrium. In that case, the price formation mechanism induces trade if and only if $\Phi_{\alpha_{\bar{\mathbf{e}}}^*}(v, \bar{e}_B) \geq \min_{i \in \mathcal{M}} \Gamma_{i, \alpha_{\bar{\mathbf{e}}}^*}(c_i, \bar{e}_i)$, whereas the social planner's objective is trade if and only if $v \geq \min_{i \in \mathcal{M}} c_i$. As a result, the agents' objectives and hence payoff maximizing investments differ from those of the social planner.

Thus, we have the following result:

Proposition 5. *With symmetric bargaining weights, efficient investments $\bar{\mathbf{e}}$ are a Nash equilibrium of the simultaneous investment game if and only if $\alpha_{\bar{\mathbf{e}}}^* = 0$.*

Proof. See Appendix B.

As shown in Proposition 5, when $\alpha_{\bar{\mathbf{e}}}^* = 0$, the buyer's optimal choice of e_B and seller i 's optimal choice of e_i are identical to the choices that the social planner would make. In other words, *efficient price formation implies efficient investments*. Intuitively, given that the allocation rule is efficient and involves full trade, each agent is the residual claimant

²³For $M \geq 2$, we can write this more simply as $\min_{j \in \mathcal{M} \setminus \{i\}} \{v, c_j\}$.

to the surplus that its investment generates. Private information about its type protects the agent from hold-up.²⁴

Proposition 5 contrasts with the hold-up that arises in the G-H-M literature. Accordingly, the implications for institutional design differ sharply between incomplete information models in which the efficiency properties of price formation are endogenous and complete information models that assume efficient bargaining. In the latter, the social planner would aim to align, say, property rights with how the agents’ investments affect social surplus. In contrast, in incomplete information models, the planner would choose designs that render price formation efficient. Once price formation is efficient, efficient investments follow.

Hatfield et al. (2018) provide an equivalence result between efficient dominant-strategy mechanisms under incomplete information and efficient investments, which is obviously tightly related to Proposition 5. Efficient dominant strategy mechanisms are equivalent to the Vickrey-Clarke-Groves (VCG) mechanism, and with independent private values, there is a well-known equivalence between Bayesian incentive compatibility and dominant strategy incentive compatibility (see e.g. Gershkov et al., 2013). In this way, Proposition 5 connects to the equivalence result of Hatfield et al. (2018) and to earlier work by Milgrom (1987) and Rogerson (1992).²⁵ However, the no-deficit constraint in our setting implies that the VCG mechanism is not admissible when we have overlapping supports.

As is perhaps clear from the analysis above, the efficiency result of Proposition 5 continues to hold if instead of investments in cost reduction, each supplier can invest in the “quality” of its product. Specifically, suppose that when supplier i makes investment $\theta_i \geq 0$ in the quality of its product, the buyer then has value $\theta_i v$ for supplier i ’s product. In this setup, both the planner and supplier i only value supplier i ’s investment when the buyer trades with supplier i . Because the VCG mechanism gives supplier i its social marginal product, accounting for the investment θ_i , efficient investment levels continue to be a Nash equilibrium. This result contrasts with that of Che and Hausch (1999), who study a contracting setup in which investments by suppliers in cost reduction are efficient, but investments by suppliers that benefit the buyer need not be. Importantly, however, there is no incomplete information at the price formation stage in their model.

²⁴In a setup where efficient bargaining is possible because of shared ownership (rather than the absence of any allocation-relevant private information), Schmitz (2002, p. 176) notes that “Intuitively, ... a party’s ex ante expected utility from an ex post efficient mechanism is (up to a constant) equal to the total expected surplus, so that each party is residual claimant on the margin from his or her point of view.”

²⁵Related to this, Lauermaun (2013) considers a dynamic search model and finds that it is easier/possible to converge to Walrasian efficiency with private information, but without private information, hold up prevents convergence to efficiency. These results are consistent with ours when one interprets search as investment.

Market structure and the efficiency of investment

Armed with Proposition 5, we can analyze the effect of a change in market structure, such as vertical integration, on the efficiency of investment. With one seller in the pre-integration market and overlapping supports, we have $\alpha_{\bar{e}}^* > 0$, implying that equilibrium investments are inefficient. But, by assumption, trade, and thus investments, are efficient after vertical integration. Thus, with overlapping supports, vertical integration promotes efficient investment insofar as there is an equilibrium with efficient investments after integration but not before. In contrast, with two or more sellers and nonoverlapping supports, price formation is efficient ($\alpha_{\bar{e}}^* = 0$) without vertical integration, and so investments are efficient without vertical integration. But following vertical integration, the price formation process has $\alpha_{\bar{e}}^* > 0$, and so investments are no longer efficient.²⁶ Thus, with two or more sellers and nonoverlapping supports, vertical integration disrupts efficient investment insofar as there is no equilibrium with efficient investments after integration whereas there was one before integration.

Corollary 2. *With one seller and overlapping supports, vertical integration promotes efficient investment; but with two or more sellers and nonoverlapping supports, vertical integration disrupts efficient investment.*

5 Extensions

In this section, we discuss the implications for bargaining breakdown of having a price formation process whose efficiency properties are endogenous, extend the model to allow for multi-object demand by the buyer and supplier-specific preferences, and show that bargaining externalities naturally arise in the thus extended setup.

²⁶In addition, the vertically integrated firm's objective is further differentiated from the social planner's objective because it may trade internally. Specifically, letting $\hat{\Phi}_a(c, e)$ denote the vertically integrated firm's weighted virtual value function (associated with type c drawn from $G_1(\cdot, e)$) and $\hat{\Phi}_a^{-1}(x, e)$ denote the inverse with respect to the first argument, the vertically integrated firm's objective when the mechanism assumes efficient investments is (dropping the subscript \bar{e} on α^* to ease notation):

$$\begin{aligned} & \mathbb{E}_{v, c} \left[\max\{0, v - c_1\} \cdot \mathbf{1}_{c_1 < \hat{\Phi}_{\alpha^*}^{-1}(\min_{i \in \mathcal{M} \setminus \{1\}} \Gamma_{i, \alpha^*}(c_i, \bar{e}_i), \bar{e}_B)} \right. \\ & \left. + \left(v - \hat{\Phi}_{\alpha^*}^{-1}\left(\min_{i \in \mathcal{M} \setminus \{1\}} \Gamma_{i, \alpha^*}(c_i, \bar{e}_i), \bar{e}_B\right) \right) \cdot \mathbf{1}_{c_1 > \hat{\Phi}_{\alpha^*}^{-1}(\min_{i \in \mathcal{M} \setminus \{1\}} \Gamma_{i, \alpha^*}(c_i, \bar{e}_i), \bar{e}_B)} \right] - \Psi_B(e_B) - \Psi_1(e_1), \end{aligned}$$

which for $\alpha_{\bar{e}}^* > 0$, implies first-order conditions for the vertically integrated firm's optimal e_B and e_1 that differ from the corresponding first-order conditions in the planner's problem. The inefficiency relates to the difficulties that arise in principal-agent problems when agents' actions are multi-dimensional (e.g., Holmström and Milgrom, 1991).

5.1 Bargaining breakdown

Bargaining has come to the forefront of many areas in applied economics. A pervasive feature of real-world bargaining is that negotiations often break down. Anecdotal examples range from the U.S. government shut down, to the British coal miners’ and the U.S. air traffic controllers’ strikes in the 1980s, to failures to form coalition governments in countries with proportional representation, to, possibly, Brexit.²⁷ There is also systematic evidence that negotiations break down on the equilibrium path. For example, in a data set covering 25 million observations of bilateral negotiations on eBay, Backus et al. (2018) find a breakdown probability of roughly 55 percent.

In the price formation process that we study, negotiations break down on the equilibrium path for two reasons. First, it may be that the buyer’s value is below the seller’s cost, but because of private information, the two parties do not know this before they sit down at the negotiating table. Second, by the Myerson-Satterthwaite theorem, even if the buyer’s value exceeds the seller’s cost, the constraints imposed by incentive compatibility, individual rationality, and no deficit may prevent ex post efficient trade from taking place.

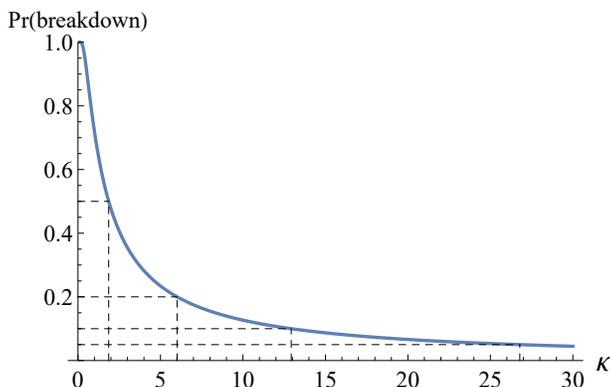


Figure 3: Probability bargaining breakdown as a function of κ assuming $F(v) = 1 - (1 - v)^{1/\kappa}$ and $G_1(c) = c^{1/\kappa}$, both with support $[0, 1]$, and assuming equal bargaining weights.

Importantly, under the assumption that negotiations are the outcome of our price formation mechanism with equal bargaining weights, one can use observed frequencies of negotiation breakdowns to back out the parameters of the distributions from which the buyer and the seller draw their types. To illustrate, assume that the buyer’s value v is drawn from the distribution $F(v) = 1 - (1 - v)^{1/\kappa}$ and the seller’s cost c is drawn from the distribution $G(c) = c^{1/\kappa}$, whose supports are $[0, 1]$, where $\kappa \in [0, \infty)$ has the interpretation of a “capacity.” Figure 3 plots the probability that negotiations break down as a function

²⁷As described by Crawford (2014), there are regular blackouts of broadcast television stations on cable and satellite distribution platforms due to the breakdown of negotiations over the terms for retransmission of the broadcast signal.

of κ under the assumptions stated. For example, if, as in the data set of Backus et al. (2018), 55 percent of all negotiations break down, eyeballing the figure indicates that κ must be around 1.5.²⁸ Rather than treating negotiation breakdowns as measurement error, which is difficult to justify if breakdown occurs more than fifty percent of the time in 25 million observation, the frequency of those breakdowns is valuable information that can be used for estimation in the incomplete information framework.

Connecting bargaining breakdown with vertical integration

To illustrate how initial market conditions, particularly the probability of breakdown, can effect the efficiency consequences of vertical integration, consider a pre-integration market with two sellers and one buyer. Parameterize the type distributions so that $G_i(c) = c^{1/\kappa_i}$ for $\kappa_1, \kappa_2 > 0$ and $F(v) = 1 - (1 - v)^{1/\kappa}$ for $\kappa > 0$, and assume equal bargaining weights. As an identifying assumption, assume that the cost distributions are uniform on average (alternatively one might use, e.g., margin data for identification). We show in Figure 4(a) the results of the calibration of these parameterized distributions given data on seller market shares and the probability of bargaining breakdown.

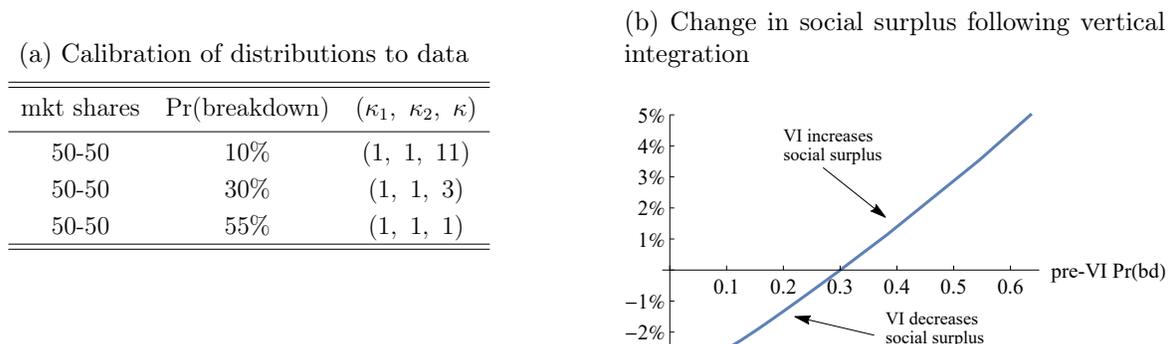


Figure 4: Interaction between breakdown probabilities and the effects of vertical integration on social surplus. Panel (a): Calibration of distributional parameters based on market shares and breakdown probabilities assuming one buyer and two sellers with $F(v) = 1 - (1 - v)^{1/\kappa}$ and for $i \in \{1, 2\}$, $G_i(c) = c^{1/\kappa_i}$, and assuming equal bargaining weights and that the sellers’ cost distributions are uniform on average. Panel (b): Change in social surplus as a result of vertical integration based on the analogous calibration to that of Panel (a) with symmetric suppliers and equal bargaining weights before and after integration, but varying the probability of breakdown in the pre-integration market (“pre-VI Pr(bd)” denotes the pre-integration probability of breakdown).

Now consider the effect on social surplus of vertical integration assuming two pre-integration sellers with equal market shares and that pre-integration and post-integration

²⁸More precisely, for the case considered, a breakdown probability of 55 percent corresponds to $\kappa = 1.6090$.

bargaining weights are symmetric—before integration $b = s_1 = \dots = s_M = 1/n$ and after integration between the buyer and seller 1, $b = s_2 = \dots = s_M = 1/(n-1)$. As illustrated in Figure 4(b), in markets where before integration the probability of breakdown is low, the change in social surplus from vertical integration is negative. In that case, the reduced efficiency of price formation with the independent seller dominates the gain in efficiency associated with internal transactions, and vertical integration reduces social surplus. In contrast, when the probability of breakdown is high prior to integration, then the increased efficiency of internal transactions dominates, and social surplus increases as a result of vertical integration.

5.2 Multi-object demand and seller-specific preferences

We now extend the model to allow the buyer to have preferences over sellers and to have demand for multiple objects. To this end, we let $\boldsymbol{\theta} = (\theta_1, \dots, \theta_M)$ be a commonly known vector of taste parameters of the buyer, with the meaning that the value to the buyer of trade with seller i when the buyer's type is v is $\theta_i v$. Assuming $D \geq M$ for the moment, the buyer's payoff when receiving the object from seller i with probability q_i and making the payment p is $\sum_{i \in \mathcal{M}} \theta_i v q_i - p$. Adjusting for the buyer's maximum demand D , under (ex post) efficiency, trade should occur between the buyer and seller i if and only if $\theta_i v - c_i$ is positive and among the D highest values of $(\theta_j v - c_j)_{j \in \mathcal{M}}$. The problem is trivial if $\max_{i \in \mathcal{M}} \theta_i \bar{v} \leq \underline{c}$ because then it is never ex post efficient to have trade with any seller. From now on, we therefore assume that $\max_{i \in \mathcal{M}} \theta_i \bar{v} > \underline{c}$.

This setup encompasses (i) differentiated products by letting the seller-specific taste parameters differ; (ii) a one-buyer version of the Shapley and Shubik (1972) model by setting $D = 1$; and (iii) a version of the Shapley-Shubik model in which the buyer has demand for multiple products of the sellers by setting $D > 1$. For an extension of the one-to-many setup that encompasses additional models, see Appendix C.

In order to define the price formation mechanism for the generalized setup, we define the virtual surplus $\Lambda_{i,\alpha}$ associated with trade between the buyer and seller i , accounting for the agents' bargaining weights and the mechanism adjustment parameter α :

$$\Lambda_{i,\alpha}(v, c_i; b, s_i) \equiv \theta_i \Phi_{\max\{\alpha, 1 - bn(1-\alpha)\}}(v) - \Gamma_{i, \max\{\alpha, 1 - s_i n(1-\alpha)\}}(c_i).$$

Let $\mathbf{\Lambda}(v, \mathbf{c}; b, \mathbf{s}) \equiv (\Lambda_{1,\alpha}(v, c_1; b, s_1), \dots, \Lambda_{M,\alpha}(v, c_M; b, s_M))$, and let $\mathbf{\Lambda}(v, \mathbf{c}; b, \mathbf{s})_{(D)}$ denote the D -th highest element of $\mathbf{\Lambda}(v, \mathbf{c}; b, \mathbf{s})$. As before, in order to save notation, we ignore ties.

Lemma 4. *In the generalized setup, the price formation process has the allocation rule*

$$q_{i,\alpha}(v, \mathbf{c}; b, \mathbf{s}) \equiv \begin{cases} 1 & \text{if } \Lambda_{i,\alpha}(v, c_i; b, s_i) \geq \max\{0, \Lambda(v, \mathbf{c}; b, \mathbf{s})_{(D)}\}, \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

where α is the smallest $\alpha \in [0, 1]$ such that no deficit is satisfied.

Proof. See Appendix B.

We can now use this generalized setup to consider bargaining externalities between suppliers.

5.3 Bargaining externalities

We consider the case of one buyer and two sellers with equal bargaining weights. Assuming that F , G_1 , and G_2 are the uniform distribution on $[0, 1]$, and assuming that $\theta_2 = 1$, we allow the buyer's preference for seller 1, θ_1 , and the buyer's total demand, D , to vary.

Table 1: Outcomes for one-to-many price formation for the case of one buyer and two sellers with equal bargaining weights and with all agents drawing their types from the uniform distribution on $[0, 1]$ and $\theta_2 = 1$. The values of D and θ_1 vary as indicated in the table.

	$D = 1$		$D = 2$	
θ_1	1	2	1	2
α^*	0.27	0.24	0.33	0.28
u_B	0.13	0.34	0.14	0.38
$u_{1,S}$	0.05	0.21	0.07	0.22
$u_{2,S}$	0.05	0.01	0.07	0.08

As shown in Table 1, focusing on the case with $D = 1$, an increase in the buyer's preference for seller 1 from $\theta_1 = 1$ to $\theta_1 = 2$ benefits seller 1 ($u_{1,S}$ increases) but harms seller 2 ($u_{2,S}$ decreases). The increase in the buyer's preference for seller 1 means that seller 2 is less likely to trade. As a result, seller 2 is harmed by the increase in the buyer's preference for seller 1.

When $D = 2$, the results differ. Seller 1 again benefits from being preferred by the buyer, but in this case seller 2 also benefits, albeit less than seller 1. The increase in the buyer's value from trade with seller 1 means that the value of α^* is reduced, so seller 2 trades more often. As a result of the change from $\theta_1 = 1$ to $\theta_1 = 2$, both $u_{1,S}$ and $u_{2,S}$ increase.

The effect that we observe in this example when $D = 2$ is general in the sense that it holds whenever $M = D \geq 2$. In this case, the probability that seller i trades, $\Pr(\theta_i \Phi_{\max\{\alpha^*, 1 - bn(1 - \alpha^*)\}}(v) \geq \Gamma_{i, \max\{\alpha^*, 1 - s_i n(1 - \alpha^*)\}}(c_i))$, does not depend on the preference parameters of the other sellers except through their effect on α^* . Because an increase in a rival seller’s preference parameter causes a decrease in α^* , it increases the probability of trade and so benefits the seller. Thus, we have the following result:

Proposition 6. *If $M = D$, then an increase in the preference parameter for one seller increases the payoffs for all sellers.*

The result of Proposition 6 no longer holds when $M > D$, as shown in the example of Table 1 with $D = 1$. In that case, even though the increase in θ_1 from 1 to 2 reduces α^* , which benefits rivals, it also reduces the probability that rivals are among the set of at-most D sellers that trade.

6 Related literature

George Stigler provided early and forceful arguments that private information held by economic agents is a major obstacle to achieving efficient outcomes, noting that “important aspects of economic organization take on a new meaning when they are considered from the viewpoint of the search for information” (Stigler, 1961, p. 213). While Stigler emphasized price dispersion and the problem of uncertainty about price cuts faced by cartels (Stigler, 1964), the relevance of private information in connection to prices applies, of course, generally.

Viewed from this angle, we use the Myersonian mechanism design machinery (Myerson, 1981) to elicit—search for, as it were—agents’ private information and determine prices. Indeed, our framework builds on the bilateral trade model of Myerson and Satterthwaite (1983), augmented by bargaining weights and possibly multiple sellers, and thereby combines elements of Myerson and Satterthwaite (1983), Williams (1987), and Gresik and Satterthwaite (1989). Specifically, our procurement model allows for multiple sellers without imposing restrictions on the supports of the buyer’s value and the sellers’ costs other than assuming that all cost distributions have the same support.²⁹ We generalize Williams’ approach of maximizing an objective that assigns differential weights in a bilateral trade problem by allowing for multiple agents. In light of the quote from New Palgrave Dictionary in the introductory paragraph, our paper re-interprets Myerson and

²⁹While Gresik and Satterthwaite (1989) also allow for multiple buyers, they restrict attention to identical cost distributions. In that regard, our setup thus shares similarities with the optimal auction setting of Myerson (1981), with the important difference that our setup has two-sided private information.

Satterthwaite (1983) as a bilateral monopoly problem and shows that it is tractable and has all the required features.

In particular, the independent private values setting with continuous distributions has the virtue that, for a given objective, the mechanism that maximizes this objective, subject to incentive compatibility, individual rationality, and no-deficit constraints, is well defined and pinned down (up to a constant in the payments) by the allocation rule, which is unique. Of particular interest to industrial organization and antitrust economics, it also has the feature that, quite generally, there is a tradeoff between allocating efficiently and extracting rents. This tradeoff is at the heart of both industrial organization and Myerson's optimal auction. This tradeoff is the reason why the Williams frontier is typically not identical to the 45-degree line and, therefore, the basis from which the possibility of social surplus increasing countervailing power emerges.

Privacy of information endows economic agents with information rents and thereby protects them from hold up, as discussed in our analysis of investments. Even without investment, this protection implies, for example, that first-degree price discrimination is not possible. Rather than being an assumption, the impossibility of first-degree price discrimination is an implication in this setup.³⁰ Moreover, the aforementioned assumptions are essentially the *only* assumptions that permit a tractable approach that maintain the basic tradeoff between profit and social surplus.³¹

Snyder (2008) notes that the concept of countervailing power has been the subject of controversies ever since it was introduced by Galbraith (1952). As we have shown, the possibility of countervailing power arises naturally in an incomplete information setup exactly because of its inherent tradeoff between efficiency and rent extraction. In this regard, one important contribution of the present paper is the observation that *buyer power* and *countervailing power* are distinct things: making a buyer powerful by increasing its bargaining power has socially desirable countervailing effects if the increase in buyer power levels the playing field by making all bargaining weights more equal; if the change

³⁰Likewise, setting a uniform market clearing price is the optimal mechanism for a monopoly with constant marginal costs facing a continuum of buyers, so under these conditions uniform pricing is a conclusion rather than an assumption; if the monopoly has increasing marginal costs and the revenue function that it faces is not concave, then setting non-market-clearing prices may be optimal (see Loertscher and Muir, 2019).

³¹Dropping the assumption of risk neutrality, Maskin and Riley (1984) and Matthews (1984) show that optimal mechanisms depend on the nature of risk aversion, are not easily characterized, and, among other things, may require payments to and/or from losers. Without independence, as foreshadowed by Myerson (1981), Crémer and McLean (1985, 1988) show that there is no tradeoff between profit and social surplus. Without private values, additional and, therefore, in some sense arbitrary, restrictions may be required to maintain tractability and/or the tradeoff between profit and social surplus (Mezzetti, 2004, 2007). Notwithstanding recent progress, with multi-dimensional private information and multiple agents, the optimal mechanism is not known (see, e.g., Daskalakis et al., 2017). With discrete types, there is no payoff equivalence theorem. In other words, the mechanism is not pinned down by the allocation rule.

moves bargaining weights in the opposite directions, matters worsen.³²

With regard to vertical integration, our model has the feature that, because the efficiency of the price formation process is endogenous, vertical integration can decrease social surplus. That is, in this setting there is no basis for the presumption that vertical integration increases social surplus.³³ Related to Judge Leon’s recent ruling in the AT&T-Time-Warner merger, our model has thus the property that vertical integration can be detrimental to social surplus without relying on complete information bargaining.

The tight connection between incentives for efficient investment and incentives for efficiency in incomplete information models has its roots in the seminal works of Vickrey (1961), Clarke (1971), and Groves (1973) and the subsequent uniqueness results of Green and Laffont (1977) and Holmström (1979). Essentially, dominant strategy incentive compatibility under incomplete information requires each agent to be a price taker, and efficiency then further requires this price to be equal to the agent’s social marginal product (or cost). But, as demonstrated by Milgrom (1987), Rogerson (1992), Hatfield et al. (2018), and Loertscher and Riordan (2019), this is precisely the set of conditions that have to be satisfied for incentives for investment to be aligned with efficiency. In contrast, in complete information models in the tradition of Grossman and Hart (1986) and Hart and Moore (1990), efficient bargaining typically creates hold up, which is an impediment to efficient investment. The intuition behind these differences is that, as mentioned, privacy of information provides protection against hold up. Following the Dow-DuPont merger decision, there has been a recent upsurge of interest in industrial organization relating to market structure and the incentives to invest (see, e.g., Federico et al., 2017, 2018; Jullien and Lefouili, 2018; Loertscher and Marx, 2019). Thus, our results pertaining to mergers and vertical integration and investment relate to this strand of literature as well.

Partly based on the availability of large scale microlevel datasets, there has been an even larger upsurge of interest in bargaining (see, for example, Larsen, 2018; Backus et al., 2018, 2019; Zhang et al., 2019). Bargaining has also come to the forefront of the empirical IO literature, in particular in analyses of bundling and vertical integration such as Crawford and Yurukoglu (2012) and Crawford et al. (2018). Collard-Wexler et al. (2019) provide a recent theoretical foundation for the widely used Nash-in-Nash bargaining model. Ho and Lee (2017) apply this framework to the question of countervailing power by insurers when negotiating with hospitals and find evidence that consolidation

³²Loertscher and Marx (2019) consider mergers in a procurement setup with one-sided private information, assuming that either bargaining weights are equal or that $b = 1$, and that bargaining weights do not change with a merger (with the exception of noting that a merger increases a buyer’s incentive to acquire bargaining power). Because bargaining weights are fixed, countervailing power does not arise.

³³For an overview of the literature on the competitive effects of vertical integration, see Riordan and Salop (1995). As described there, the literature has taken the view that most vertical mergers lead to some efficiencies.

among insurers improves their bargaining position vis-à-vis hospitals. Our paper contributes to this literature by showing, among other things, that in incomplete information models, bargaining breakdown occurs on the equilibrium path and that the probability of breakdown can, under suitable assumptions, be used to estimate distributions.

7 Conclusions

We analyze a procurement setup with incomplete information that pertains to both sides of the market in which price formation and its efficiency properties are endogenous and depend, among other things, on the bargaining power of the buyer and the suppliers. Social surplus increasing countervailing power and socially harmful vertical integration arise naturally in this setting. We also discuss the relation between the efficiency of the price formation process and the incentives to invest, which differs fundamentally from what obtains in complete information models that are based on the assumption that efficient trade is always possible.

Our paper shows that countervailing power and buyer power are conceptually distinct insofar as increasing buyer power has socially desirable effects if and only if it makes all bargaining powers more equal. It also shows that an economic agent's strength or weakness has two dimensions that are, conceptually, independent. The first one, which may be thought of as the agent's productive strength or power, refers to the agent's productivity. Is the agent likely to have a high value if it is a buyer or a low cost if it is a seller? The second dimension captures the agent's bargaining power, that is, its ability (or inability) to affect the price formation process in its favour.

This distinction has the implication that bargaining power is, per se, independent of prices and distributions. For example, a buyer who has all the bargaining power facing a single supplier makes higher or lower take-it-or-leave-it offers depending on the realization of its value, and on average these offers will be higher if the buyer's distribution is stronger, say, in the sense of stochastic dominance. What is indicative of the relative bargaining powers is then not so much the level of prices but rather the price formation process itself. For example, in a bilateral trade setting, if the buyer (seller) always makes the price offer, one would conclude that the buyer (seller) has all the bargaining power, indicating that there is scope for countervailing power. In contrast, if the buyer and seller participate in a k -double-auction with $k = 1/2$, this may be indicative of equal bargaining powers, suggesting that there is no scope for welfare increasing countervailing power.

Avenues for future research are many. For example, one could augment the setup to have multiple buyers and multiple sellers, which may give rise to a raising rivals' costs effect of vertical integration. More fundamentally, developing a better understanding of what

determines bargaining power would add considerable value. Hopefully, the distinction between productive strength and bargaining power brought to light in the present paper will prove useful in that regard.

A Appendix: Mechanism design concepts

In this appendix, we assume that $M = 1$. The extension to $M > 1$ is straightforward.

A.1 Incentive compatibility and individual rationality

Take as given a direct mechanism (q, \mathbf{m}) mapping buyer and seller reports to a probability of trade and payments, $q : [\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}] \rightarrow [0, 1]$ and $\mathbf{m} : [\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}] \rightarrow \mathbb{R}^2$, where $q(v, c) \in [0, 1]$ is the probability with which the seller trades with the buyer given reports v and c .

Let $\hat{q}_B(z)$ be the buyer's expected quantity if it reports z and the seller reports truthfully, and let $\hat{m}_B(z)$ be the buyer's expected payment if it reports z and the seller reports truthfully:

$$\hat{q}_B(z) = \mathbb{E}_c[q(z, c)] \quad \text{and} \quad \hat{m}_B(z) = \mathbb{E}_c[m_B(z, c)].$$

The mechanism is incentive compatible for the buyer if for all $v, z \in [\underline{v}, \bar{v}]$,

$$U_B(v) \equiv \hat{q}_B(v)v - \hat{m}_B(v) \geq \hat{q}_B(z)v - \hat{m}_B(z). \quad (11)$$

Defining \hat{q}_S and \hat{m}_S analogously, where \hat{m}_S is the expected payment to the seller, the mechanism is incentive compatible for the seller if for all $c, z \in [\underline{c}, \bar{c}]$,

$$U_S(c) \equiv \hat{m}_S(c) - \hat{q}_S(c)c \geq \hat{m}_S(z) - \hat{q}_S(z)c. \quad (12)$$

Individual rationality is satisfied for the buyer if for all $v \in [\underline{v}, \bar{v}]$, $U_B(v) \geq 0$, and for the seller if for all $c \in [\underline{c}, \bar{c}]$, $U_S(c) \geq 0$.

A.2 Myerson-Satterthwaite impossibility result

Under the assumption that $\underline{v} < \bar{c}$, which together with the assumption that $\bar{v} > \underline{c}$ implies that trade is sometimes but not always ex post efficient, Myerson and Satterthwaite (1983) show that when $M = 1$, there is no mechanism that simultaneously satisfies ex post efficiency, incentive compatibility, and individual rationality and that does not run a budget deficit. This is the famous *impossibility* result of Myerson and Satterthwaite (1983) (see Appendix A.2 for details). Their result depends on $\underline{v} < \bar{c}$ because, without this assumption, ex post efficiency subject to incentive compatibility and individual rationality can easily be achieved without running a deficit. For example, the *posted price* mechanism that has the buyer pay $p = (\underline{v} + \bar{c})/2$ to the seller achieves this.

For the purpose of making the paper self-contained, we provide a statement and proof of the impossibility theorem of Myerson and Satterthwaite (1983). Under the IPV as-

sumptions and the assumption that $\underline{v} < \bar{c}$, Myerson and Satterthwaite (1983) show that there is no mechanism satisfying incentive compatibility and individual rationality that allocates ex post efficiently and that does not run a deficit.

By now, the proof of this result can be provided in a couple of lines (see, e.g, Krishna, 2002). Consider the dominant strategy implementation in which the buyer pays $p_B = \max\{c, \underline{v}\}$ and the seller receives $p_S = \min\{v, \bar{c}\}$ whenever there is trade, and no payments are made otherwise. Notice that $U_B(\underline{v}) = 0 = U_S(\bar{c})$. Thus, the individual rationality constraints are satisfied. Further, notice that $p_B - p_S \leq 0$, with a strict inequality for almost all type realizations. This implies that the mechanism runs a deficit in expectation. By the payoff equivalence theorem, any other ex post efficient mechanism satisfying incentive compatibility and individual rationality will run a deficit of at least that size (and a larger one if one or both of the individual rationality constraints are slack).

To see how this impossibility result rests on the assumption $\underline{v} < \bar{c}$, assume to the contrary that $\underline{v} \geq \bar{c}$. Then the mechanism described above continues to satisfy incentive compatibility and individual rationality, but for all type realizations $p_B = \underline{v} \geq \bar{c} = p_S$, which implies that the mechanism does not run a deficit.

Assuming that the virtual value and virtual cost functions are increasing (the assumption of regularity), Myerson and Satterthwaite (1983) also derive the second-best mechanism and show that it is characterized by the allocation rule that induces trade if and only if

$$\Phi_{\alpha^*}(v) \geq \Gamma_{\alpha^*}(c),$$

where α^* is the smallest number $\alpha \in [0, 1]$ such that:³⁴

$$\Pi_\alpha \equiv \mathbb{E}_{v,c} [(\Phi(v) - \Gamma(c)) \cdot \mathbf{1}_{\Phi(v) \geq \Gamma(c)}] \geq 0. \quad (13)$$

³⁴As discussed in Appendix A.3, the left side of (13) is the expected deficit of a mechanism that induces trade if and only if $\Phi_\alpha(v) \geq \Gamma_\alpha(c)$. Note that $\mathbb{E}_v[\Phi(v)] = \underline{v}$ and $\mathbb{E}_c[\Gamma(c)] = \bar{c}$. Thus, if $\alpha = 0$ and $\underline{v} \geq \bar{c}$, then the conditioning event in (13) always holds, and the left side is simply $\underline{v} - \bar{c}$, which is nonnegative in this case. Thus, $\underline{v} \geq \bar{c}$ implies that $\alpha^* = 0$. But if $\underline{v} < \bar{c}$, then we know from the Myerson-Satterthwaite impossibility result that efficient trade is not possible without running a deficit, implying that the left side of (13) is negative when $\alpha = 0$. When $\alpha = 1$, trade only occurs when $\Phi(v) \geq \Gamma(c)$, so the expectation in (13) is positive. It then follows from continuity that when $\underline{v} < \bar{c}$, α^* is well defined and satisfies $\alpha^* \in (0, 1)$.

A.3 Mechanism surplus (deficit) and interim expected payoffs

As we now show, standard arguments imply that in any incentive compatible, interim individually rational mechanism, the mechanism's expected budget surplus is

$$\mathbb{E}_{v,c} [(\Phi(v) - \Gamma(c)) \cdot q(v, c)] - U_B(\underline{v}) - U_S(\bar{c}).$$

By the Revelation Principle, we can focus attention on direct mechanisms (q, \mathbf{m}) , where $q : [\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}] \rightarrow [0, 1]$ and $\mathbf{m} : [\underline{v}, \bar{v}] \times [\underline{c}, \bar{c}] \rightarrow \mathbb{R}^2$. Standard arguments (see, e.g., Krishna, 2002, Chapter 5.1) proceed as follows:

Define

$$\hat{q}_B(z) = \int_{[\underline{c}, \bar{c}]} q(z, c)g(c)dc \quad \text{and} \quad \hat{q}_S(z) = \int_{[\underline{v}, \bar{v}]} q(v, z)f(v)dv$$

to be the probability that the buyer trades when it reports z and the seller reports its type truthfully and the probability that the seller trades when it reports z and the buyer reports truthfully. Similarly, define

$$\hat{m}_B(z) = \int_{[\underline{c}, \bar{c}]} m_B(z, c)g(c)dc \quad \text{and} \quad \hat{m}_S(z) = \int_{[\underline{v}, \bar{v}]} m_S(v, z)f(v)dv$$

to be the expected payment made by the buyer when it reports z and the seller reports truthfully and the expected payment received by the seller when it reports r and the buyer reports truthfully. Because we assume independent draws, for $i \in \{B, S\}$, $\hat{q}_i(z)$ and $\hat{m}_i(z)$ depend only on the report z and not on the reporting agent's true type. The expected payoff of a buyer with type v that reports z is then $\hat{q}_B(z)v - \hat{m}_B(z)$, and the expected payoff of a seller with type c that reports z is $\hat{m}_S(z) - \hat{q}_S(z)c$.

The direct mechanism is incentive compatible and individual rational under the conditions described in Appendix A.1.

Focusing on the buyers, incentive compatibility implies that

$$U_B(v) = \max_{z \in [\underline{v}, \bar{v}]} \{\hat{q}_B(z)v - \hat{m}_B(z)\},$$

i.e., U_B is a maximum of a family of affine functions, which implies that U_B is convex and so absolutely continuous and differentiable almost everywhere in the interior of its domain.³⁵ In addition, incentive compatibility implies that $U_B(z) \geq \hat{q}_B(v)z - \hat{m}_B(v) =$

³⁵A function $h : [\underline{v}, \bar{v}] \rightarrow \mathbb{R}$ is absolutely continuous if for all $\varepsilon > 0$ there exists $\delta > 0$ such that whenever a finite sequence of pairwise disjoint sub-intervals (v_k, v'_k) of $[\underline{v}, \bar{v}]$ satisfies $\sum_k (v'_k - v_k) < \delta$, then $\sum_k |h(v'_k) - h(v_k)| < \varepsilon$. One can show that absolute continuity on compact interval $[a, b]$ implies that h has a derivative h' almost everywhere, the derivative is Lebesgue integrable, and that $h(x) = h(a) + \int_a^x h'(t)dt$ for all $x \in [a, b]$.

$U_B(v) + \hat{q}_B(v)(z - v)$, which for $\delta > 0$ implies

$$\frac{U_B(v + \delta) - U_B(v)}{\delta} \geq \hat{q}_B(v)$$

and for $\delta < 0$ implies

$$\frac{U_B(v + \delta) - U_B(v)}{\delta} \leq \hat{q}_B(v),$$

so taking the limit as δ goes to zero, at every point v where U_B is differentiable, $U'_B(v) = \hat{q}_B(v)$. Because U_B is convex, this implies that $\hat{q}_B(v)$ is nondecreasing. Because every absolutely continuous function is the definite integral of its derivative,

$$U_B(v) = U_B(\underline{v}) + \int_{\underline{v}}^v \hat{q}_B(t) dt,$$

which implies that, up to an additive constant, a buyer's expected payoff in an incentive compatible direct mechanism depends only on the allocation rule. By an analogous argument, $U'_S(c) = -\hat{q}_S(c)$, $\hat{q}_S(c)$ is nonincreasing, and

$$U_S(c) = U_S(\bar{c}) + \int_c^{\bar{c}} \hat{q}_S(t) dt.$$

Using the definitions of U_B and U_S in (11) and (12), we can rewrite these as

$$\hat{m}_B(v) = \hat{q}_B(v)v - \int_{\underline{v}}^v \hat{q}_B(t) dt - U_B(\underline{v}) \tag{14}$$

and

$$\hat{m}_S(c) = \hat{q}_S(c)c + \int_c^{\bar{c}} \hat{q}_S(t) dt + U_S(\bar{c}). \tag{15}$$

The expected payment by the buyer is then

$$\begin{aligned}
\mathbb{E}_v [\hat{m}_B(v)] &= \int_{\underline{v}}^{\bar{v}} \hat{m}_B(v) f(v) dv \\
&= \int_{\underline{v}}^{\bar{v}} \left(\hat{q}_B(v)v - \int_{\underline{v}}^v \hat{q}_B(t) dt \right) f(v) dv - U_B(\underline{v}) \\
&= \left(\int_{\underline{v}}^{\bar{v}} \hat{q}_B(v)v f(v) dv - \int_{\underline{v}}^{\bar{v}} \int_t^{\bar{v}} \hat{q}_B(t) f(v) dv dt \right) - U_B(\underline{v}) \\
&= \left(\int_{\underline{v}}^{\bar{v}} \hat{q}_B(v)v f(v) dv - \int_{\underline{v}}^{\bar{v}} \hat{q}_B(t) (1 - F(t)) dt \right) - U_B(\underline{v}) \\
&= \int_{\underline{v}}^{\bar{v}} \hat{q}_B(v) \left(v - \frac{1 - F(v)}{f(v)} \right) f(v) dv - U_B(\underline{v}) \\
&= \int_{\underline{v}}^{\bar{v}} \hat{q}_B(v) \Phi(v) f(v) dv - U_B(\underline{v}) \\
&= \mathbb{E}_v [\hat{q}_B(v) \Phi(v)] - U_B(\underline{v}),
\end{aligned}$$

where the first equality uses the definition of the expectation, the second uses (14), the third switches the order of integration, the fourth integrates, the fifth collects terms, the sixth uses the definition of the virtual value Φ , and the last equality uses the definition of the expectation. Similarly, using (15), the expected payment to the seller is

$$\mathbb{E}_c [\hat{m}_S(c)] = \int_{\underline{c}}^{\bar{c}} \hat{m}_S(c) g(c) dc = \mathbb{E}_c [\hat{q}_S(c) \Gamma(c)] + U_S(\bar{c}).$$

Thus, imposing $q_B(v, c) = \bar{q}_S(v, c)$, we have the result that the expected budget surplus to the mechanism is

$$\mathbb{E}_{v,c} [(\Phi(v) - \Gamma(c)) \cdot q(v, c)] - U_B(\underline{v}) - U_S(\bar{c}).$$

As mentioned, it is straightforward to extend these results to the case of $M > 1$.

A.4 Implementation

In this appendix, we show that when the value and cost distributions are not uniform, as long as sellers are symmetric, both in terms of distributions and bargaining weights, the price formation process with parameter α can be implemented by a fee-setting mechanism in which trade occurs via a fee-setting broker (Loertscher and Niedermayer, 2019).

To begin, assume that $M = 1$ and drop the seller subscript on the virtual cost function, which, as before, is assumed to be increasing. In the fee-setting mechanism, the buyer

sets a price p , which the seller can accept or reject. If the seller accepts p , the broker pays p to the seller and the broker charges $\tau_{\alpha^*(b)}(p)$ to the buyer. If the seller rejects, no payments are made.³⁶

By setting the fee

$$\tau_{\alpha^*(b)}(p) = \mathbb{E}_c \left[\Phi_{\max\{\alpha^*(b), 1-2b(1-\alpha^*(b))\}}^{-1} (\Gamma_{\max\{\alpha^*(b), 1-2(1-b)(1-\alpha^*(b))\}}(c)) \mid c \leq p \right],$$

the broker can induce the buyer of type v to set the price

$$p^*(v) = \Gamma_{\max\{\alpha^*(b), 1-2(1-b)(1-\alpha^*(b))\}}^{-1} (\Phi_{\max\{\alpha^*(b), 1-2b(1-\alpha^*(b))\}}(v)),$$

which implements the allocation rule in (2).³⁷

When there are $M \geq 2$ suppliers drawing their costs independently from the same distribution G with virtual cost function Γ , fee setting with the same fee $\tau_{\alpha^*(b)}(\tilde{p})$ still implements the price formation mechanism when all sellers have equal weights. The only changes are that the sellers now participate in a second-price auction with reserve p set by the buyer and that the fee is now levied on the transaction price \tilde{p} , which is the minimum of the reserve and the second-lowest bid of a supplier.³⁸

In a nutshell, the intuition for this invariance result is the well-known result that with private values and identically distributed types, the optimal reserve does not vary with the number of bidders. When the supports are not the same, the fee may need to be augmented by the constant payments for the worst-off types. When the sellers' distributions or bargaining weights differ, more complicated mechanisms are required for implementation.

³⁶Strictly speaking, the fee the intermediary charges is, of course, the difference $\tau_{\alpha^*(b)}(p) - p$.

³⁷The buyer's problem is to choose to p to maximize $(v - \tau_{\alpha^*(b)}(p))G(p)$. Plugging in the formula for $\tau_{\alpha^*(b)}(p)$ and solving the first-order condition yields $p^*(v)$. Moreover, because Γ is increasing, the problem is quasiconcave, implying that the first-order condition is sufficient for a maximum, that is, $p^*(v)$ is the maximizer.

³⁸To see this, notice that the buyer's optimization problem becomes

$$\max_p (v - \tau_{\alpha^*(b)}(p))M(1 - G(p))^{M-1}G(p) + \int_{\underline{c}}^p (v - \tau_{\alpha^*(b)}(x))dG_{[2]}(x),$$

where $G_{[2]}(c) = 1 - (1 - G(c))^M - M(1 - G(c))^{M-1}G(c)$ is the distribution of the second-lowest of M independent draws from G . Plugging in the formula for $\tau_{\alpha^*(b)}$ and maximizing gives the result.

B Appendix: Proofs

Proof of Lemma 1. The price formation mechanism with bargaining weights (b, \mathbf{s}) maximizes

$$\mathbb{E}_{v, \mathbf{c}} [\alpha R(v, \mathbf{c}) + (1 - \alpha)W(v, \mathbf{c}; b, \mathbf{s})],$$

subject to incentive compatibility and individual rationality, and with the smallest $\alpha \in [0, 1]$ such that the no deficit condition holds. Using the mechanism design concepts from Appendix A, for a given α , the allocation rule maximizes

$$\begin{aligned} & \mathbb{E}_{v, \mathbf{c}} \left[\alpha \sum_{i \in \mathcal{M}} (\Phi(v) - \Gamma_i(c_i)) q_i(v, \mathbf{c}) \right. \\ & + \left. (1 - \alpha) \sum_{i \in \mathcal{M}} ((\Phi(v) - \Gamma_i(c_i)) + \min\{bn, 1\}(v - \Phi(v)) + \min\{s_i n, 1\}(\Gamma_i(c_i) - c_i)) q_i(v, \mathbf{c}) \right] \\ & = \mathbb{E}_{v, \mathbf{c}} \left[\sum_{i \in \mathcal{M}} ((1 - \alpha) \min\{bn, 1\}v + (1 - \min\{bn, 1\}(1 - \alpha))\Phi(v)) q_i(v, \mathbf{c}) \right] \\ & \quad - \mathbb{E}_{v, \mathbf{c}} \left[\sum_{i \in \mathcal{M}} ((1 - \alpha) \min\{s_i n, 1\}c_i + (1 - \min\{s_i n, 1\}(1 - \alpha))\Gamma_i(c_i)) q_i(v, \mathbf{c}) \right] \\ & = \mathbb{E}_{v, \mathbf{c}} \left[\sum_{i \in \mathcal{M}} [\Phi_{1 - \min\{bn, 1\}(1 - \alpha)}(v) - \Gamma_{i, 1 - \min\{s_i n, 1\}(1 - \alpha)}(c_i)] q_i(v, \mathbf{c}) \right], \end{aligned}$$

which is maximized with $q_i(v, \mathbf{c}) = 1$ when

$$\Phi_{1 - \min\{bn, 1\}(1 - \alpha)}(v) > \Gamma_{i, 1 - \min\{s_i n, 1\}(1 - \alpha)}(c_i) = \min_{j \in \mathcal{M}} \Gamma_{j, 1 - \min\{s_j n, 1\}(1 - \alpha)}(c_j),$$

and zero otherwise. To complete the proof, note that

$$\begin{aligned} 1 - \min\{bn, 1\}(1 - \alpha) & = \begin{cases} 1 - bn(1 - \alpha) & \text{if } b \in [0, \frac{1}{n}], \\ \alpha & \text{if } b \in (\frac{1}{n}, 1], \end{cases} \\ & = \max\{\alpha, 1 - bn(1 - \alpha)\}, \end{aligned}$$

and similarly

$$\begin{aligned} 1 - \min\{s_i n, 1\}(1 - \alpha) & = \begin{cases} 1 - s_i n(1 - \alpha) & \text{if } s_i \in [0, \frac{1}{n}], \\ \alpha & \text{if } s_i \in (\frac{1}{n}, 1], \end{cases} \\ & = \max\{\alpha, 1 - s_i n(1 - \alpha)\}. \end{aligned}$$

■

Proof of Lemma 2. The second-best mechanism maximizes

$$\mathbb{E}_{v, \mathbf{c}} \left[\sum_{i \in \mathcal{M}} [\alpha (\Phi(v) - \Gamma_i(c_i)) + (1 - \alpha) (v - c_i)] q_i(v, \mathbf{c}) \right],$$

subject to incentive compatibility and individual rationality, and has the smallest $\alpha \in [0, 1]$ such that no deficit holds. For a given α , the second-best mechanism has the allocation rule

$$q_{i, \alpha}^*(v, \mathbf{c}) \equiv \begin{cases} 1 & \text{if } \Phi_\alpha(v) > \Gamma_{i, \alpha}(c_i) = \min_{j \in \mathcal{M}} \Gamma_{j, \alpha}(c_j), \\ 0 & \text{otherwise.} \end{cases}$$

Using the definition of $q_{i, \alpha}(v, \mathbf{c}; b, \mathbf{s})$, one can see that $q_{i, \alpha}(v, \mathbf{c}; \frac{1}{M+1}, \dots, \frac{1}{M+1}) = q_{i, \alpha}^*(v, \mathbf{c})$, which then implies that the smallest α is the same for the second-best mechanism and for our price formation mechanism, and so the allocation rules are the same. ■

Proof of Lemma 3. The proof uses two intermediate steps, Lemmas 5 and 6:

Lemma 5. For all $y \in [\underline{v}, \bar{v}]$, $\int_y^{\bar{v}} \Phi(x) f(x) dx = y(1 - F(y))$, and for all $y \in [\underline{c}, \bar{c}]$, $\int_{\underline{c}}^y \Gamma(x) g(x) dx = yG(y)$.

Proof. Given $y \in [\underline{v}, \bar{v}]$,

$$\begin{aligned} \int_y^{\bar{v}} \Phi(x) f(x) dx &= \int_y^{\bar{v}} (f(x)x - 1 + F(x)) dx \\ &= \bar{v} - F(y)y - \int_y^{\bar{v}} F(x) dx - (\bar{v} - y) + \int_y^{\bar{v}} F(x) dx \\ &= y(1 - F(y)), \end{aligned}$$

and given $y \in [\underline{c}, \bar{c}]$,

$$\begin{aligned} \int_{\underline{c}}^y \Gamma(x) g(x) dx &= \int_{\underline{c}}^y (xg(x) + G(x)) dx \\ &= yG(y) - \int_{\underline{c}}^y G(x) dx + \int_{\underline{c}}^y G(x) dx \\ &= yG(y). \end{aligned}$$

□

Lemma 6. *For symmetric or asymmetric sellers,*

$$\mathbb{E}_{v,\underline{c}} \left[\sum_{i \in \mathcal{M}} (\Phi(v) - \Gamma_i(c_i)) \cdot \mathbf{1}_{\Gamma_i(c_i) \leq \min_j \{v, \Gamma_j(c_j)\}} \right] \begin{cases} = 0 & \text{if } \underline{c} \geq \underline{v}, \\ > 0 & \text{otherwise} \end{cases}$$

and, assuming symmetric sellers, then

$$\mathbb{E}_{v,\underline{c}} \left[(\Phi(v) - \Gamma_i(c_i)) \cdot \mathbf{1}_{\underline{c} \leq \min_{j \in \mathcal{M}} \{\Phi(v), c_j\}} \right] \begin{cases} = 0 & \text{if } \bar{v} \leq \bar{c}, \\ > 0 & \text{otherwise.} \end{cases}$$

Proof. Letting L be the distribution of $\min_{i \in \mathcal{M}} \Gamma_i(c_i)$, with density ℓ , then

$$\begin{aligned} & \mathbb{E}_{v,\underline{c}} \left[\sum_{i \in \mathcal{M}} (\Phi(v) - \Gamma_i(c_i)) \cdot \mathbf{1}_{\Gamma_i(c_i) \leq \min_j \{v, \Gamma_j(c_j)\}} \right] \\ &= \mathbb{E}_{v,\underline{c}} \left[\Phi(v) - \min_{i \in \mathcal{M}} \Gamma_i(c_i) \mid v \geq \min_{i \in \mathcal{M}} \Gamma_i(c_i) \right] \Pr \left(v \geq \min_{i \in \mathcal{M}} \Gamma_i(c_i) \right) \\ &= \int_{\underline{c}}^{\max\{\underline{c}, \min\{\bar{c}, \bar{v}\}\}} \int_{\max\{\underline{v}, y\}}^{\bar{v}} (\Phi(v) - y) f(v) \ell(y) dv dy \\ &= \int_{\underline{c}}^{\max\{\underline{c}, \min\{\bar{c}, \bar{v}\}\}} (\max\{\underline{v}, y\} - y) (1 - F(\max\{\underline{v}, y\})) \ell(y) dy, \end{aligned}$$

where the final equality uses Lemma 5. If $\underline{c} \geq \underline{v}$, then for all $y \in [\underline{c}, \max\{\underline{c}, \min\{\bar{c}, \bar{v}\}\}]$, $\max\{\underline{v}, y\} = y$ and so the integrand is zero for all relevant values of y , and so the expression is zero. If $\underline{c} < \underline{v}$, then we can write the expression as (limiting the bounds of integration so that $y \leq \underline{v}$):

$$\int_{\underline{c}}^{\max\{\underline{c}, \min\{\bar{c}, \underline{v}\}\}} (\underline{v} - y) (1 - F(\underline{v})) \ell(y) dy = \int_{\underline{c}}^{\min\{\bar{c}, \underline{v}\}} (\underline{v} - y) \ell(y) dy > 0,$$

where the inequality follows because, under the assumption that $\underline{c} < \underline{v}$, the set $[\underline{c}, \min\{\bar{c}, \underline{v}\}]$ is open and for $y \in (\underline{c}, \min\{\bar{c}, \underline{v}\})$, the integrand is positive.

Assuming symmetric sellers with virtual cost function Γ , and now letting L be the

distribution of $\min_{i \in \mathcal{M}} c_i$, with density ℓ , then

$$\begin{aligned}
& \mathbb{E}_{v, \mathbf{c}} \left[(\Phi(v) - \Gamma(c_i)) \cdot \mathbf{1}_{c_i \leq \min_{j \in \mathcal{M}} \{\Phi(v), c_j\}} \right] \\
&= \mathbb{E}_{v, \mathbf{c}} \left[\left(\Phi(v) - \Gamma\left(\min_{j \in \mathcal{M}} c_j\right) \right) \mid \min_{j \in \mathcal{M}} c_j < \Phi(v) \right] \Pr \left(\min_{j \in \mathcal{M}} c_j < \Phi(v) \right) \\
&= \int_{\min\{\bar{v}, \max\{\underline{v}, \Phi^{-1}(\underline{c})\}}^{\bar{v}}} \int_{\underline{c}}^{\min\{\bar{c}, \Phi(v)\}} (\Phi(v) - \Gamma(\hat{c})) \ell(\hat{c}) f(v) d\hat{c} dv \\
&= \int_{\min\{\bar{v}, \max\{\underline{v}, \Phi^{-1}(\underline{c})\}}^{\bar{v}}} (\Phi(v) - \min\{\bar{c}, \Phi(v)\}) L(\min\{\bar{c}, \Phi(v)\}) f(v) dv,
\end{aligned}$$

where the final equality uses Lemma 5. If $\Phi(\bar{v}) \leq \bar{c}$, i.e., if $\bar{v} \leq \bar{c}$, then for all $v \in [\min\{\bar{v}, \max\{\underline{v}, \Phi^{-1}(\underline{c})\}, \bar{v}\}]$, $\Phi(v) \leq \Phi(\bar{v}) = \bar{v} \leq \bar{c}$, and so the integrand is zero, and so the expression is zero. If $\bar{v} > \bar{c}$, then we can write the expression as (limiting the bounds of integration so that $\bar{c} < \Phi(v)$):

$$\begin{aligned}
\int_{\min\{\bar{v}, \max\{\underline{v}, \Phi^{-1}(\underline{c}), \Phi^{-1}(\bar{c})\}}^{\bar{v}}} (\Phi(v) - \bar{c}) L(\bar{c}) f(v) dv &= \int_{\max\{\underline{v}, \Phi^{-1}(\bar{c})\}}^{\bar{v}} (\Phi(v) - \bar{c}) f(v) dv \\
&> 0,
\end{aligned}$$

where the inequality follows because, under the assumption that $\bar{v} > \bar{c}$, the set $[\max\{\underline{v}, \Phi^{-1}(\bar{c})\}, \bar{v}]$ is open and for $v \in (\max\{\underline{v}, \Phi^{-1}(\bar{c})\}, \bar{v})$, the integrand is positive. \square

Continuation of the Proof of Proposition 3. Recall that $\alpha^*(b, \mathbf{s})$ is the smallest value of $\alpha \in [0, 1]$ such that the following is nonnegative:

$$\begin{aligned}
& \mathbb{E}_{v, \mathbf{c}} \left[\sum_{i \in \mathcal{M}} (\Phi(v) - \Gamma_i(c_i)) \right. \\
& \left. \cdot \mathbf{1}_{\Gamma_{i, 1-s_i(1-\alpha)(M+1)}(c_i) \leq \min_{j \in \mathcal{M}} \{\Phi_{1-b(1-\alpha)(M+1)}(v), \Gamma_{j, 1-s_j(1-\alpha)(M+1)}(c_j)\}} \right]. \tag{16}
\end{aligned}$$

Note that (16) is strictly increasing in α (α is the weight on budget surplus in the mechanism's objective, so increasing α leads to an increase in expected budget surplus).

Consider the case of $b = 1$, $s_1 = \dots = s_M = 0$, and $\alpha = \frac{M}{M+1}$. Then (16) is

$$\mathbb{E}_{v, \mathbf{c}} \left[\sum_{i \in \mathcal{M}} (\Phi(v) - \Gamma_i(c_i)) \cdot \mathbf{1}_{\Gamma_i(c_i) \leq \min_{j \in \mathcal{M}} \{v, \Gamma_j(c_j)\}} \right],$$

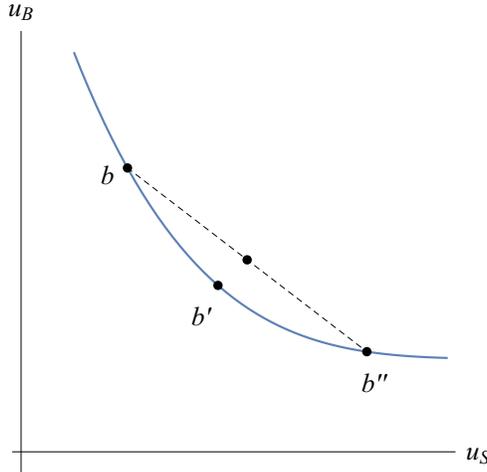
which by Lemma 6 is zero when $\underline{c} \geq \underline{v}$ and positive otherwise. Thus, we can conclude

that $\alpha^*(1, 0, \dots, 0) = \frac{M}{M+1}$ when $\underline{c} \geq \underline{v}$ (and $\alpha^*(1, 0, \dots, 0) < \frac{M}{M+1}$ otherwise). For the case of symmetric suppliers, $b = 0$, $s_1 = \dots = s_M = \frac{1}{M}$, and $\alpha = \frac{1}{M+1}$, (16) is

$$\mathbb{E}_{v, c} \left[\sum_{i \in \mathcal{M}} (\Phi(v) - \Gamma(c_i)) \cdot \mathbf{1}_{c_i \leq \min_{j \in \mathcal{M}} \{\Phi(v), c_j\}} \right],$$

which by Lemma 6 is zero when $\bar{v} \leq \bar{c}$ and positive otherwise. Thus, we can conclude that $\alpha^*(0, \frac{1}{M}, \dots, \frac{1}{M}) = \frac{1}{M+1}$ when $\bar{v} \leq \bar{c}$ (and $\alpha^*(0, \frac{1}{M}, \dots, \frac{1}{M}) < \frac{1}{M+1}$ otherwise). ■

Proof of Proposition 2. To begin, note that because u_B is strictly increasing in the buyer's bargaining weight b , each point on the frontier is associated with a unique b . Given buyer bargaining weight b , we denote the associated point on the frontier as $(u_S(b), u_B(b))$, where u_S is the sum of all seller payoffs. As illustrated in the figure below, suppose that the Williams frontier is not concave.



Then there exist points on the frontier, which we denote by their associated buyer bargaining weights $b > b' > b''$, such that $(u_S(b) + u_S(b''))/2 > u_S(b')$ and $(u_B(b) + u_B(b''))/2 > u_B(b')$. Let $\eta(b)$ be the price formation mechanism associated with buyer bargaining weight b . Then consider the mechanism that is a 50-50 mixture of $\eta(b)$ and $\eta(b'')$. By the construction of the price formation process, the expected budget surplus to the mechanism in $\eta(b)$, $\eta(b')$, $\eta(b'')$ is zero. The expected weighted welfare under $\eta(b')$ satisfies

$$b' u_B(b') + \sum_{i \in \mathcal{M}} \frac{1-b'}{M} u_{S_i}(b') < b' \frac{u_B(b) + u_B(b'')}{2} + \sum_{i \in \mathcal{M}} \frac{1-b'}{M} \frac{u_{S_i}(b) + u_{S_i}(b'')}{2},$$

where the right side is the expected weighted welfare given b' under the mechanism that is a 50-50 mixture of $\eta(b)$ and $\eta(b'')$, which since the no deficit constraint is satisfied,

contradicts the optimality of $\eta(b')$. This contradiction completes the proof. ■

Proof of Proposition 5. Suppose that the price formation process is the one associated with efficient investments \bar{e} . Then the price formation mechanism has trade if and only if $\Phi_{\alpha^*}(v, \bar{e}_B) > \min_{i \in \mathcal{M}} \Gamma_{i, \alpha^*}(c_i, \bar{e}_i)$, whereas the social planner's objective is trade if and only if $v > \min_{i \in \mathcal{M}} c_i$. As a result, the agents' objectives and hence payoff maximizing investments differ from those of the social planner. To see this, note that the buyer's problem is (dropping the subscript \bar{e} on α^* for ease of notation):

$$\begin{aligned} & \mathbb{E}_{v, \mathbf{c}} \left[\left(v - \Phi_{\alpha^*}^{-1} \left(\min_{i \in \mathcal{M}} \Gamma_{i, \alpha^*}(c_i, \bar{e}_i), \bar{e}_B \right) \right) \cdot \mathbf{1}_{v > \Phi_{\alpha^*}^{-1}(\min_{i \in \mathcal{M}} \Gamma_{i, \alpha^*}(c_i, \bar{e}_i), \bar{e}_B)} \right] - \Psi_B(e_B) \\ &= \int_{\underline{v}}^{\bar{v}} \tilde{L}(x, \mathbf{e}_S) (1 - F(x, e_B)) dx - \Psi_B(e_B), \end{aligned}$$

where $\tilde{L}(\cdot, \mathbf{e}_S)$ is the distribution of $\Phi_{\alpha^*}^{-1}(\min_{i \in \mathcal{M}} \Gamma_{i, \alpha^*}(c_i, \bar{e}_i), \bar{e}_B)$. Thus, the buyer's first-order condition is

$$- \int_{\underline{v}}^{\bar{v}} \tilde{L}(x, \mathbf{e}_S) \frac{\partial F(x, e_B)}{\partial e_B} dx = \Psi'_B(e_B),$$

which differs from (6) because $\alpha^* > 0$ implies that $\tilde{L}(x, \mathbf{e}_S)$ differs from $L(x, \mathbf{e}_S)$ for all x in an open subset of (\underline{v}, \bar{c}) , which itself must be an open set because $\alpha^* > 0$ implies that $\underline{v} < \bar{c}$. ■

Proof of Lemma 4. The extension to allow seller specific quality parameters follows by analogous arguments to Lemma 1 noting that the buyer's value for seller i 's good is $\theta_i v$, which has distribution $\hat{F}(x) \equiv F(x/\theta_i)$ on $[\theta_i \underline{v}, \theta_i \bar{v}]$ with density $\hat{f}(x) = \frac{1}{\theta_i} f(v/\theta_i)$. Thus, the virtual type when the buyer's value is v is

$$\theta_i v - \frac{1 - \hat{F}(\theta_i v)}{\hat{f}(\theta_i v)} = \theta_i v - \theta_i \frac{1 - F(v)}{f(v)} = \theta_i \Phi(v).$$

Thus, the parameter θ_i “factors out” of the virtual type function.

The extension to multi-object demand follows by standard mechanism design arguments. ■

C Appendix: Generalization

Let \mathcal{P}_M be the set of subsets of M with no more than D elements (including the empty set) and let $\boldsymbol{\theta} = \{\theta_X\}_{X \in \mathcal{P}_M}$ be a commonly known vector of taste parameters of the buyer

satisfying the “size dependent discounts” condition of Delacrétaz et al. (2019). Specifically, let there be seller-specific preferences $\{\hat{\theta}_i\}_{i \in \mathcal{M}}$ and size dependent discounts $\{\delta_i\}_{i \in \mathcal{M}}$ with $0 = \delta_0 = \delta_1 \leq \delta_2 \leq \dots \leq \delta_M$ such that for all $X \in \mathcal{P}_M$, $\theta_X = \sum_{i \in X} \hat{\theta}_i - \delta_{|X|}$. Thus, the buyer’s value for purchasing from sellers in $X \in \mathcal{P}_M$ when its type is v is $\theta_X v$, which depends on the buyer’s value, the buyer’s preferences for standalone purchases from the sellers in X , and a discount that depends on the total number of units purchased. Note that $\theta_\emptyset = 0$, so that the value to the buyer of no trade is zero.

This setup encompasses (i) the homogeneous good model with constant marginal value or decreasing marginal value by setting $\hat{\theta}_i = \theta$ for some common θ and for $i \in \mathcal{M}$, δ_i either all zero for constant marginal value or increasing in i for decreasing marginal value; (ii) differentiated products by letting $\hat{\theta}_i$ differ by i and setting all δ_i to zero; (iii) a one-buyer version of the Shapley-Shubik model by setting $D = 1$; and (iv) a version of the Shapley-Shubik model in which the buyer has demand for multiple products of the sellers by setting $D > 1$.

Define

$$X_\alpha^*(v, \mathbf{c}) \in \arg \max_{X \in \mathcal{P}_M} \theta_X \Phi_\alpha(v) - \sum_{i \in X} \Gamma_{i, \alpha}(c_i),$$

i.e., $X_\alpha^*(v, \mathbf{c})$ is the set of trading partners for the buyer that maximizes the difference between the ironed α -weighted virtual value, scaled by $\theta_{X_\alpha^*(v, \mathbf{c})}$, and the ironed α -weighted virtual costs of the trading partners. We then define α^* to be the smallest $\alpha \in [0, 1]$ such that

$$\mathbb{E}_{v, \mathbf{c}} \left[\theta_{X_\alpha^*(v, \mathbf{c})} \Phi(v) - \sum_{i \in X_\alpha^*(v, \mathbf{c})} \Gamma_i(c_i) \right] = 0.$$

Given the type realization (v, \mathbf{c}) , the one-to-many α^* -mechanism induces trade between the buyer and sellers in $X_{\alpha^*}^*(v, \mathbf{c})$. The expected payoff of the buyer is

$$u_B = \mathbb{E}_v \left[U_B(\underline{v}) + \int_{\underline{v}}^v \sum_{X \in \mathcal{P}_M} \theta_X \Pr_{\mathbf{c}}(X \in X_{\alpha^*}^*(x, \mathbf{c})) dx \right],$$

and the expected payoff of seller i is

$$u_{i, S} = \mathbb{E}_{c_i} \left[U_{i, S}(\bar{c}) + \int_{c_i}^{\bar{c}} \Pr_{v, \mathbf{c}_{-i}}(i \in X_{\alpha^*}^*(v, x, \mathbf{c}_{-i})) dx \right].$$

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