On Cheating and Whistleblowing*

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Abstract

We study the role of whistleblowing in the following inspection game. Two agents who compete for a prize can either behave legally or illegally. After the competition, a controller investigates the agents' behavior. This inspection game has a unique Bayesian equilibrium in mixed strategies. We then add a whistleblowing stage, where the controller asks the loser to blow the whistle. This extended game has a unique perfect Bayesian equilibrium in which only a cheating loser accuses the winner of cheating and the controller tests the winner if and only if the winner is accused of cheating. Whistleblowing reduces the frequencies of cheating, is less costly in terms of test frequencies, and leads to a strict Pareto-improvement if punishments for cheating are sufficiently large.

Keywords: Whistleblowing, leniency, inspection games, signalling.

JEL-Classification: C72, D82, K42, K21.

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1 Introduction

We address the following problem. Two agents who compete for a prize can choose between two strategies: play fair or cheat. If both agents play the same strategy, either agent wins the prize with an exogenously given probability. If only one player cheats, the probability that he wins increases. This simple game has a unique Pareto-inefficient equilibrium in dominant strategies, where both agents cheat if the cost of cheating are sufficiently small.[1] This inefficiency calls for a mechanism to eliminate cheating. We first investigate a standard inspection regime, where a third agent controls the winner of the contest. The controller faces the problem that the agents' actions are private information, and that detecting cheating is costly. This inspection game has a unique perfect Bayesian equilibrium, in which both agents randomize between cheating and not cheating. Thus, this control regime cannot eliminate cheating.

In the popular press it is increasingly recognised that whistleblowing and leniency clauses, offering legal immunity to whistleblowers, can play an important role in combating illegal behavior.[2] The question thus arises whether in our cheating game a mechanism that would allow the loser to blow the whistle can improve the outcome. In order to study this question, we add a whistleblowing stage after the contest. In this stage, the controller asks the loser whether the winner has cheated. If the loser says yes, the controller inspects the winner, otherwise no inspection takes place. If the inspection reveals that the winner has cheated, punishment takes place.

[1] An obvious example is doping in sports, where players can increase their winning chance by using performance enhancing drugs. Other examples are public procurement auctions where bidders can either bid honestly or cheat by bribing government officials for example.

[2] The year 2002 has been extraordinarily successful for whistleblowers. First, the Time magazine featured three women as "persons of the year 2002" for whistleblowing. They were responsible for the disclosure of fraudulent accounting practices at Enron and Worldcom, and neglected terrorist warnings prior September 11 at the FBI. Second, in response to several accounting scandals in the US, the Sarbanes-Oxley act of 2002 not only prohibits retaliation against whistleblowers, but also solicits, encourages and reinforces the very act of whistleblowing.

We show that our whistleblowing mechanism improves the efficiency of controls. In fact, our results make a strong case for the use of whistleblowing. First, if punishments for detected cheaters are sufficiently large, then the cheating probabilities are smaller than in the inspection game. Second, whistleblowing allows for a strict Pareto-improvement relative to the inspection game. Third, whistleblowing reduces control costs since the frequency of tests is smaller with the whistleblowing mechanism than without it.

Despite the growing importance of whistleblowing in practice,[3] the economic literature on whistleblowing and leniency clauses is rather small. A recent literature on antitrust legislation investigates whistleblowing as an instrument to fight cartels (e.g. Motta and Polo, 2003; Rey, 2001; and Spagnolo, 2002). The objective of the antitrust authorities is to design a leniency program that deters the formation of cartels (Spagnolo, 2002) or to extract information from cartel members once the antitrust authority has opened an investigation (Motta and Polo, 2003). Apesteguia et al. (2003) examine the impact of leniency laws experimentally. Our approach is also broadly related to the economics of law enforcement, initiated by Becker (1968). With respect to leniency programs, this literature provides ample evidence that lowering sanctions for self-reporting agents is welfare improving (e.g. Innes, 1999a,b; Malik, 1993; Kaplow et al., 1994).

The paper is structured as follows. In Section 2 we introduce the inspection game. Section 3 investigates the whistleblowing game, and Section 4 compares the two mechanisms. In Section 5, we allow in turn for the possibilities that cheating is less than perfectly effective and that tests are not perfectly reliable. Section 6 concludes. All proofs are in the Appendix.

[3] For example, the U.S Antitrust Division created a "leniency policy" in 1993 and the European Commission took similar steps, first in 1996, and subsequently in 2002 when a legislation which closely mimics the U.S. policy was adopted (see Aspeguia et al. (2003)). According to Salem and Franze (2002), in the US "the first whistleblower statute was enacted in 1863, at the height of the Civil War. The False Claims Act was aimed at punishing corrupt defense contractors who were overcharging the Union Army for its supplies."

2 The inspection game

We describe our inspection game in the context of sports competition, where in the first stage two athletes compete for a prize only one of them can win. They face the choice between playing "clean" or playing "doped", where the use of performance enhancing drugs increases an agent's chances of winning the competition. In the second stage a third agent, called the controller, performs doping tests. If a player is caught, the athlete is punished. Given the examples and illustrations discussed in the introduction, the game is amenable to a variety of interpretations outside of sports.

2.1 The doping stage

Consider the doping stage first. The athletes, labelled 1 and 2, compete for a prize of value $w = 1$. Agents' sets of pure strategies are $\{c, d\}$, where c stands for playing clean and d for doping. Doping improves the performance of the players at cost γ where $0 < \gamma < 1$. Cost γ reflects the notion that people do not like to cheat: Each athlete prefers winning clean to winning doped, so that $w > w - \gamma$. However, each athlete also prefers winning doped to losing clean, i.e., $w - \gamma > 0$. The cost may also represent expected health cost and monetary cost of drugs.

The probability that agent 1 wins the prize if both agents are clean or if both agents are doped is $\sigma \geq \frac{1}{2}$, so that agent 1 is the better player. The probability that agent 2 wins is accordingly $1-\sigma$. The winning probabilities are exogenous and common knowledge.[4] We assume for now that doping is completely effective: a doped agent wins with certainty if the opponent is clean. In Section 5 we relax this assumption.[5]

The payoffs are $(\sigma - \gamma, 1 - \sigma - \gamma)$ for the strategy profile (d, d), $(1 - c, 0)$ for (d, c), $(0, 1 - c)$ for (c, d), and $(\sigma, 1 - \sigma)$ for (c, c). One can show that

[4] In practice, the odds set by bookmakers could be used to approximate σ.

[5] In a standard inspection game there is typically one inspector and one inspectee (see e.g. Güth and Pethig, 1992) or one controller and many isolated acting inspectees (an overview of such games is given by Avenhaus et al., 2002). In contrast, our model describes the situation of one inspector and two interacting agents.

this simple doping game has a unique Nash equilibrium with the following equilibrium strategies: if $1-\sigma > \gamma$, both athletes dope with certainty; if $\sigma < \gamma$, they are clean and if $\sigma > \gamma > 1-\sigma$ there exists a mixed-strategy equilibrium where the probability that agent 1 dopes α and the probability that agent 2 dopes β are [6]

$$\alpha = \frac{\sigma - \gamma}{2\sigma - 1} \text{ and } \beta = \frac{\gamma - (1-\sigma)}{2\sigma - 1}, \text{ respectively.}$$

The welfare property of the equilibrium provides a strong reason for implementing anti-doping measures: if $\sigma > \gamma$, the equilibrium is Pareto-inefficient. This is most apparent when $1-\sigma > \gamma$, in which case the game is a Prisoner's Dilemma: doping is a dominant strategy and the equilibrium is Pareto-inefficient. To see this recall that if both players dope the expected payoffs are $\sigma - \gamma$ and $1 - \sigma - \gamma$ and if both players play clean, they are σ and $1-\sigma$.

For the rest of the paper we restrict the range of parameter values as follows. First, we assume that $1-\sigma > \gamma$ so that the two athletes have a dominant strategy to dope. Second, to simplify the exposition, we let the doping costs γ become arbitrarily small.

2.2 The control stage

After the competition, in the second stage, the controller decides whether to test the winner. The controller's set of pure strategies is $\{T, NT\}$, where T stands for test and NT for no test. Consistent with the assumption that σ is common knowledge, the controller knows which agent is the better one. Therefore, the controller can use different testing probabilities for either agent. The probability that agent 1 is tested if he wins is denoted by t_1 and the probability that agent 2 is tested after winning is t_2. Thus, t_1 and t_2 denote the controller's mixed strategy of testing winner 1 and winner 2, respectively. For now we assume that the tests are reliable, i.e. there are no test errors. In Section 5 we relax this assumption.

[6] For a game theoretic analysis of this doping game and the effects of the anti-doping measures of the International Olympic Committee see Berentsen (2002).

In many inspection games, it is assumed that the controller commits to a certain inspection frequency.[7] We depart from this assumption for the following reason. With commitment and with sufficiently severe punishments, doping is trivially eliminated. Nevertheless, the costly inspections must be carried out even though the controller knows that nobody cheats. This is certainly not sequentially rational. We therefore assume that the testing probabilities and the doping behavior are mutually best responses.

We assume the following exogenously given payoff for the controller. If the test indicates that the winner is doped, the controller receives the reward $S > 0$. The disutility of testing is $K \in (0, S)$, so that the payoff is $S - K > 0$. If we normalize his payoff by S, we get $1 - k > 0$, where $k = \frac{K}{S}$.[8] If the test indicates that the winner is clean, the controller gets $-k < 0$. Finally, if the controller makes no test, his payoff is zero.

If a player is tested positive after winning, he gets the exogenously given punishment $P \geq -1$. If $P < 0$, the winner's payoff is reduced but still strictly positive. If $P = 0$, the punishment is disqualification so that the winner does not receive the prize. Finally, if $P > 0$, the winner's payoff is strictly negative. It implies that the controller cannot only confiscate the prize but can impose some additional punishment in terms of utility to a cheater. The loser's payoff is always zero.[9] The punishment P leaves the payoffs of the controller unaffected. Such a deadweight loss penalty reflects the fact that in sports the penalty consists of disqualification and a ban from further competitions. It is also accurate in situations where detected cheaters are fired or sent to jail.[10] Finally, throughout the paper we assume

[7] Examples for inspection games with commitment are fare dodging in public transport (Avenhaus, 1997) and tax avoidance (Greenberg, 1984).

[8] One can also consider the inspection game as a game in which the controller also competes for the prize. He wins the prize of value one if he proves that the winner has cheated. In this setting the controller would never test the loser because his payoff would be $-k$.

[9] We assume that the loser does not inherit the prize. The strategic effect of inheritance on the doping behavior is analyzed in Berentsen (2002).

[10] In some control situations the penalty is a transfer from the cheating agent to the controller as for example for fare dodgers (as in Avenhaus, 1997). It is straightforward to change our framework to cover situations where the penalty is a transfer from the cheating

that in contrast to the testing probabilities punishments and rewards cannot be conditioned on individual agents.

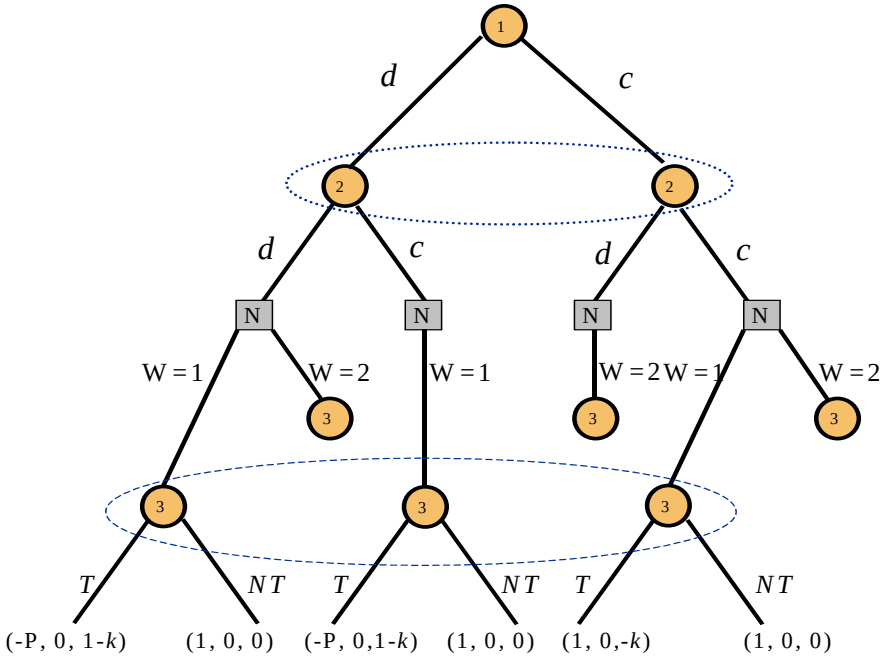

Figure 1: The inspection game.

The inspection game is depicted in Figure 1. In order to keep the figure simple, the controller's information set when agent 2 is the winner ($W = 2$) is not drawn. The controller (player 3) only observes which agent has won, but he has no information about the actions carried out by either player.

2.3 Equilibrium

Before we investigate the equilibrium, it is useful to derive the winning probabilities and the conditional probabilities that the winner is doped. Let $\Pr(W = 1)$ and $\Pr(W = 2)$ denote the winning probabilities of agent 1 or agent 2 in the inspection game. They are

$$\begin{aligned} \Pr(W = 1) &= \sigma(1-\alpha)(1-\beta) + \alpha(1-\beta) + \sigma\alpha\beta \quad \text{and} \\ \Pr(W = 2) &= (1-\sigma)(1-\alpha)(1-\beta) + (1-\alpha)\beta + (1-\sigma)\alpha\beta. \end{aligned}$$

agent to the controller.

Consider the winning probability of agent 1. With probability $(1-\alpha)(1-\beta)$ both agents are clean and with probability $\alpha\beta$ both are doped. In either case he wins with probability σ. With probability $\alpha(1-\beta)$ only he is doped in which case he wins with probability 1.

Let $\Pr(d \mid W = 1)$ and $\Pr(d \mid W = 2)$ denote the conditional probabilities that agents 1 and 2, respectively, are doped if they win. These probabilities are

$$\begin{aligned}
\Pr(d \mid W = 1) &= \frac{\alpha(1-\beta) + \sigma\alpha\beta}{(1-\beta)\left(\sigma(1-\alpha) + \alpha\right) + \sigma\alpha\beta} \quad \text{and} \\
\Pr(d \mid W = 2) &= \frac{(1-\sigma)\alpha\beta + (1-\alpha)\beta}{(1-\alpha)\left((1-\sigma)(1-\beta) + \beta\right) + (1-\sigma)\alpha\beta}.
\end{aligned}$$

In equilibrium all agents must be indifferent between their pure strategies, which implies that for the controller and the agents the following conditions must hold.

The controller's expected payoff of testing winner 1 is the conditional probability that winner 1 is doped minus the costs of testing, that is $\Pr(d \mid W = 1) - k$, while his (expected) payoff of not testing is zero.[11] In equilibrium both strategies have to yield the same (expected) outcome, so that the controller's equilibrium condition for testing winner 1 or winner 2 are

$$\Pr(d \mid W = 1) - k = 0 \tag{1}$$

$$\Pr(d \mid W = 2) - k = 0. \tag{2}$$

The expected payoff for agent 1 of playing c is $E_1[c] = \sigma(1-\beta)$. With probability $(1-\beta)$ player 2 is clean in which case he wins with probability σ. In all other cases he loses. The expected payoff of playing d is $E_1[d] = (1-t_1)(\sigma\beta + (1-\beta)) - t_1(\sigma\beta + (1-\beta))P$. With probability $1-\beta$ player 2 is clean in which case he wins with certainty and with probability β player 2 is doped in which case he wins with probability σ. With probability $(1-t_1)$ he is not tested by the controller in which case he receives the prize of value 1 and with probability t_1 he is tested and receives punishment P. Thus the

[11] Throughout the paper, we call agent 1 (2) winner 1 (2) if he has won the game.

equilibrium condition for agent 1 is

$$\sigma(1-\beta) = (1 - t_1(1+P))(\sigma\beta + (1-\beta)). \tag{3}$$

The equilibrium condition for agent 2, derived in the same way, is

$$(1-\sigma)(1-\alpha) = (1 - t_2(1+P))((1-\sigma)\alpha + (1-\alpha)). \tag{4}$$

Lemma 1 *In any equilibrium the following is true:*

(i) the favorite player (agent 1) is more likely to dope than the underdog ($\alpha \geq \beta$; with strict inequality if $\sigma > \frac{1}{2}$);

(ii) the underdog is more likely to be tested than agent 1 ($t_2 \geq t_1$; with strict inequality if $\sigma > \frac{1}{2}$);

(iii) $\Pr(W = 1) = \sigma$ *and* $\Pr(W = 2) = 1 - \sigma$.

The fact that in any equilibrium $\alpha \geq \beta$ is a consequence of the equilibrium conditions (1) and (2). They imply that in any equilibrium the conditional probabilities that agent 1 and 2 are doped after winning are the same. If both agents dope with the same probability, and because agent 2 is more likely to lose if both agents are doped, he is more likely to be doped after winning. Consequently, α must be greater than β. Agent 2 is more likely to be tested if he wins because, all else equal, doping is relatively more attractive for agent 2 than for agent 1.

Interestingly, the possibility of cheating does not affect the probabilities of winning the game, i.e. $\Pr(W = 1) = \sigma$ and $\Pr(W = 2) = 1 - \sigma$.[12] Thus, the winning probabilities are identical in games (i) without doping opportunities, in games (ii) with doping opportunities without controls, and (iii) in games with doping opportunities and controls and punishments. This is reminiscent of a result reported by Snyder (1989) and Rosen (1986) in the

[12] This result relies on our assumption that there are no direct cost of doping (e.g. health).

context of contests where effort choice does not affect the winning probabilities in equilibrium.[13]

Proposition 1 *The strategy profile* $(t_1^*; t_2^*; \alpha^*; \beta^*)$ *is the unique Bayesian Nash Equilibrium of the inspection game. The doping probabilities are*

$$\alpha^* = \frac{1 - \Psi - k(1 - 2\sigma)}{2\sigma}, \beta^* = \frac{1 - \Psi + k(1 - 2\sigma)}{2(1 - \sigma)}$$

and the testing probabilities

$$t_1^* = \frac{\beta^*}{k(1 + P)}, t_2^* = \frac{\alpha^*}{k(1 + P)},$$

where $\Psi = \sqrt{(1 - k)(1 - k(1 - 2\sigma)^2)}$.

The equilibrium strategies have the following comparative static properties. First, the favorite player's cheating probability is increasing in σ, $\frac{\partial \alpha^*}{\partial \sigma} > 0$, while the underdog's is decreasing, $\frac{\partial \beta^*}{\partial \sigma} < 0$. Second, for the testing probabilities we have $\frac{\partial t_1^*}{\partial \sigma} < 0$ and $\frac{\partial t_2^*}{\partial \sigma} > 0$. Consequently, the differences in the cheating probabilities $\alpha^* - \beta^*$ and the testing probabilities $t_2^* - t_1^*$ are both increasing in σ. Third, the agents' cheating probabilities are increasing in k.

Finally, the agents' equilibrium strategies are independent of the punishment P.[14] The punishment P has no impact on the cheating probabilities because both agents randomize so as to keep the controller indifferent between testing and not testing and the controller's payoff is independent of P. Increasing P, however, reduces the testing probabilities because the controller must keep the agents indifferent between cheating and not cheating. The same argument explains also why the agents increase their doping probabilities if k increase, i.e. $\frac{\partial \alpha^*}{\partial k} > 0$ and $\frac{\partial \beta^*}{\partial k} > 0$.

[13] In this literature, the contest success function is $H(x, y) = \frac{ah(x)}{ah(x)+(1-a)h(y)}$, where $H(x, y)$ denotes the winning probability of agent X if his effort is x and the opponent's is y. The function $h(.)$ satisfies $h(0) = 0$, $h'(0) > 0$ and $h'' \leq 0$ and $a \in (0, 1)$ is the natural (dis)advantage of X. In any pure strategy equilibrium with identical effort costs, Snyder shows that $H(x^*, y^*) = a$. We thank Gerd Mühlheusser for pointing this out to us.

[14] For this result the assumption that P is a deadweight loss is crucial. If, in contrast, punishment P is a transfer from the agents to the controller, the agents' equilibrium strategies would depend negatively on P.

3 The whistleblowing game

This raises the question whether an improvement is possible if the controller can extract information that is available to the agents but not to him. For that purpose, we now extend the game with a whistleblowing stage. We model this stage as a signalling game between the controller and the loser, where after the contest, the loser sends a message to the controller. The message space is $\{D, I\}$, where D is the message "The winner is doped" and I is the message "I don't know". Thus, when sending message D, the loser "blows the whistle". After receiving a message, the controller decides whether to test the winner, and the game ends. Note that this kind of whistleblowing is to be understood normatively as something that might be used to reduce the use of doping rather than as a positive description of existing real world whistleblowing mechanisms.

In what follows, we assume that while the equilibrium strategies α and β are inferred by (and thus in a sense "known" to) every player in the game, the actual play (i.e. c or d) is private information of the respective agent. This private information is used by the loser to update his beliefs about the winner's behavior. In particular, due to the effectiveness of doping, a doped loser can infer with certainty that the winner is doped, which is an inference the controller cannot make because he does not observe the play of the game.

Our goal is to design an incentive-compatible reward and punishment scheme, which allows the controller to extract the loser's private information about the winner's behavior in the contest. More specifically, we want to design a "whistleblowing mechanism" between the loser and the controller, where the loser and the controller behave as follows: The doped loser, who infers the winner's behavior with perfect accuracy, "blows the whistle" and the clean loser sends the message "I don't know". The controller tests the winner if and only if the loser blows the whistle.

3.1 Strategies and beliefs

An agent's strategy in the whistleblowing game consists of three choices. For example, for player 1 it consists of the probability α with which he dopes, the probability that he sends message D given that he is clean, denoted by $m_1(c)$, and the probability that he sends message D given that he is doped, denoted by $m_1(d)$. Thus, a strategy for agent 1 is

$$\delta_1 = (\alpha, m_1(c), m_1(d)).$$

For example, the strategy $\delta_1 = (\alpha, 0, 1)$ means that agent 1 dopes with probability α, sends the message I with certainty if he is a clean loser, and sends the message D with certainty if he is a doped loser. Likewise, a strategy for agent 2 is $\delta_2 = (\beta, m_2(c), m_2(d))$.

We denote the beliefs of agent 1 after losing the competition by $\mu_1(d \mid c)$ and $\mu_1(d \mid d)$.[15] For example, after losing the competition, $\mu_1(d \mid c)$ is agent 1's belief that the winner (agent 2) is doped given that he himself is clean. Note that our assumption on the effectiveness of doping implies that no clean agent will ever win against a doped player. Therefore, $\mu_1(d \mid d) = 1$. Agent 2's beliefs are denoted accordingly.

The controller's strategy still consists of the testing probabilities t_1 and t_2. In contrast to the inspection game, these testing probabilities are now contingent on the loser's message. Therefore, a mixed strategy for the controller is now denoted as

$$\delta_3 = (t_1(I), t_1(D); t_2(I), t_2(D)).$$

If player 1 wins the game, the controller's beliefs are $\mu_3(d \mid W = 1, D)$ and $\mu_3(d \mid W = 1, I)$. For example, $\mu_3(d \mid W = 1, D)$ is the controller's belief that winner 1 is doped if player 2 sends the message D. The controller's beliefs when player 2 wins the game are denoted accordingly.

[15] The beliefs of the winner are irrelevant for the game so we do not state them.

3.2 The whistleblowing stage

We first describe the payoffs in the whistleblowing stage and then we consider the incentive constraints that have to hold in the whistleblowing equilibrium. The loser's payoff is $-\Phi(D, C) \leq 0$ if he sends message D and the test indicates that the winner is clean. It is $\Phi(D, D) \geq 0$ if he sends message D and the test indicates that the winner is doped. In all other cases we set the loser's payoff equal to zero because we do not want to punish or reward a loser whose message is "I don't know." Likewise, we do not wish to reward or punish a loser when there is no test.[16]

We want to implement the whistleblowing mechanism, which is a separating equilibrium of the whistleblowing game defined as follows: First, the loser sends message D if and only if he is doped. Second the controller tests if and only if he receives message D. Thus, the whistleblowing mechanism requires that the expected payoff of a doped loser from sending message D, denoted as $E_i\,(D \mid d)$, has to be greater than the expected payoff of sending message I, which is $E_i\,(I \mid d) = 0$. That is, for $i = 1, 2$[17]

$$\mu_i\,(d \mid d)\,\Phi(D, D) + (1 - \mu_i\,(d \mid d))\,(-\Phi(D, C)) > 0. \tag{5}$$

The mechanism also requires that the expected payoff of a clean loser from sending message D, $E_i\,(D \mid c)$, is smaller than the expected payoff of sending message I, $E_i\,(I \mid c) = 0$. That is, for $i = 1, 2$

$$\mu_i\,(d \mid c)\,\Phi(D, D) + (1 - \mu_i\,(d \mid c))\,(-\Phi(D, C)) \leq 0. \tag{6}$$

There are four conditions for the controller. The first two conditions make sure that he does not test given message I, i.e. $E_3(T|I) \leq E_3(NT|I) = 0$, implying for $i = 1, 2$

$$\mu_i\,(d \mid c)\,(1 - k) + (1 - \mu_i\,(d \mid c))\,(-k) \leq 0. \tag{7}$$

[16] Note that the controller infers the behavior of the loser from the loser's message. The whistleblowing mechanism therefore involves a "courageous" leniency clause (Spagnolo, 2002), which involves a reward for the cheating loser who blows the whistle, rather than merely reducing his expected sanctions.

[17] Recall that for now we assume that tests are completely reliable.

Note that (7) takes into account that in equilibrium if agent j wins and the controller receives message I, then the controller's beliefs $\mu_3(d \mid W = j, I)$ equal loser i's beliefs $\mu_i(d \mid c)$, i.e. $\mu_3(d \mid W = j, I) = \mu_i(d \mid c)$. This equality of beliefs is the crucial element of the whistleblowing mechanism: In equilibrium, the controller extracts the private information of the loser, which allows for a more efficient testing scheme.

The other two constraints ensure that the controller tests the winner given message D, i.e. $E_3(T|D) \geq E_3(NT|D) = 0$, implying for $i = 1, 2$

$$\mu_i(d \mid d)(1 - k - \Phi(D, D)) + (1 - \mu_i(d \mid d))(-k + \Phi(D, C)) \geq 0. \tag{8}$$

Note again that (8) takes into account that in equilibrium if agent j wins and the controller receives message D, then the controller's beliefs are

$\mu_3(d \mid W = j, D) = \mu_i(d \mid d)$, i.e. he has the same beliefs as the loser.

We impose the following restrictions for $\Phi(D, D)$ and $\Phi(D, C)$:

$$\Phi(D, D) > 0, \tag{9}$$

$$1 - k > \Phi(D, D), \text{ and } \Phi(D, C) > k. \tag{10}$$

These restrictions imply the following. First, since doping is perfectly effective, we have $\mu_i(d \mid d) = 1$ and $\mu_i(c \mid d) = 0$. Consequently, the incentive constraint (5) simplifies to $\Phi(D, D) > 0$. Consequently, (9) implies that (5) is satisfied in any equilibrium in which both agents dope with strictly positive probability. Second, assumptions (10) imply that the controller has a dominant strategy to test given message D, i.e. (8) holds in any equilibrium. Third, assumptions (10) also imply that if constraint (7) holds, then (6) holds, too. Consequently, if both agents dope with strictly positive probability, the binding constraints are (7), which can be written as

$$\mu_i(d \mid c) \leq k \quad \text{for } i = 1, 2. \tag{11}$$

The whistleblowing stage is depicted in Figure 2. In order to keep the figure simple, we only consider the case where agent 2 is the winner so that the signaling game is between loser 1 (the sender) and the controller (the

receiver). In the first information set, which contains two nodes, player 1 is clean and his beliefs are $\mu_1(. \mid c)$. In the upper node winner 2 is doped and in the lower node he is clean. However, loser 1 cannot distinguish these two nodes. In the second information set, loser 1 is doped and his beliefs are $\mu_1(d \mid d) = 1$ because doping is perfectly effective.

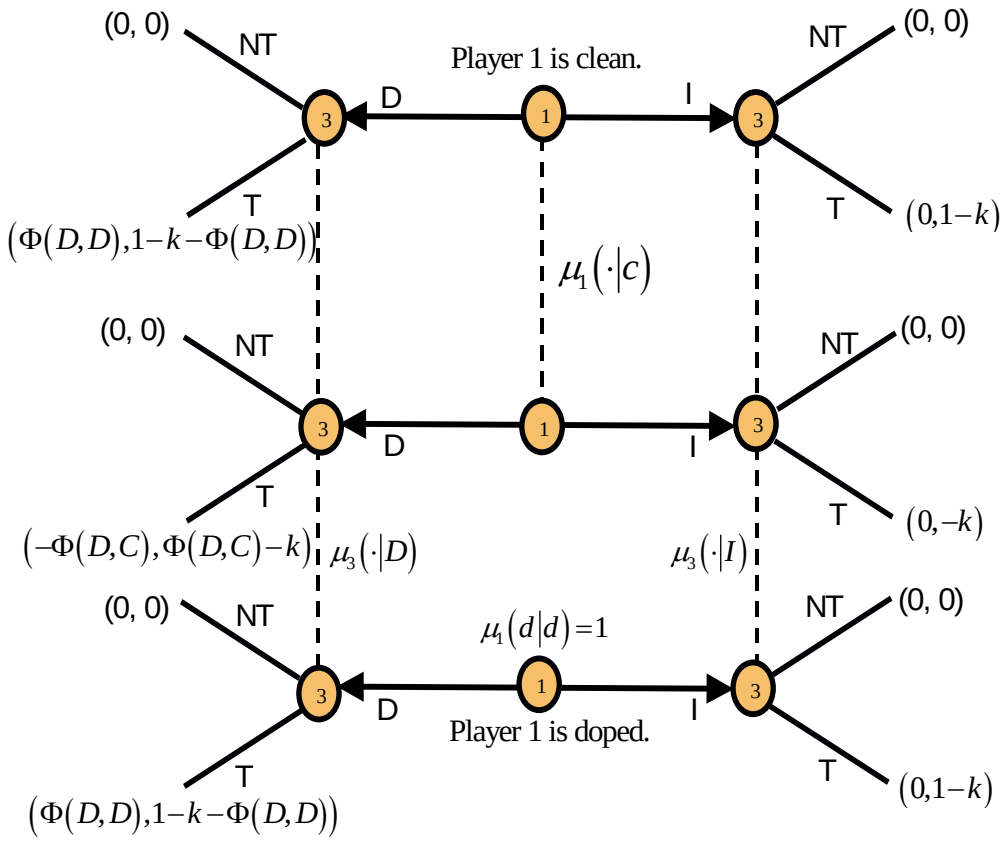

Figure 2. Whistleblowing stage.

3.3 Whistleblowing equilibrium

In the whistleblowing equilibrium, the loser's message is D if he has cheated and I if he is clean, and the controller tests the winner if and only if the message is D. The agents still randomize between doping and playing clean.

Consequently, the equilibrium conditions for the agents require that the expected payoff of playing clean is the same as the expected payoff of doping. That is, for $i = 1, 2$, in equilibrium $E_i(c) = E_i(d)$, which implies

$$\sigma(1-\beta) = (1-\beta) - \sigma\beta P + (1-\sigma)\beta\Phi(D,D) \quad \text{and} \tag{12}$$

$$(1-\sigma)(1-\alpha) = (1-\alpha) - (1-\sigma)\alpha P + \sigma\alpha\Phi(D,D). \tag{13}$$

Equality (12) is agent 1's indifference relation. The left-hand side is the expected payoff of playing clean. With probability $1-\beta$ player 2 is clean as well and agent 1 wins the contest with probability σ. Note that agent 1 does not blow the whistle after losing because he is clean. The right-hand side is the expected payoff for cheating. According to the first term, with probability $1-\beta$ agent 2 is clean. In this case agent 1 wins with certainty (since he is doped) and receives the prize of value one and agent 2 does not blow the whistle because he is clean. According to the second term, with probability β agent 2 is also doped. Consequently, agent 1 wins the contest with probability σ. In this case agent 2 blows the whistle so that agent 1's payoff is $-P$. Finally, according to the third term, agent 2 is doped with probability β and wins the contest with probability $1-\sigma$. In this case agent 1 blows the whistle yielding payoff $\Phi(D,D)$.

Solving for α and β yields the equilibrium doping probabilities of agent 1 and 2, respectively. They satisfy

$$\alpha^{**} = \frac{\sigma}{\sigma\left[1-\Phi(D,D)\right] + (1-\sigma)P} \text{ and } \beta^{**} = \frac{1-\sigma}{(1-\sigma)\left[1-\Phi(D,D)\right] + \sigma P}.$$

Note that $\alpha^{**} \geq \beta^{**}$ because $\sigma \geq \frac{1}{2}$. Then, we can state the following Proposition.

Proposition 2 *The whistleblowing game has a unique perfect Bayesian equilibrium if*

$$P > \frac{\sigma}{1-\sigma}\left(\Phi(D,D) + \frac{1-k}{\sigma k}\right) \tag{14}$$

holds. The behavioral strategy profile is

$$\delta_1 = (\alpha^{**}, 0, 1),\ \delta_2 = (\beta^{**}, 0, 1),\ \textit{and}\ \delta_3 = (0, 1; 0, 1).$$

The equilibrium beliefs are

$$\mu_1(d \mid c) = \frac{\beta^{**}}{\beta^{**} + (1-\sigma)(1-\beta^{**})}, \quad \mu_2(d \mid c) = \frac{\alpha^{**}}{\alpha^{**} + \sigma(1-\alpha^{**})}$$
$$\mu_3(d \mid W = 2, I) = \mu_1(d \mid c), \qquad \mu_3(d \mid W = 1, I) = \mu_2(d \mid c)$$
$$\mu_3(d \mid W = 2, D) = \mu_1(d \mid d) = \mu_3(d \mid W = 1, D) = \mu_2(d \mid d) = 1.$$

In contrast to the inspection game, the equilibrium probabilities of doping now depend negatively on P, so that increasing P reduces the frequency of doping. The reason for this result is that, in contrast to the inspection game, the players choose their doping probabilities to make each other indifferent. If an agent increases his doping probability, he also increases the probability that the other agent is tested when he wins and receives punishment P. The equilibrium probabilities of doping depend positively on the whistleblowing reward $\Phi(D, D)$ because an agent obtains this reward only if he is a doped loser and sends the message D. Because the controller plays a pure strategy (which is contingent on the message received), the costs of testing, k, do not affect the agents' equilibrium strategies or beliefs. As in the inspection game, the favorite player is more likely to be doped than the underdog ($\alpha^{**} \geq \beta^{**}$), with $\frac{\partial \alpha^{**}}{\partial \sigma} > 0$, $\frac{\partial \beta^{**}}{\partial \sigma} < 0$, and $\frac{\partial(\alpha^{**} - \beta^{**})}{\partial \sigma} > 0$.

Condition (14) is derived from the controller's incentive constraint (7), respectively (11), which guarantees that he does not test after receiving message I. This constraint only holds if P is sufficiently large. If P is too small, the doping probabilities α^{**} and β^{**} are so large that the controller wants to control after receiving message I. Obviously, testing after message I leads to a break-down of the whistleblowing equilibrium.

4 Comparing the games

We now compare the doping frequencies, the testing frequencies, and the expected payoffs of the agents and the controller in the whistleblowing game with those in the inspection game.

Proposition 3 *There exist critical values $P_\alpha \geq P_\beta > 0$ defined in the proof such that if $P \geq P_\alpha$, $\alpha^{**} \leq \alpha^*$ and if $P \geq P_\beta$, $\beta^{**} \leq \beta^*$.*

According to Proposition 3 if the punishment is sufficiently large, the cheating probabilities are lower in the whistleblowing mechanism than in the inspection game. Moreover, the more talented player must be punished harsher than the underdog ($P_\alpha \geq P_\beta$) in order to reduce his cheating frequency below the one in the inspection game. The intuition behind Proposition 3 is that increasing P reduces the doping probabilities in the whistleblowing game but not in the inspection game. In the inspection game, increasing P only reduces the equilibrium probabilities of testing.

We now consider under which conditions the whistleblowing equilibrium Pareto dominates the equilibrium of the inspection game.

Proposition 4 *The unique equilibrium of the whistleblowing game Pareto-dominates the unique equilibrium of the inspection game if $P > P_\alpha$.*

Proposition 4 makes a strong case for the use of the whistleblowing mechanism: All participants are ex ante better off if $P > P_\alpha$. The intuition behind this result is that the controller's payoff is strictly positive in the whistleblowing game. In contrast, it is zero in the inspection game. Second, the expected utilities of both agents are larger in the whistleblowing mechanism compared to the inspection game if $P > P_\alpha$.

Finally, we compare the testing frequencies. In the inspection game the probabilities that player i is tested is $\Pr(W = i)t_i^*$ for $i = 1, 2$, which by Lemma 1 is equivalent to $t_1^*\sigma$ and $t_2^*(1-\sigma)$, respectively. So the testing frequency is

$$F_{SC} = t_1^*\sigma + t_2^*(1-\sigma) = \frac{\sigma\beta^* + (1-\sigma)\alpha^*}{k(1+P)}.$$

Note that F_{SC} strictly increases in k, since both t_1^* and t_2^* increase in k. Hence, F_{SC} is smallest as k approaches 0.

In the whistleblowing game the controller tests if and only if both players are doped, i.e. the testing frequency is

$$F_{WB} = \alpha^{**}\beta^{**}.$$

Note that F_{WB} strictly increases in $\Phi(D, D)$, so that F_{WB} is smallest as $\Phi(D, D)$ approaches 0. In the Appendix we show that for any $P > 1$, there is a $\Phi(D, D)$ such that $F_{WB} < F_{SC}$ whenever the whistleblowing equilibrium exists. This result further strengthens the case for the use of the whistleblowing mechanism since lower testing frequencies reduce the cost of implementing controls. It is worth mentioning that $P > 1$ is a sufficient condition, which is derived under the assumption that the control costs approach zero ($k \to 0$), which makes the standard inspection game most effective. For higher values of k, P can be reduced below 1 and we still have $F_{WB} < F_{SC}$.

5 Extensions

In this section we investigate the effects of less than perfectly effective doping and the effects of unreliable tests.

5.1 Less effective doping

We now allow for the possibility that doping is less than perfectly effective. For simplicity, we assume that both agents are equally talented, i.e. we assume $\sigma = \frac{1}{2}$. We denote by $s \in (\frac{1}{2}, 1]$ the probability that an agent wins the contest if he has doped and the other one has not. When $s < 1$ a doped loser faces some uncertainty about the behavior of the winner. In contrast, when $s = 1$ as in the previous section, then a doped loser knows for sure that the winner has doped. We first consider the inspection game and then the whistleblowing mechanism.

5.1.1 The inspection game

Since both agents are equally talented in this section, in equilibrium the conditional probabilities $\Pr(d \mid W = 1)$ and $\Pr(d \mid W = 2)$ and the doping probabilities α and β must be equal. Consequently, the indifference relations (1) and (2) for the controller satisfy

$$\Pr(d \mid W = 1) = \Pr(d \mid W = 2) = k,$$

where

$$\Pr(d \mid W = i) = 2\alpha(1-\alpha)s + \alpha^2.$$

Solving for α yields the equilibrium doping probabilities

$$\alpha' = \beta' = \frac{s - \sqrt{k - 2ks + s^2}}{2s - 1}.$$

The doping probabilities are decreasing in s. For $s = 1$ we have $\alpha' = 1 - \sqrt{1-k}$, and for $s \to \frac{1}{2}$, $\alpha' \to k$. Note also that α' is increasing in k. Of course, the testing probabilities are identical too, i.e. $t_1 = t_2 = t$. For agents to be indifferent between d and c, an equality analogous to (3) and (4) must be satisfied. Therefore,

$$\frac{1}{2}(1-\alpha') + (1-s)\alpha' = (1 - t\,(1+P))[s(1-\alpha') + \frac{1}{2}\alpha']$$

holds. Solving for t yields

$$t' = \frac{1}{1+P}\frac{\alpha'\,(2s-1)}{k}.$$

Note that at $s = \frac{1}{2}$, $t' = 0$. In summary, increasing the effectiveness of doping (i.e. increasing s) yields lower doping probabilities and higher testing probabilities in the inspection game with less than perfectly effective doping.

5.1.2 The whistleblowing game

Allowing for less than perfectly effective doping changes the whistleblowing models as follows. First, it affects the doped loser's beliefs $\mu_i(d \mid d)$ and $\mu_i(d \mid c)$ and hence the expected payoff of blowing the whistle. In particular, a doped loser is uncertain about the behavior of the winner because an agent can win against a doped opponent without the use of performance enhancing drugs. Second, since in the whistleblowing equilibrium the controller's beliefs are equal to the loser's beliefs, the controller's beliefs are affected as well. Third, the equilibrium doping probabilities change because the expected payoffs of doping and playing clean are modified as shown below.

The agents' expected payoffs of playing clean respectively doped are equal because both agents are equally talented. The equilibrium strategies satisfy

$$\begin{aligned} E_i(c) &= (1-\alpha)/2 + (1-s)\alpha \text{ and} \\ E_i(d) &= \frac{\alpha}{2}\left[(-P) + \Phi(D,D)\right] + (1-\alpha)\left[s - (1-s)\Phi(D,C)\right]. \end{aligned}$$

Then, $E_i(c) = E_i(d)$ yields the equilibrium doping probabilities

$$\alpha'' = \beta'' = \frac{2s - 1 - 2(1-s)\Phi(D,C)}{1 + P - \Phi(D,D) - 2(1-s)\Phi(D,C)}.$$

In contrast to the inspection game with less than perfectly effective doping, the doping probabilities are increasing in s. As before, the loser's incentive constraints (5) and (6) and the controller's incentive constraints (7) and (8) must hold.[18]

Proposition 5 *If agents are equally talented (i.e. $\sigma = \frac{1}{2}$) and doping is less than perfectly effective (i.e. $s \in (\frac{1}{2}, 1)$), then there exist values for P, $\Phi(D,C)$ and $\Phi(D,D)$ with $P > -1, 0 < \Phi(D,D) < 1-k$ and $\Phi(D,C) < k$ such that the whistleblowing game has a perfect Bayesian equilibrium with the behavioral strategy profile*

$$\delta_i = (\alpha'', 0, 1) \text{ for } i = 1, 2, \text{ and } \delta_3 = (0, 1; 0, 1).$$

The beliefs for $i = 1, 2$, $j \neq i$ are

$$\begin{aligned} \mu_i(d \mid d) &= \frac{\alpha''}{\alpha'' + 2(1-\alpha'')(1-s)}, \ \mu_i(d \mid c) = \frac{2s\alpha''}{2s\alpha'' + (1-\alpha'')}, \\ \mu_3(W = i \mid D) &= \mu_j(d \mid d) \text{ and } \mu_3(W = i \mid I) = \mu_j(d \mid c). \end{aligned}$$

5.2 Unreliable tests

We now consider unreliable tests. We assume that the controller gets the premium of value one if and only if he tests the winner and the test indicates

[18] Recall that due to the symmetry of agents, strategies, beliefs and the constraints are the same for both.

that the winner is doped. Likewise, agents get their payments $P, \Phi(D, D)$ and $\Phi(D, C)$ contingent on the result of the test. The power of the test (i.e. the probability that the test indicates that an agent is doped if he has played d) is denoted as θ_{DD}, and the size of the test (i.e. the probability that the test indicates that the agent is doped if he is clean) is $1 - \theta_{CC}$. We assume that

$$1 \geq \theta_{DD}, \theta_{CC} \geq \frac{1}{2}.$$

These restrictions are without much loss of generality because if $\frac{1}{2} \geq \theta_{CC}, \theta_{DD} \geq 0$ we could simply invert the interpretation of the test. To ensure that the expected control costs are smaller than the potential benefit of testing a doped winner we impose

$$k < \theta_{DD}.$$

We first consider the inspection game and then the whistleblowing mechanism.

5.2.1 The inspection game

When tests are less than perfectly reliable, the equilibrium conditions (1), (2), (3), and (4) must be modified as follows. For the controller they are

$$\Pr(d \mid W = 1)\theta_{DD} + [1 - \Pr(d \mid W = 1)]\,(1 - \theta_{CC}) - k = 0 \quad (15)$$

$$\Pr(d \mid W = 2)\theta_{DD} + [1 - \Pr(d \mid W = 2)]\,(1 - \theta_{CC}) - k = 0. \quad (16)$$

Then, the equilibrium doping probabilities are

$$\widetilde{\alpha} = \frac{1 - \widetilde{\Psi} - \widetilde{k}(1 - 2\sigma)}{2\sigma}, \quad \widetilde{\beta} = \frac{1 - \widetilde{\Psi} + \widetilde{k}(1 - 2\sigma)}{2(1 - \sigma)}$$

where $\widetilde{k} = \frac{k + \theta_{CC} - 1}{\theta_{DD} + \theta_{CC} - 1}$ and $\widetilde{\Psi} = \sqrt{(1 - \widetilde{k})(1 - \widetilde{k}(1 - 2\sigma)^2)}$. From Section 2 we know that $\frac{\partial \alpha}{\partial k} > 0$ and $\frac{\partial \beta}{\partial k} > 0$. Hence, if $\widetilde{k} > k$, then $\alpha > \alpha^*$ and $\widetilde{\beta} > \beta^*$. Making tests more reliable by increasing θ_{DD} decreases the doping probabilities. In contrast, making tests more reliable by increasing θ_{CC} increases the doping probabilities.

The equilibrium conditions for the agents are

$$\sigma(1-\widetilde{\beta})\,(1-t_1P_C) = (\sigma\widetilde{\beta}+(1-\widetilde{\beta}))(1-t_1P_D) \quad (17)$$

$$(1-\sigma)(1-\widetilde{\alpha})\,(1-t_2P_C) = ((1-\sigma)\widetilde{\alpha}+(1-\widetilde{\alpha}))(1-t_2P_D) \quad (18)$$

where we have defined $P_C = (1-\theta_{CC})\,(1+P)$ and $P_D = \theta_{DD}\,(1+P)$. The testing probabilities satisfy

$$t_1 = \left(\frac{1}{1+P}\right)\left(\frac{\beta'}{\theta_{DD}k+(1-\theta_{CC})\left(\beta'-k\right)}\right) \text{ and}$$

$$t_2 = \left(\frac{1}{1+P}\right)\left(\frac{\alpha'}{\theta_{DD}k+(1-\theta_{CC})\left(\alpha'-k\right)}\right).$$

5.2.2 The whistleblowing game

Unreliable tests change the agents' equilibrium strategies through two channels. First, the expected punishment for a doped winner who is tested is smaller since

$$\widehat{P} = \theta_{DD}P - (1-\theta_{DD}) < P.$$

With probability θ_{DD} the test is positive and he receives punishment P. With probability $1-\theta_{DD}$ the test is negative and he receives the prize of value one.

Second, the modified expected transfers for whistleblowing are

$$\widehat{\Phi}\,(D,D) = \theta_{DD}\Phi(D,D) - (1-\theta_{DD})\,\Phi(D,C)$$

$$\widehat{\Phi}(D,C) = \theta_{CC}\Phi(D,C) - (1-\theta_{CC})\,\Phi(D,D)$$

where $\widehat{\Phi}\,(D,D)$ is the expected reward for whistleblowing if the winner is doped, and $\widehat{\Phi}(D,C)$ is the expected punishment for whistleblowing if the winner is clean. For example, with probability θ_{DD} the test is positive and the whistleblower receives the reward $\Phi(D,D)$. With probability $1-\theta_{DD}$ the test is negative and he receives $\Phi(D,C)$.

The following comparisons with the whistleblowing game under perfectly reliable tests are insightful. First, since $\widehat{\Phi}\,(D,D) < \Phi(D,D)$, a doped loser has a smaller incentive to send message D, the expected reward for doing

so being smaller. Second, because $\widehat{\Phi}(D,C) < \Phi(D,C)$, a clean loser has a greater incentive to send message D, since the expected punishment is smaller. Third, the controller's expected payoff of testing given message D is smaller with unreliable tests. In contrast, his incentive to test given message I is larger since he has a chance that a clean agent tests positive.

Given these modifications, the agents' equilibrium strategies satisfy

$$\widehat{\alpha} = \frac{\sigma}{\sigma\left[1-\widehat{\Phi}\left(D,D\right)\right]+(1-\sigma)\,\widehat{P}} \text{ and } \widehat{\beta} = \frac{1-\sigma}{(1-\sigma)\left[1-\widehat{\Phi}\left(D,D\right)\right]+\sigma\widehat{P}}.$$

It is interesting to note that the effects of increasing θ_{DD} on $\widehat{\alpha}$ and $\widehat{\beta}$ are ambiguous because $\widehat{\Phi}\left(D,D\right)$ enters negatively and $\widehat{P}$ positively in the equations for $\widehat{\alpha}$ and $\widehat{\beta}$. As in the whistleblowing game with perfectly reliable tests, the equilibrium doping probabilities $\widehat{\alpha}$ and $\widehat{\beta}$ are strictly decreasing in P.

Our findings are summarized in the following proposition.

Proposition 6 *If tests are less than perfectly reliable, then for* $\widehat{\Phi}(D,D) > 0$ *and for*

$$\widehat{P} > \frac{\sigma}{1-\sigma}\left(\widehat{\Phi}(D,D)+\frac{1}{\sigma}\frac{\theta_{DD}-k}{k-(1-\theta_{CC})}\right), \tag{19}$$

the whistleblowing game has a perfect Bayesian equilibrium with the behavioral strategy profile

$$\delta_1 = (\widehat{\alpha},0,1),\ \delta_2 = (\widehat{\beta},0,1),\ \textit{and}\ \delta_3 = (0,1;0,1).$$

The equilibrium beliefs are

$$\mu_1(d \mid c) = \frac{\widehat{\beta}}{\widehat{\beta}+(1-\sigma)(1-\widehat{\beta})}, \quad \hat{\mu}_2(d \mid c) = \frac{\widehat{\alpha}}{\widehat{\alpha}+\sigma(1-\widehat{\alpha})},$$
$$\mu_3(d \mid W=2,I) = \hat{\mu}_1(d \mid c), \quad \mu_3(d \mid W=1,I) = \mu_2(d \mid c),$$
$$\mu_3(d \mid W=2,D) = \mu_1(d \mid d) = \mu_3(d \mid W=1,D) = \mu_2(d \mid d) = 1.$$

Condition (19) is derived from the controller's incentive constraint (7) using $\widehat{\alpha}$ and $\widehat{\beta}$, which guarantees that he does not test after receiving message

I. This constraint only holds if P and hence $\widehat{P}$ is sufficiently large. If P is too small, the doping probabilities $\widehat{\alpha}$ and $\widehat{\beta}$ are so large that the controller wants to control after receiving message I.

6 Conclusions

In this paper, we have analyzed the role of whistleblowing in an inspection game with two agents and a controller. We have shown that whistleblowing improves the efficiency of controls by allowing the controller to extract private information from the agents. In fact, our results make a strong case for the use of the whistleblowing mechanism. First, if punishments for detected cheaters are sufficiently large, the cheating probabilities are smaller than in an inspection game. In contrast to the inspection game, cheating is eliminated as punishments get arbitrarily large. Second, whistleblowing allows for a strict Pareto-improvement relative to the inspection game. Third, the frequency of tests are smaller under the whistleblowing mechanism than in the inspection game. Thus, whistleblowing reduces control costs since testing is costly.

We consider the following extensions of the model worthwhile. Allowing for $n > 2$ agents might be useful in the case where doping is less than perfectly effective. If doping is not perfectly effective, the doped loser faces some uncertainty as to the strategy played by the winner. If there are two or more losers, the controller might want to test only if two (or more) losers send the message D. Thus, increasing the number of agents might partially outweigh the effects of less effective doping. Another way to extend the model consists of using fines, i.e. transfer payments, rather than deadweight loss penalties, and to analyze cheating in tournaments that involve more than one round.

We think our inspection game captures many relevant features of cheating and the fight against cheating, though we have not explicitly modelled the dynamic issues inherent in many cheating situations. For example, in sports new performance enhancing drugs are being developed that allow

cheating athletes to be a step ahead of the controlling authorities. As a consequence, cheating will never be fully eliminated.

7 Appendix

Proof of Lemma 1. Equations (1) and (2) imply $\Pr(d \mid W = 1) = \Pr(d \mid W = 2)$. Then, $\Pr(d \mid W = i) = \frac{\Pr(W=i \cap d)}{\Pr(W=i)}$, $i = 1, 2$, implies that

$$\frac{\Pr(W = 2)}{\Pr(W = 1)} = \frac{\Pr(W = 2 \cap d)}{\Pr(W = 1 \cap d)}. \tag{20}$$

Using the fact that $\Pr(W = i) = \Pr(W = i \cap d) + \Pr(W = i \cap c)$ to replace $\Pr(W = 1)$ and $\Pr(W = 2)$ in (20), we get

$$\frac{\Pr(W = 2 \cap d) + \Pr(W = 2 \cap c)}{\Pr(W = 1 \cap d) + \Pr(W = 1 \cap c)} = \frac{\Pr(W = 2 \cap d)}{\Pr(W = 1 \cap d)}$$

or be re-arranging

$$\Pr(W = 2 \cap d) \Pr(W = 1 \cap c) = \Pr(W = 1 \cap d) \Pr(W = 2 \cap c).$$

Using the definitions for these probabilities and re-arranging once more yields

$$\frac{1 - \sigma}{\sigma} = \left(\frac{\beta}{1 - \beta}\right)\left(\frac{1 - \alpha}{\alpha}\right), \tag{21}$$

which implies that in any equilibrium $\alpha \geq \beta$ because $\sigma \geq \frac{1}{2}$. This is the proof of part (i) of Lemma 1.

To see that the underdog is more often tested, define the relative attractiveness of doping for agent i as $\frac{E_i[d]}{E_i[c]}$, which from (3) and (4) is equal to one in any equilibrium. Therefore,

$$\begin{aligned} \frac{E_1[d]}{E_1[c]} &= [1 - t_1 (1 + P)] \left(\frac{\beta}{1 - \beta} + \frac{1}{\sigma}\right) \\ &= [1 - t_2 (1 + P)] \left(\frac{\alpha}{1 - \alpha} + \frac{1}{1 - \sigma}\right) = \frac{E_2[d]}{E_2[c]}. \end{aligned}$$

But since $\left(\frac{\beta}{1-\beta} + \frac{1}{\sigma}\right) < \left(\frac{\alpha}{1-\alpha} + \frac{1}{1-\sigma}\right)$, we have $t_1 < t_2$, which proves part (ii).

Finally, using equation (21) in equation (20) we get after simplifying $\frac{\Pr(W=2)}{\Pr(W=1)} = \frac{1-\sigma}{\sigma}$. Replacing $\Pr(W = 2)$ by $1 - \Pr(W = 1)$ proves part (iii) of Lemma 1.■

Proof of Proposition 1. >From (1), (2), (3), and (4), the equilibrium conditions are

$$\frac{\alpha(1-\beta)+\sigma\alpha\beta}{\sigma(1-\alpha)(1-\beta)+\alpha(1-\beta)+\sigma\alpha\beta} = k \tag{22}$$

$$\frac{(1-\alpha)\beta+(1-\sigma)\alpha\beta}{(1-\sigma)(1-\alpha)(1-\beta)+\beta(1-\alpha)+(1-\sigma)\alpha\beta} = k \tag{23}$$

$$(1-t_1(1+P))(\sigma\beta+(1-\beta)) = \sigma(1-\beta) \tag{24}$$

$$(1-t_2(1+P))((1-\sigma)\alpha+(1-\alpha)) = (1-\sigma)(1-\alpha). \tag{25}$$

This system of four equations has the unique solution $(t_1^*, t_2^*, \alpha^*, \beta^*)$.■

Proof of Proposition 2. The proof involves two steps. We first show that the strategy profile and beliefs described by Proposition 2 are a perfect Bayesian Nash equilibrium (PBE). After this we prove that the equilibrium is unique.

Existence: Assume first that the cheating probabilities as defined in the text are $\alpha = \alpha^{**}$ and $\beta = \beta^{**}$. Then, if (9) and (10) hold, and if the controller has the same beliefs as the loser, the only binding constraint is (11) where

$$\mu_3(d \mid W=1, I) = \mu_2(d \mid c) = \frac{\alpha^{**}}{\alpha^{**}+\sigma(1-\alpha^{**})} \quad \text{and}$$

$$\mu_3(d \mid W=2, I) = \mu_1(d \mid c) = \frac{\beta^{**}}{\beta^{**}+(1-\sigma)(1-\beta^{**})}.$$

Consequently, we need

$$k \geq \max\left\{\frac{\alpha^{**}}{\alpha^{**}+\sigma(1-\alpha^{**})}, \frac{\beta^{**}}{\beta^{**}+(1-\sigma)(1-\beta^{**})}\right\}. \tag{26}$$

Inserting the equilibrium values α^{**} and β^{**} into condition (26) we get

$$k \geq \max\left\{\frac{1}{1+P-\sigma(P+\Phi(D,D))}, \frac{1}{1+\sigma(P+\Phi(D,D))-\Phi(D,D)}\right\}.$$

Note that $1+P-\sigma(P+\Phi(D,D)) \leq 1+\sigma(P+\Phi(D,D))-\Phi(D,D)$ because $\sigma \geq \frac{1}{2}$. Therefore,

$$k \geq \frac{1}{1+P-\sigma(P+\Phi(D,D))}, \tag{27}$$

which can be re-arranged to get condition (14) of Proposition 2.

Assume now that all incentive constraints hold. Then, α and β satisfy the expression given in the Proposition. Therefore, the strategy profile and the beliefs in Proposition 2 constitute a PBE.

Uniqueness: We now show that the equilibrium is unique. We first show that there is no pooling equilibrium, i.e. that there is no equilibrium in which one or both agents send the same message regardless of whether they are doped or not. Note first that testing given message D is a dominant strategy for the controller because $\Phi(D, C) > k$ and $0 < \Phi(D, D) < 1 - k$. Consequently, for a doped loser sending message I is not sequentially rational: The doped loser is sure that the winner is doped and will be tested if he sends message D, so that his expected payoff of sending D is $\Phi(D, D) > 0$, while the payoff of sending I is 0. Hence, sending message I is not sequentially rational for a doped loser.

Therefore, the only candidates for pooling equilibria consist of strategy profiles where the pooling loser always says D. Because testing is a dominant strategy given message D, the winner (who may or may not pool himself) will be tested with probability one if he wins. But if an agent is certain to be tested if he wins, he will never dope: If tested with probability one, his expected payoff of doping is negative, while the expected payoff of playing clean is positive. But given that his opponent never dopes, the expected payoff of always saying D is negative. Thus, it is not a best response for the pooling agent to always say D. Hence, there are no pooling equilibria.

We next show that there are no separating and no hybrid equilibria. The plan is as follows: In step 1 we show that there are no equilibria in which a clean loser sends message D with positive probability. We then show in step 2 that there are no equilibria in which the controller tests both agents with a positive probability when receiving message I. In step 3 we show that there is no PBE, in which the controller tests at least one agent with strictly positive probability given message I.

Step 1: We first show that there are no equilibria in which $m_i(c) > 0$ for

at least one agent i. To see this, note that $m_i(c) > 0$ implies that

$$\mu_i(d \mid c)\Phi(D, D) + \mu_i(c \mid c)\,(-\Phi(D, C)) \geq 0 \tag{28}$$

$i = 1, 2$. But if (28) holds, then

$$\mu_i(d \mid c)(1 - k) + \mu_i(c \mid c)\,(-k) > 0$$

because in equilibrium $\mu_3(d \mid W = j, I) = \mu_i(d \mid c)$.[19] Hence, if the expected payoff of clean agent i of sending message D is non-negative, then the controller's expected payoff of testing given message I is strictly positive. But this implies that winner j, for $j \neq i$, is tested with certainty. Consequently, he will never dope and therefore (28) cannot hold. Thus in any equilibrium $m_i(c) = 0$, $i = 1, 2$.

Step 2: We now show that there is no separating PBE, in which the controller's testing probabilities are $0 < t_i(I) \leq 1$ for both i.

First, consider the case with $t_i(I) = 1$ for at least one i. Given that sequential rationality requires the doped loser to say D and that testing is a dominant strategy given message D (i.e. $t_i(D) = 1$, $i = 1, 2$), $t_i(I) = 1$ implies that agent i will always be tested, which cannot be an equilibrium.

Next consider the case with $0 < t_i(I) < 1$ for $i = 1, 2$. This cannot be an equilibrium. For such a strategy to be part of an equilibrium strategy profile, the controller's expected payoff of testing the winner given that the loser has sent message I must be equal to the payoff of not testing, which is zero. Thus, for the controller we need

$$\frac{\alpha}{\sigma(1-\alpha)+\alpha} - k = 0 \quad \text{and} \quad \frac{\beta}{(1-\sigma)(1-\beta)+\beta} - k = 0$$

yielding $\widehat{\alpha} = \frac{k\sigma}{1-k(1-\sigma)}$ and $\widehat{\beta} = \frac{k(1-\sigma)}{1-k\sigma}$.

The payoff to the loser if the winner is tested positive is $\Phi(D, D)$. Because agents 1 and 2 must be indifferent between the two (pure) behavioral

[19] Note that if the message I is sent under the strategy profile considered, the loser is clean. Therefore, the controller shares the belief of the clean loser if the message is I.

strategies, we need

$$\begin{aligned}(1-\beta)(1-t_1(1+P)) - \sigma\beta P + (1-\sigma)\beta\Phi(D,D) &= \sigma(1-\beta) \text{ and} \\ (1-\alpha)(1-t_2(1+P)) - (1-\sigma)\alpha P + \sigma\alpha\Phi(D,D) &= (1-\sigma)(1-\alpha).\end{aligned}$$

Using $\widehat{\alpha}$ and $\widehat{\beta}$ and re-arranging yields

$$t_1 = \frac{1-\sigma}{(1-k)(1+P)}(1-k\left[1-\Phi(D,D)+\sigma(\Phi(D,D)+P)\right]) \quad (29)$$

$$t_2 = \frac{\sigma}{(1-k)(1+P)}(-1+k\left[1+P-\sigma(\Phi(D,D)+P)\right]). \quad (30)$$

By assumption t_1 and t_2 are strictly positive probabilities. Because the fraction in equation (29) is positive, the term in brackets in equation (29) also has to be positive. Thus we must have $k\left[1-\Phi(D,D)+\sigma(\Phi(D,D)+P)\right] < 1$. On the other hand, because the fraction in (30) is positive, the term in brackets needs to be greater than zero. Thus, $k\left[1+P-\sigma(\Phi(D,D)+P)\right] > 1$. But this can never simultaneously be the case, because

$$k\left[1+P-\sigma(\Phi(D,D)+P)\right] > k\left[1-\Phi(D,D)+\sigma(\Phi(D,D)+P)\right]$$

implies $\sigma < \frac{1}{2}$, which is a contradiction. Hence, $0 < t_i(I) < 1$ for $i = 1,2$ cannot be part of an equilibrium.

Step 3. From step 2 we know that both $t_1(I) > 0$ and $t_2(I) > 0$ is not compatible with equilibrium. By sequential rationality of the doped loser and the dominant strategy to test given message D of the controller, we know also that $t_i(I) = 1$ for $i = 1,2$ is not possible in an equilibrium. Therefore, we are left to show that there is no equilibrium with $t_i(I) = 0$ and $0 < t_j(I) < 1$ for $j \neq i$. Assume to the contrary $t_2(I) = 0$ and $0 < t_1(I) < 1$. Then, agent 1 must keep the controller indifferent. That is, α is such that

$$\frac{\alpha}{\alpha+\sigma(1-\alpha)} - k = 0. \quad (31)$$

On the other hand, agent 1 must also keep agent 2 indifferent. Hence, α also solves

$$(1-\alpha) + \sigma\alpha\Phi(D,D) - (1-\sigma)\alpha P = (1-\alpha)(1-\sigma). \quad (32)$$

But α solves (31) and (32) if and only if $\Phi(D,D) = \frac{1-\sigma}{\sigma}\left(P - \frac{1-k}{k(1-\sigma)}\right)$, which contradicts (14). Hence, this is not an equilibrium. Analogous reasoning rejects the case with $t_1(I) = 0$ and $0 < t_2(I) < 1$.

This concludes the proof that the equilibrium is unique.■

Proof of Proposition 3. The proof is straightforward and only involves the comparison of α^{**} with α^* and β^{**} with β^*. The critical values are

$$P_\alpha = \frac{\sigma}{1-\sigma}\left(\frac{1-\alpha^*}{\alpha^*} + \Phi(D,D)\right) \text{ and} \tag{33}$$

$$P_\beta = \frac{1-\sigma}{\sigma}\left(\frac{1-\beta^*}{\beta^*} + \Phi(D,D)\right). \tag{34}$$

Next, use (21) to replace $\frac{1-\alpha^*}{\alpha^*}$ in (33) to get

$$P_\alpha = \left(\frac{1-\beta^*}{\beta^*}\right) + \frac{\sigma}{1-\sigma}\Phi(D,D) \geq \frac{1-\sigma}{\sigma}\left(\frac{1-\beta^*}{\beta^*}\right) + \frac{1-\sigma}{\sigma}\Phi(D,D) = P_\beta,$$

because $\sigma \geq \frac{1}{2}$. Finally, note that α^{**} and β^{**} are strictly decreasing in P. In contrast, α^* and β^* are independent of P. Consequently, if $P \geq P_\alpha$, $\alpha^{**} \leq \alpha^*$ and if $P \geq P_\beta$, $\beta^{**} \leq \beta^*$.■

Proof of Proposition 4. In the unique equilibrium of the inspection game, the expected payoff of the controller is zero because in equilibrium he is indifferent between testing and not testing, where not testing yields zero payoff. In contrast, in the whistleblowing game the controller's expected payoff is strictly positive, because with positive probability he receives the accurate message D and tests the winner, which yields a positive payoff. Consequently, the controller is strictly better off in the whistleblowing game compared to the inspection game.

In both games, the agents are indifferent between doping and not doping. Consequently, in both games the expected payoffs must be equal to the expected payoff of playing clean, which are $\sigma(1-\beta^*)$ and $\sigma(1-\beta^{**})$ for agent 1 and $(1-\sigma)(1-\alpha^*)$ and $(1-\sigma)(1-\alpha^{**})$ for agent 2, respectively. Consequently, in the whistleblowing game if $\beta^{**} < \beta^*$, agent 1 is strictly

better off and if $\alpha^{**} < \alpha^*$, agent 2 is strictly better off. Proposition 3 implies that if $P > P_\alpha$, $\beta^{**} < \beta^*$ and $\alpha^{**} < \alpha^*$. Thus, if $P > P_\alpha$, the expected utilities of both agents and the controller are strictly larger in the whistleblowing game than in the inspection game. ■

Proof that $F_{SC} > F_{WB}$ if $P > 1$. We show that the limit of F_{SC} as k goes to 0 is greater than the limit of F_{WB} as $\Phi(D, D)$ approaches 0 (where the whistleblowing equilibrium does not necessarily exist). Because F_{SC} strictly increases in k and F_{WB} is independent of it, $F_{SC} > F_{WB}$ for any greater k then follows immediately. Therefore, $F_{SC} > F_{WB}$ will also be feasible through appropriate (i.e. sufficiently small) choice of $\Phi(D, D)$ for any k for which the whistleblowing equilibrium exists. The limit

$$\lim_{\Phi(D,D)\to 0} F_{wb} = \frac{\sigma(1-\sigma)}{(1-\sigma+\sigma P)(\sigma+P-\sigma P)}$$

is straightforward to find. By L'Hôpital's rule, the limit of F_{SC} is

$$\lim_{k\to 0} F_{sc} = 2\frac{\sigma(1-\sigma)}{1+P}.$$

Now, for $P > 1$ we have

$$2\frac{\sigma(1-\sigma)}{1+P} > \frac{\sigma(1-\sigma)}{(1-\sigma+\sigma P)(\sigma+P-\sigma P)}, \tag{35}$$

since inequality (35) can be simplified to yield

$$(1-P)^2\sigma(1-\sigma) > \frac{1}{2}(1-P), \tag{36}$$

which is always true because for $P > 1$, the right-hand side of (36) is negative, while the left-hand side is positive.■

Proof of Proposition 5. The incentive constraints of the agents are

$$\mu_i(d \mid d)\Phi(D,D) - (1-\mu_i(d \mid d))\,\Phi(D,C) \geq 0 \quad \text{and} \tag{37}$$

$$\mu_i(d \mid c)\Phi(D,D) - (1-\mu_i(d \mid c))\,\Phi(D,C) \geq 0. \tag{38}$$

Solving (37) and (38) for $\frac{\Phi(D,D)}{\Phi(D,C)}$ yields

$$\frac{1-\mu_i(d \mid d)}{\mu_i(d \mid d)} \leq \frac{\Phi(D,D)}{\Phi(D,C)} \leq \frac{1-\mu_i(d \mid c)}{\mu_i(d \mid c)}. \tag{39}$$

The incentive constraints of the controller are

$$\mu_i(d \mid d)(1-k-\Phi(D,D)) - (1-\mu_i(d \mid d))\,(k-\Phi(D,C)) \geq 0 \quad \text{and} \tag{40}$$

$$\mu_i(d \mid c)(1-k) - (1-\mu_i(d \mid c))\,k \leq 0. \tag{41}$$

implying

$$\frac{1-\mu_i(d \mid d)}{\mu_i(d \mid d)} \leq \frac{1-k-\Phi(D,D)}{k-\Phi(D,C)} \leq \frac{1-\mu_i(d \mid c)}{\mu_i(d \mid c)} \quad \text{and} \tag{42}$$

$$\frac{1-\mu_i(d \mid d)}{\mu_i(d \mid d)} \leq \frac{1-k}{k} \leq \frac{1-\mu_i(d \mid c)}{\mu_i(d \mid c)}. \tag{43}$$

Now let $\Phi(D,C) = \frac{k}{1-k}\Phi(D,D)$. Then, $\frac{\Phi(D,D)}{\Phi(D,C)} = \frac{1-k-\Phi(D,D)}{k-\Phi(D,C)} = \frac{1-k}{k}$. Consequently, conditions (39), (42), and (43) collapse to

$$\mu_i(d \mid c) \leq k \leq \mu_i(d \mid d). \tag{44}$$

The first inequality guarantees that the controller does not test given message I and the second guarantees that the controller test given message D.

For $s > 1/2$, the beliefs $\mu_i(d \mid c)$ and $\mu_i(d \mid d)$ are continuous, increasing and concave functions in the opponent's doping probability, with $\mu_i(d \mid c) < \mu_i(d \mid d)$ if the doping probability is strictly greater than zero and strictly less than one. Moreover, symmetry implies that $\alpha = \beta$, $\mu_1(d \mid c) = \mu_2(d \mid c)$, and $\mu_1(d \mid d) = \mu_2(d \mid d)$. Then, since $\frac{\partial\beta}{\partial P} = \frac{\partial\alpha}{\partial P} < 0$, we have $\frac{d\mu_1(d|d)}{dP} = \frac{\partial\mu_1(d|d)}{\partial\beta}\frac{\partial\beta}{\partial P} < 0$ and $\frac{d\mu_2(d|c)}{dP} = \frac{\partial\mu_2(d|c)}{\partial\alpha}\frac{\partial\alpha}{\partial P} < 0$.

To see that it is always possible to find a $\Phi(D,D) > 0$ and a $P > -1$ such that (44) holds, we let $\mu_i(d \mid c) = k$ because $\mu_i(d \mid c) < \mu_i(d \mid d)$. This implies from the definition of the beliefs that

$$\alpha'' = \frac{k}{2s\,(1-k)+k}. \tag{45}$$

The equilibrium doping probability is

$$\alpha'' = \frac{s - \frac{1}{2} - (1-s)\Phi(D,C)}{\frac{1}{2}[1 + P - \Phi(D,D)] - (1-s)\Phi(D,C)}.$$

Use $\Phi(D,C) = \frac{k}{1-k}\Phi(D,D)$ to get

$$\alpha'' = \frac{(2s-1)(1-k) - 2(1-s)k\Phi(D,D)}{[1 + P - \Phi(D,D)](1-k) - 2(1-s)k\Phi(D,D)}. \tag{46}$$

Then (45) and (46) imply

$$\frac{k}{2s(1-k)+k} = \frac{(2s-1)(1-k) - 2(1-s)k\Phi(D,D)}{[1 + P - \Phi(D,D)](1-k) - 2(1-s)k\Phi(D,D)}$$

If we let $\Phi(D,D) \to 0$ we get

$$\frac{k}{2s(1-k)+k} = \frac{2s-1}{1+P}.$$

Solving for P yields

$$P = (2s-1)\frac{2s(1-k)+k}{k} - 1$$

Evidently, if $s \to \frac{1}{2}$, then $P \to -1$. Otherwise, $P > -1$.■

Proof of Proposition 6. Assume first that the cheating probabilities are $\alpha = \widehat{\alpha}$ and $\beta = \widehat{\beta}$. The whistleblowing mechanism requires that the expected payoff of a doped loser from sending message D, $E_i(D \mid d)$, has to be greater than the expected payoff of sending message I, $E_i(I \mid d) = 0$. That is, for $i = 1, 2$

$$\mu_i(d \mid d)\,\widehat{\Phi}(D,D) - \mu_i(c \mid d)\,\widehat{\Phi}(D,C) > 0. \tag{47}$$

The mechanism also requires that the expected payoff of a clean loser from sending message D, $E_i(D \mid c)$, is smaller than the expected payoff of sending message I, $E_i(I \mid c)$, which is also zero. That is, for $i = 1, 2$

$$\mu_i(d \mid c)\,\widehat{\Phi}(D,D) - \mu_i(c \mid c)\,\widehat{\Phi}(D,C) \leq 0. \tag{48}$$

There are four conditions for the controller. The first two conditions make sure that he does not test given message I, i.e. $E_3(T|I) \leq E_3(NT|I) = 0$, implying for $i = 1, 2$

$$\mu_3\left(d \mid W = i, I\right) \Phi(T \mid I, d) + \mu_3\left(c \mid W = i, I\right) \Phi(T \mid I, c) \leq 0, \tag{49}$$

where $\Phi(T \mid I, d) = \theta_{DD}(1-k) + (1-\theta_{DD})(-k)$ is the expected payoff if the controllers tests a doped loser given message I and $\Phi(T \mid I, c) = (1-\theta_{CC})(1-k) + \theta_{CC}(-k)$ is the expected payoff if the controller tests a clean loser given message I. Note that $\Phi(T \mid I, d) - \Phi(T \mid I, c) > 0$.

The other two constraints ensure that the controller tests the winner given message D, i.e. $E_3(T|D) \geq E_3(NT|D) = 0$, implying for $i = 1, 2$

$$\mu_3\left(d \mid W = i, D\right) \Phi(T \mid D, d) + \mu_3\left(c \mid W = i, D\right) \Phi(T \mid D, c) > 0, \tag{50}$$

where $\Phi(T \mid D, d) = \Phi(T \mid I, d) - \widehat{\Phi}(D, D)$ is the expected payoff if the controllers tests a doped loser given message D. Likewise $\Phi(T \mid D, c) = \Phi(T \mid I, c) - \widehat{\Phi}(D, C)$ is the expected payoff if the controller tests a clean loser given message D.

We impose the following restrictions for $\widehat{\Phi}(D, D)$ and $\widehat{\Phi}(D, C)$:

$$\widehat{\Phi}(D, D) > 0, \text{ and} \tag{51}$$

$$1 - k > \widehat{\Phi}(D, D), \text{ and } \widehat{\Phi}(D, C) > k. \tag{52}$$

The reminder of the proof is similar as the proof of Proposition 2. The restrictions (51) and (52) require

$$\frac{\widehat{\Phi}(D, D)}{\widehat{\Phi}(D, C)} > \frac{1 - \theta_{DD}}{\theta_{DD}}.$$

They imply the following: First, because doping is perfectly effective, we have $\mu_i\left(d \mid d\right) = 1$ and $\mu_i\left(c \mid d\right) = 0$. Consequently, the incentive constraint (47) simplifies to $\widehat{\Phi}(D, D) > 0$. Thus, given restriction (51), (47) is satisfied. Second, (52) implies that (50) holds, i.e. the controller has a dominant strategy to test given message D. Third, in equilibrium the controller beliefs $\mu_3\left(d \mid W = i, I\right)$ are equal to loser j's beliefs, i.e. $\mu_3\left(d \mid W = i, I\right) =$

$\mu_j(d \mid c)$. This equality of beliefs and (52) imply that if constraint (49) holds, then (48) holds, too.

Consequently, (49) is the only relevant constraint. It can be written for $i = 1, 2$

$$\mu_3(d \mid W = i, I) \leq \widehat{k} = \frac{-(1-\theta_{CC}) + k}{\theta_{CC} + \theta_{DD} - 1}. \tag{53}$$

The controller's beliefs satisfy

$$\begin{aligned} \mu_3(d \mid W = 1, I) &= \mu_2(d \mid c) = \frac{\widehat{\alpha}}{\widehat{\alpha} + \sigma(1-\widehat{\alpha})} \text{ and} \\ \mu_3(d \mid W = 2, I) &= \mu_1(d \mid c) = \frac{\widehat{\beta}}{\widehat{\beta} + (1-\sigma)(1-\widehat{\beta})} \end{aligned}$$

Consequently, we need

$$\widehat{k} > \max\left\{\frac{\widehat{\alpha}}{\widehat{\alpha} + \sigma(1-\widehat{\alpha})}, \frac{\widehat{\beta}}{\widehat{\beta} + (1-\sigma)(1-\widehat{\beta})}\right\}. \tag{54}$$

Inserting the equilibrium values $\widehat{\alpha}$ and $\widehat{\beta}$ into condition (54) we get

$$\widehat{k} > \max\left\{\frac{1}{1 - \sigma\widehat{\Phi}(D,D) + (1-\sigma)\widehat{P}}, \frac{1}{1 - (1-\sigma)\widehat{\Phi}(D,D) + \sigma\widehat{P}}\right\}.$$

Note that $1 - \sigma\widehat{\Phi}(D,D) + (1-\sigma)\widehat{P} \leq 1 - (1-\sigma)\widehat{\Phi}(D,D) + \sigma\widehat{P}$ because $\sigma \geq \frac{1}{2}$. Therefore, we need

$$\widehat{k} > \frac{1}{1 - \sigma\widehat{\Phi}(D,D) + (1-\sigma)\widehat{P}} \tag{55}$$

which can be re-arranged to get the condition in the Proposition (19).

Finally, the doping probabilities $\widehat{\alpha}$ and $\widehat{\beta}$ satisfy $E_1(c) = E_1(d)$. The beliefs of the agents and the controller are derived as before. We conclude that a perfect Bayesian equilibrium exists when tests are not perfectly reliable if (19) holds.■

References

Apesteguia, José, Martin Dufwenberg, and Reinhard Selten (2003). Blowing the Whistle. Discussion Paper No. 9/2003, Bonn Graduate School of Economics, Universität Bonn.

Avenhaus, Rudolf (1997). Entscheidungstheoretische Analyse der Fahrgast-Kontrollen. Der Nahverkehr 9, 27-30.

Avenhaus, Rudolf, Bernhard Von Stengel, and Shmuel Zamir (2002). Inspection Games. In Aumann, Robert J., and Sergiu Hart (Eds.), Handbook of Game Theory, Vol. 3.

Becker, Gary (1968). Crime and Punishment: An Economic approach. Journal of Political Economy 76(2), 169-217.

Benoît, Jean-Pierre and Juan Dubra (2004), Why Do Good Cops Defend Bad Cops?, International Economic Review (forthcoming).

Berentsen, Aleksander (2002). The Economics of Doping. European Journal of Political Economy 18(1), 109-127.

Greenberg, Joseph (1984). Avoiding Tax Avoidance: A (Repeated) Game-Theoretic Approach. Journal of Economic Theory 32(1), 1-13.

Güth, Werner and Rüdiger Pethig (1992). Illegal Pollution and Monitoring of Unknown Quality. In Pethig, Rüdiger (Ed.), Conflicts and Cooperation in Managing Environmental Resources, Chapter 10, 275 - 330. Springer Verlag.

Innes, Robert (1999a). Remediation and Self-Reporting in Optimal Law Enforcement. Journal of Public Economics 72(3), 379-393.

Innes, Robert (1999b). Self-Policing and Optimal Law Enforcement When Violator Remediation Is Valuable. Journal of Political Economy 107(6), 1305-1325.

Kaplow, Louis and Steven Shavell (1994). Optimal Law Enforcement with Self-Reporting of Behavior. Journal of Political Economy 102(3), 583-606.

Malik, Arun S. (1993). Self-Reporting and the Design of Policies for Regulating Stochastic Pollution. Journal of Environmental Economics and Management 24(3), 241-257.

Motta, Massimo and Michele Polo (2003). Leniency Programs and Cartel Prosecution. International Journal of Industrial Organization 21(3), 347-379.

Rey, Patrick (2001). Towards a Theory of Competition. University of Toulouse.

Rosen, Sherwin (1986). Prizes and Incentives in Elimination Tournaments. American Economic Review 76(4), 701-715.

Salem, George R. and Laura M. Franze (2002). The Whistleblower Provisions of the Sarbanes-Oxley Act of 2002.
http://www.niri.org/regulations/SarbanesOxley/
WhistleblowerProvisions2002.pdf

Snyder, James (1989). Election Goals and the Allocation of Campaign Resources, Econometrica 57(3), 637-660.

Spagnolo, Giancarlo (2002). Self-Defeating Antitrust Laws: How Leniency Programs Solve Bertrand's Paradox and Enforce Collusion in Auctions. University of Mannheim.
⟨http://www.cepr.org/meets/wkcn/6/6607/papers/spagnolo.pdf⟩