

# MARKET MAKING OLIGOPOLY\*

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This paper analyzes price competition between market makers who set costly capacity constraints before they intermediate between producers and consumers. The unique equilibrium outcome with pure strategies at the capacity stage is the Cournot outcome. The paper thus provides a rationale for Cournot-type competition between market makers. This contrasts with previous findings in the literature, where due to the absence of capacity constraints that are set *ex ante* the Bertrand result typically obtains.

## I. INTRODUCTION

IN MANY INDUSTRIES, firms act as price setters both on the input and on the output market. For example, commercial banks set both deposit rates on the input market and loan and mortgage rates on the output market. Similarly, retailers like Wal-mart take neither input nor output prices as given, as witnessed by the much publicized complaints of farmers and Wal-mart's less efficient competitors alike. Likewise, producers who use labor as input to produce a given output set both wages and output prices.

As arbitrageurs these firms bring together supply and demand much in the same way a Walrasian auctioneer does. Therefore, they are called market makers.<sup>1</sup> Quite naturally, a monopolistic market maker sets a lower bid price on the input market and a higher ask price on the output market than a Walrasian auctioneer would, which allows the monopoly to net a positive profit. Setting bid and ask prices, competing market

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makers engage in Bertrand competition and net zero profits. This is true even if the number of competitors is but two, as first observed by Stahl [1988] in a model where market makers first bid for the inputs that then constrain the quantity of output that can be sold. Thus, as in price competition on product markets, two is enough for perfect competition. Insofar as increasing the number of market makers is naturally expected to narrow the bid-ask spread steadily, this result is counter-intuitive. It also leads to the conclusion that market making is a natural monopoly industry if there are fixed costs of entry Gehrig [1993, p.100]. The question thus arises under what conditions transition from monopoly to perfect competition is more continuous. This is the question we address.

We analyze the equilibrium behavior of market makers who set costly capacity constraints prior to competing in prices on the input market. In our model, market makers thus set capacity constraints *ex ante*. This contrasts with Stahl [1988], where market makers become capacity constrained only in the *interim* stage. There are two motivations for our approach. First, physical capacity constraints that are set *ex ante* are a fact of reality. Retail shops are constrained by the sale space and their capacity to transport and store the goods they buy and sell. Commercial banks are constrained by the number of branches and their sales force when intermediating between customers who lend and customers who borrow money. Firms who transform labor input into output are constrained by the size of their production plants. Second, the capacity constraints approach allows us to maintain the homogenous goods assumption typically made in the literature, yet permits a non-degenerate form of imperfect competition.

Define the Cournot model of market making as the model in which intermediaries simultaneously quote the quantities they want to trade, facing market clearing input and output prices set by a Walrasian auctioneer. Accordingly, call the equilibrium outcome emerging in this model *Cournot outcome*. Our main results are summarized as follows. When intermediaries first set costly capacities and then compete in prices, the unique equilibrium outcome with pure strategies at the capacity stage is Cournot rather than Bertrand. This holds true for a fairly wide range of alternative settings. The basic intuition for the result is that capacity constraints substantially soften price competition, as first observed by Edgeworth [1925]. If all firms face sufficiently small capacity constraints, none of them can take over the whole market, so that price competition will be less aggressive than in the Bertrand model. Since we assume efficient rationing, the residual demand and supply functions market makers face are the same as under Cournot competition. Consequently, the largest firm's expected revenue will be the same as the revenue it would earn had it set capacity equal to its Cournot best response, given the

capacity of all other firms. If capacity is costly, the largest firm's capacity best response is unique. Thus, all alternative candidate equilibria with pure strategies at the capacity stage unravel.

The paper makes three contributions. First, it shows that whether capacity constraints are set in an *interim* stage as in Stahl [1988] or *ex ante* as in this paper makes a big difference. Second, the fact that most of the findings of Kreps and Scheinkman carry over to an arbitrary number of market makers is interesting news in itself.<sup>2</sup> Because models of market making may behave quite differently from the underlying oligopoly model [see, e.g., Stahl, 1988, Yannelle, 1989, 1996]<sup>3</sup> and because the model of Kreps and Scheinkman has not been perceived by the literature as very robust with respect to seemingly small changes in the underlying assumptions [see, e.g., Davidson and Deneckere, 1986, Reynolds and Wilson, 2000], this is by no means a foregone conclusion. In light of Stahl's [1988, p.191] statement that the Kreps and Scheinkman result may be singular, our findings can be seen as restoring the initial result under the provision that *some* kind of binding capacity is set *ex ante*. Third, the paper provides a rationale for Cournot-type competition between market makers, so that the Cournot model of market making can be seen as a short-cut to a more complicated model, in which intermediaries first set capacities and then compete in prices.

Apart from the extensive literature on capacity constrained product market competition, referenced, e.g., in Footnote 12 below, the paper is closest to Stahl [1988], Gehrig [1993], Fingleton [1997] and Rust and Hall [2003]. In these models, though, there are no *ex ante* capacity constraints.<sup>4</sup> Neeman and Vulkan [2003] and Kugler, Neeman and Vulkan [2006] analyze how a given centralized market drives out trade through direct negotiations, whereas we investigate how an intermediated market operates and under what conditions it approaches the ideal or centralized market they take as given. Ju, Linn and Zhu [2006] study capacity constrained price competition between market makers, but do not consider mixed pricing strategies. Schevchenko [2004] analyzes competition between middlemen in a setting with heterogenous goods and preferences when terms of trade are determined through Nash bargaining.<sup>5</sup> Similarly, in Rubinstein and Wolinsky [1987] all trade occurs at terms that result from bargaining. Moreover, their middlemen's capacities are exogenously given. Lastly, as market making is by its very nature a two-sided activity, the paper relates also to the recent literature on two-sided markets; see, e.g., Caillaud and Jullien [2001], Rochet and Tirole [2002, 2003] and Armstrong [2006].

The remainder of the paper is structured as follows. Section II introduces the basic model, and Section III derives the equilibrium for this model. Section IV contains the extensions, which deal in turn with forward contracts, inelastic demand and simultaneous

ask and bid price setting. Section V concludes. Proofs are in the Appendix.

## II. THE MODEL

Except for the requirement that market makers have to set capacities prior to setting prices on either side of the market, the model is very similar to the one in Stahl [1988, section 3].<sup>6</sup> The assumptions are as follows.

There are  $n$  market makers, indexed as  $i = 1, \dots, n$  and occasionally also called firms. A typical market maker is indexed as  $i, j$  or  $k$ . We take the number of market makers as exogenously given here. In Section III.(v), we show that the equilibrium number of market makers can easily be derived as a function of the fixed cost of entry in a game with an additional entry stage preceding this game. Each market maker maximizes its own profit. In stage 1, market makers simultaneously set physical capacity constraints, which are denoted as  $\bar{q}_i$ . The cost of capacity  $\bar{q}_i$  is denoted as  $C(\bar{q}_i)$ , where  $C(0) = 0$ ,  $C' > 0$  and  $C'' \geq 0$  is assumed.<sup>7</sup> For the purpose of completeness, we will also report results for the model with costless capacity, i.e., for the case with  $C' = 0$ . A capacity constraint is such that trading quantity up to the constraint involves no direct costs, while beyond capacity trade is prohibitively costly.<sup>8</sup> Throughout we denote by  $\bar{q}_i$  the capacity of market maker  $i$  and by  $\bar{q}_{-i}$  the aggregate capacity of all others than  $i$ , and aggregate capacity is denoted as  $\bar{Q}$ , so that by definition  $\bar{Q} \equiv \bar{q}_i + \bar{q}_{-i}$ . In stage 2, market makers simultaneously set bid prices  $b_i$  on the input market, and in stage 3, they simultaneously set ask prices  $a_i$  on the output market. All previous actions are assumed to be observed, and in case rationing occurs, the efficient rationing rule applies. Quantity of  $i$  and aggregate quantity of all others than  $i$  are denoted as  $q_i$  and  $q_{-i}$ , respectively, and aggregate quantity is denoted as  $Q \equiv q_i + q_{-i}$ . We will make clear where necessary what quantity (stock, quantity sold or quantity demanded) is meant by  $q_i$ ,  $q_{-i}$  or  $Q$ .

Let  $A(Q)$  denote the inverse demand function, which depicts the market clearing ask price  $A(\cdot)$  as a function of aggregate quantity demanded  $Q$ , and consider Figure 1 for an illustration of the basic assumptions. The inverse supply function is denoted as  $B(Q)$ , where  $B(Q)$  is the market clearing bid price for aggregate quantity supplied  $Q$ . Let  $D(a) \equiv A^{-1}(Q)$  and  $S(b) \equiv B^{-1}(Q)$ , respectively, denote the demand and supply functions. Both functions represent the behavior of perfectly competitive agents. We assume  $A' < 0$  and  $0 \leq B(0) < A(0) < \infty$ ,  $B' > 0$ ,  $A(0) - B(0) > C'(0)$  and that the Walrasian quantity  $Q^W$ , which is given by  $A(Q^W) = B(Q^W)$ , is less than infinity. Place Figure 1 about here. Denote by  $p^W \equiv A(Q^W) = B(Q^W)$  the Walrasian price. We assume that for any  $a \geq p^W$  the ask price elasticity of demand, defined as  $\varepsilon_a(a) \equiv D'(a) \frac{a}{D(a)}$ , does

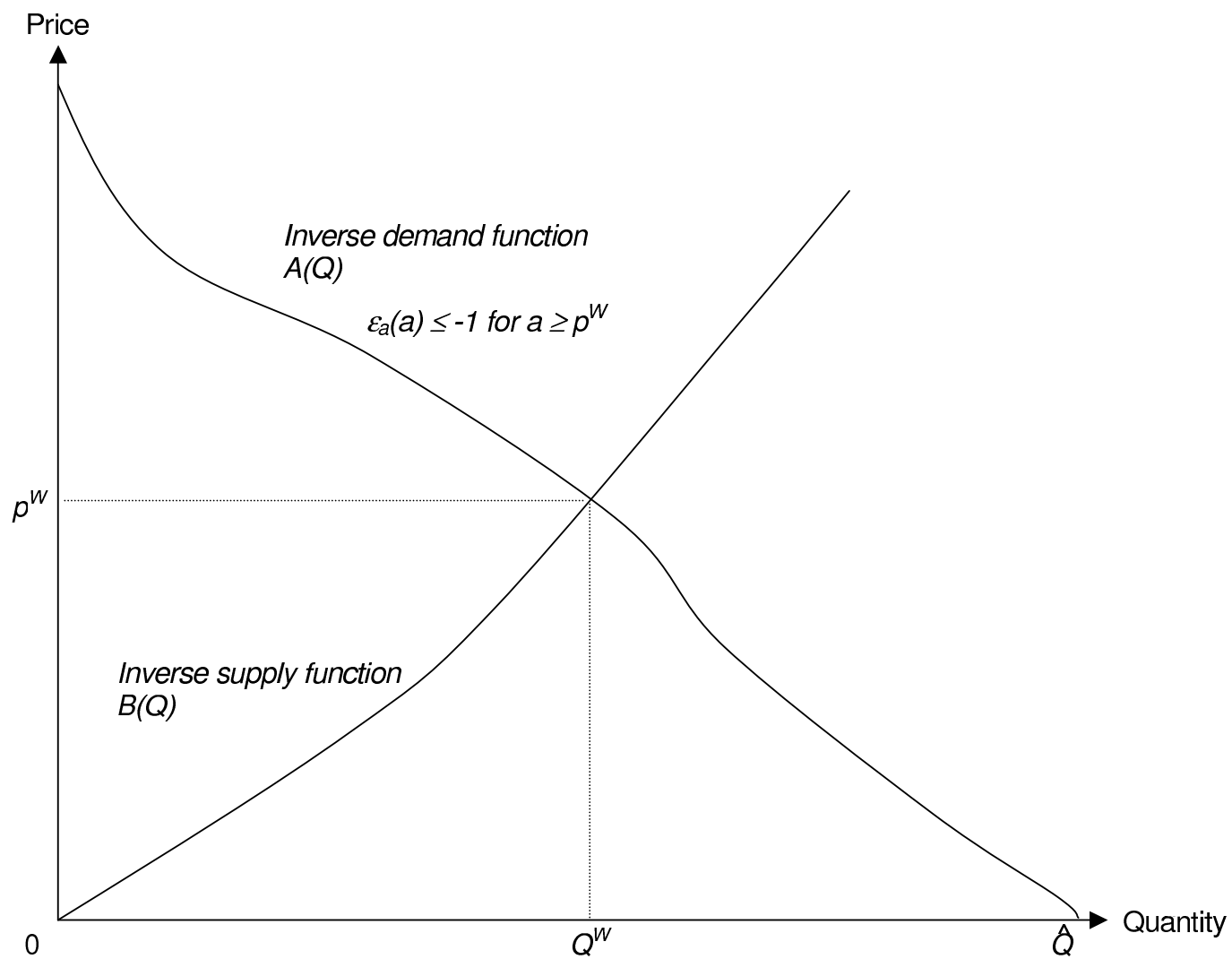


Figure 1: The Basic Setting.

not exceed minus one. Also, we assume that there is a quantity  $\hat{Q}$  such that  $A(\hat{Q}) = 0$ . For  $Q \geq \hat{Q}$ , we let  $A(Q) = 0$ . Furthermore, demand is assumed to be not too convex, i.e., to satisfy  $D'' < \frac{-2D'}{a}$  for any  $a > 0$ .<sup>9</sup> We say that demand is (price) elastic whenever  $\varepsilon_a \leq -1$ . Last, we assume  $B'' \geq A''$  for any  $Q \leq Q^W$ , which makes sure that the spread function  $Z(Q) \equiv A(Q) - B(Q)$  is weakly concave. Because  $A' < 0$  and  $B' > 0$ , we also have  $Z' < 0$ . So as to distinguish the market clearing prices given by the functions  $A(\cdot)$  and  $B(\cdot)$  from prices set by market makers, the latter are denoted by small letters and a subscript, like  $a_i$  or  $b_i$ , and we will occasionally denote the prices of all firms other than  $i$  as  $a_{-i}$  and  $b_{-i}$ .<sup>10</sup>

The rationales for these assumptions are as follows. Concavity of  $Z(\cdot)$  turns out to be very helpful and is less restrictive than assuming that  $A(\cdot)$  is concave, an assumption often made in models of product market competition. The assumption that demand is price elastic for any quantity not exceeding the Walrasian one makes sure that ask prices are market clearing in any equilibrium, which simplifies the analysis considerably.<sup>11</sup> This assumption is satisfied in many applications [e.g., Gehrig 1993, Fingleton 1997, Rust and Hall, 2003] and maintained in large parts of Stahl [1988]. To some extent, it will be relaxed in Section IV.

Efficient rationing is assumed in the largest part of the literature.<sup>12</sup> For alternative rationing schemes like proportional rationing, equilibrium behavior is likely to be more aggressive, and the equilibrium analysis prone to be more complicated, than with efficient rationing; see Davidson and Deneckere [1986]. However, the fact that the equilibrium behavior in models of capacity constrained price competition à la Kreps and Scheinkman [1983] or Levitan and Shubik [1972] is *less* competitive than Bertrand is not put into question.

As to timing, the crucial assumption is that capacity can be observed. In particular, it cannot be increased before price competition starts without having the competitors take notice. Whether this assumption is realistic depends of course on the application. It is arguably a good approximation if capacity takes the form of sale space or number and size of branches as in retail trade. It is certainly less accurate if the binding constraint is given by server capacity like, e.g., for providers of internet platforms.

The structure of the basic model is most appropriate when market makers are retailers or shops. These are capacity constrained and do not sell forward contracts but rather must have the goods in stock if they want to be able to sell.<sup>13</sup> Hence, the acquisition of stocks precedes selling.<sup>14</sup> In other instances, such as wholesale trade of agricultural produce and imported goods, forward contracts are frequently used. In Section IV we extend the model to forward contracts, which corresponds to a reversal of the input and

the output market stages. The informational assumption that all previous actions are observed is partly made for convenience and can be relaxed. For example, if the prices in stage 2 are observed, then the quantities competitors have in stock (or in the presence of forward contracts, the quantities they are obliged to buy) can be inferred from the observation of capacities and prices.

### III. EQUILIBRIUM ANALYSIS

We proceed as follows. Because concepts from Cournot competition are crucial for the analysis that follows, we first define Cournot capacities and derive the Cournot outcome for our game. Then we solve, in turn, for the equilibrium of the output market subgame, the input market subgame and the capacity subgame.

#### III.(i) *Preliminary: Cournot Competition*

As Cournot competition typically refers to competition on a product market organized by a Walrasian auctioneer, whereas we study competition between market makers, we have to make clear what we mean by Cournot competition and Cournot outcome in our setting. Assume temporarily that both on the input and on the output market a Walrasian auctioneer quotes market clearing ask and bid prices and every market maker names the quantity it wants to trade, taking as given the inverse supply and demand functions  $B(\cdot)$  and  $A(\cdot)$  and the quantities its competitors name. Then the quantities traded in equilibrium are called Cournot equilibrium quantities. That is, as a Cournot competitor  $i$  maximizes its profit by choosing its optimal quantity  $q_i^*$  given the quantities of all others  $q_{-i}$  and the inverse supply and demand functions  $B(Q)$  and  $A(Q)$  and its cost function  $C(q_i)$  with  $C' > 0$  and  $C'' \geq 0$ . Let  $\Pi_i(q_i, q_{-i})$  denote firm  $i$ 's profit when setting quantity  $q_i$ . Then, the maximization problem for  $i$  is

$$\begin{aligned} \max_{q_i} \Pi_i(q_i, q_{-i}) &= (A(q_i + q_{-i}) - B(q_i + q_{-i})) q_i - C(q_i) \\ (1) \qquad \qquad \qquad &= Z(q_i + q_{-i}) q_i - C(q_i). \end{aligned}$$

The corresponding first order condition can be written as

$$(2) \qquad r_c(q_{-i}) = \frac{Z(r_c(q_{-i}) + q_{-i}) - C'(r_c(q_{-i}))}{-Z'(r_c(q_{-i}) + q_{-i})},$$

where  $r_c(q_{-i})$  called  $i$ 's best response or reaction function to  $q_{-i}$ . Because  $Z(Q)$  has a negative slope and is weakly concave, the maximization problem (1) is a concave problem, so that (2) is the unique solution. If  $q_{-i}$  is such that  $Z(q_{-i}) - C'(0) \leq 0$ , then  $r_c(q_{-i}) = 0$ .

Since the concept is repeatedly used, let us also define the Cournot best response function with zero costs of production or trade, which is denoted as  $r(q_{-i})$  and given by

$$(3) \quad r(q_{-i}) = \frac{Z(r(q_{-i}) + q_{-i})}{-Z'(r(q_{-i}) + q_{-i})}.$$

The corner solution with  $r(q_{-i}) = 0$  arises only if  $q_{-i}$  is so large that  $Z(q_{-i}) \leq 0$ , i.e., if  $q_{-i} \geq Q^W$ . Totally differentiating (3) yields

$$(4) \quad r'(q_{-i}) = \frac{ZZ'' - (Z')^2}{-ZZ'' + 2(Z')^2},$$

where we have dropped the argument of  $Z(\cdot)$ . The property  $r' < 0$  is readily established for any concave function  $Z$ , because the numerator is negative and the denominator is positive. Moreover,  $r' > -1$ . To see this, note that  $r' > -1 \Leftrightarrow ZZ'' - (Z')^2 > -(-ZZ'' + 2(Z')^2)$ , which is clearly satisfied. The property  $0 > r' > -1$  implies also that  $r(q_{-i}) + q_{-i}$  increases in  $q_{-i}$ , i.e.,  $d(r(q_{-i}) + q_{-i})/dq_{-i} > 0$ . Moreover, the fact that for  $q_{-i} < Q^W$ ,  $r_c(q_{-i}) < r(q_{-i})$  is also readily established, using  $Z' < 0$ ,  $Z'' \leq 0$  and  $C' > 0$  to get a contradiction for  $r_c(q_{-i}) \geq r(q_{-i})$ .

Individual firms' Cournot equilibrium quantities when trade is costly are given by the unique fixed point of the equation  $q^C = r_c((n-1)q^C)$ . Aggregate Cournot quantity is denoted as  $Q^C \equiv nq^C$ , and we refer to the ask price  $A(Q^C)$  and the bid price  $B(Q^C)$  as Cournot (ask and bid) prices. For the case with zero marginal costs, equilibrium quantity  $q_Z$  is given by the fixed point of the equation  $q_Z = r((n-1)q_Z)$ . Because  $r_c(q) < r(q)$  for  $q < Q^W$ ,  $q^C < q_Z$  follows.

### III.(ii) *The Output Market Subgame*

Let  $Q$  denote the aggregate stock of market makers and assume that  $Q$  is observed. Recall that for  $Q \leq Q^W$ , the demand function is price elastic by assumption. In this case  $a_i = A(Q)$  for all  $i$  is the unique Nash equilibrium. To see this, note first that prices  $a'_i < A(Q)$  are strictly dominated by the market clearing ask price  $a_i = A(Q)$  since by setting  $a'_i$ ,  $i$  sells the same quantity as it would by setting  $a_i$  but at a lower price. Therefore, the only equilibrium candidates are prices  $a_i \geq A(Q)$ . Suppose first that all firms other than  $i$  set  $a_{-i} = A(Q)$  and let  $i$  contemplate deviation to some  $a_i > A(Q)$ . Assuming that the residual demand function a high pricing firm faces is at least as price elastic as the demand function,<sup>15</sup> increasing price by one percent will result in a reduction of residual demand by more than one percent because demand is elastic. Therefore, the deviation will not pay, and hence, given  $a_{-i} = A(Q)$ ,  $a_i = A(Q)$  is a best response for all  $i$ . Uniqueness in pure strategies follows once it is noted that for any other combination



of ask prices with  $a_i \geq A(Q)$ , at least one player could strictly increase his profit by changing his price. This proves part (ii) of the following lemma for the case  $Q \leq Q^W$  and for pure strategies. That no equilibrium in mixed strategies exists and that the first part of the lemma is true is proved in the Appendix.

*Lemma 1.* In any equilibrium, (i) aggregate quantity bought  $Q$  does not exceed  $Q^W$  and (ii) ask prices are market clearing, i.e.,  $a_i = A(Q)$  for all  $i$ .

### III.(iii) *The Input Market Subgame*

We now turn to the analysis of the bid price setting or input market subgame. We first describe the equilibrium in Stahl's model, in which there are no capacity constraints *ex ante*. Then we turn to the analysis of input market equilibrium for the model with capacity constraints *ex ante*.

Given Lemma 1, the derivation of the equilibrium outcome in the model with *interim* capacity constraints analyzed by Stahl [1988] is straightforward. Assume that there are two firms. For any aggregate quantity not exceeding the Walrasian quantity, the equilibrium output price will be market clearing. Therefore, if both firms set the same price on the input market they would share revenue on the output market. However, either firm has an incentive to slightly overbid the competitor's bid price since this discontinuously increases its profits. As in Bertrand product market competition, the unique equilibrium with two firms has thus both firms quote the Walrasian price and net zero profits. The only difference to product market competition models is that here prices are set both on the input and on the output market.

For the model with capacity constraints *ex ante*, we will first show that there is a unique Nash equilibrium in the bid price setting subgame if each market maker has a capacity no greater than the quantity given by its Cournot best response function for zero costs. In this equilibrium each  $i$  plays the pure strategy  $b_i = B(\bar{Q})$ . This establishes that given Cournot capacities,  $b_i = B(\bar{Q})$  is an equilibrium and hence that the Cournot outcome is a Nash equilibrium outcome. Second, we show that there is another region of pure strategy equilibria in which capacity constraints are by and large irrelevant, and we characterize this region. Third, we determine the expected equilibrium revenue for the largest firm for those capacity combinations for which the equilibrium of the bid price setting subgame is in mixed strategies. This expected revenue corresponds to the Stackelberg follower profit.

The regions of pure strategy equilibria, and the derivation of these regions, are completely analogous to those in Kreps and Scheinkman [1983] and Bocard and Wauthy

[2000]. Quite a different method, however, is required to show that the largest firm earns the Stackelberg follower profit in the mixed strategy region. The differences between the two models stem from the fact that in the product market competition models of Kreps and Scheinkman and Bocard and Wauthy, the profit of the firm with the highest price is independent of the actual prices set by other firms, which is due to the efficient rationing assumption. Consequently, the maximal profit of a firm who is underbid with certainty is independent of the other firms' price. Despite the assumption of efficient rationing, the analogue does not hold in our model because the higher bid prices set by other firms will determine the aggregate quantity bought, and thereby the ask prices subsequently set. Therefore, the profit of the intermediary with the lowest bid price will depend on the bid prices set by the other intermediaries. Consequently, the proof for our model differs from the one in Kreps and Scheinkman and Bocard and Wauthy. The downside to this is that new arguments have to be developed. These rely on ideas first used by Deneckere and Kovenock [1992,1996]. The upside is that the new method is more general, so that once the result is shown for two intermediaries, it will also be shown for any number of intermediaries.

There are two regions of capacity constraints where pricing equilibria are in pure strategies. When capacities are *small* (to be made precise shortly), then there is a unique equilibrium in the pricing subgames. In this equilibrium, intermediaries play pure strategies on and off the equilibrium path by setting the capacity clearing prices.

*Lemma 2.* For capacities  $\bar{q}_i \leq r(\bar{q}_{-i})$  for all  $i = 1, \dots, n$ , there is a unique Nash equilibrium in the input market subgame, in which all market makers set the market clearing bid price  $b_i = B(\bar{Q})$ .

Region I in Figure 2 depicts the region of pure strategy equilibria for the case of two firms and a linear spread function, where  $q_Z$  denotes the Cournot equilibrium quantity with zero marginal costs.<sup>16</sup>

Place Figure 2 about here.

Since setting capacity clearing bid and ask prices is an equilibrium, given capacities no larger than Cournot capacities, Lemma 2 has the following corollary.

*Corollary 1.* The Cournot outcome is a Nash equilibrium outcome.

For example, the strategy profile  $(\bar{q}_i = q^C, b_i = B(\bar{Q}), a_i = A(\bar{Q}))$  for all  $i$  is a Nash equilibrium that has Cournot actions as equilibrium outcome. The questions, however, whether Cournot is the outcome of a subgame perfect equilibrium and whether the equilibrium outcome is unique cannot be addressed without looking at the subgames

following capacities outside Region I.<sup>17</sup> Answering these questions will be the focus of the remainder of this section.

The second region of capacity constraints for which equilibria are in pure strategies basically corresponds to the one analyzed by Stahl [1988]. This region is labelled PSNE-Region II and illustrated in Figure 2.

*Lemma 3.* If  $\bar{q}_{-i} \geq Q^W$  for all  $i$ , there is always an equilibrium in which all firms play pure strategies. In this equilibrium, all firms  $i$  that buy positive quantity set  $b_i = B(Q^W)$ . All firms make zero profits in any equilibrium.

Note that there may be multiple equilibria<sup>18</sup> in Region II. Region I is exactly the same in Kreps and Scheinkman [1983], while Region II coincides with their Region II for  $n = 2$ .

If  $\bar{q}_{-j} < Q^W$  for at least one  $j$  and  $\bar{q}_k > r(\bar{q}_{-k})$  for at least one  $k$ ,<sup>19</sup> there is no pure strategy equilibrium. If  $B(\bar{Q}) < B(Q^W)$ , downward deviation from the market clearing bid price pays for  $k$ . If  $B(\bar{Q}) \geq B(Q^W)$ , the only candidate price at which quantity is traded in a pure strategy equilibrium is  $B(Q^W)$ . However, if all others set  $b_{-j} = B(Q^W)$ , downward deviation pays for  $j$  since  $\bar{q}_{-j} < Q^W$  implies that  $j$ 's residual supply is positive for some  $b_j < B(Q^W)$  and  $b_{-j} = B(Q^W)$ . When setting  $b_j \in (B(\bar{q}_{-j}), B(Q^W))$ ,  $j$  buys therefore a positive quantity on which it earns a positive spread, while with  $b_j = b_{-j} = B(Q^W)$ , its profit is zero.

The existence of an equilibrium for our game is guaranteed by Dasgupta and Maskin [1986]. The equilibrium involves non-degenerate mixed strategies. Though these mixed equilibrium strategies are hard to compute, it is possible to derive the expected equilibrium profit or revenue for the largest firm without completely characterizing these strategies. The expected equilibrium revenue is given in Lemma 4, which replicates the key finding of Kreps and Scheinkman [1983] for market makers facing a concave spread function. It states that in the mixed strategy equilibrium, the largest firm earns in expectation no more than it would have earned had it determined its capacity according to the Cournot best response function with zero costs.

*Lemma 4.* Let  $i$  be (one of) the largest firm(s). In the mixed strategy equilibrium, the expected profit of any of the largest firms equals  $R(\bar{q}_{-i}) \equiv r(\bar{q}_{-i})Z(r(\bar{q}_{-i}) + \bar{q}_{-i})$ .

The proof is in the Appendix. It consists of showing that (i) one firm, say  $i$ , is overbid with probability one when setting the lowest bid price  $\underline{b}$  over which firms randomize, (ii)  $\underline{b} \equiv B(r(\bar{q}_{-i}) + \bar{q}_{-i})$  and  $i$ 's expected profit is  $R(\bar{q}_{-i})$  and (iii)  $i$  is (one of) the largest firm(s).

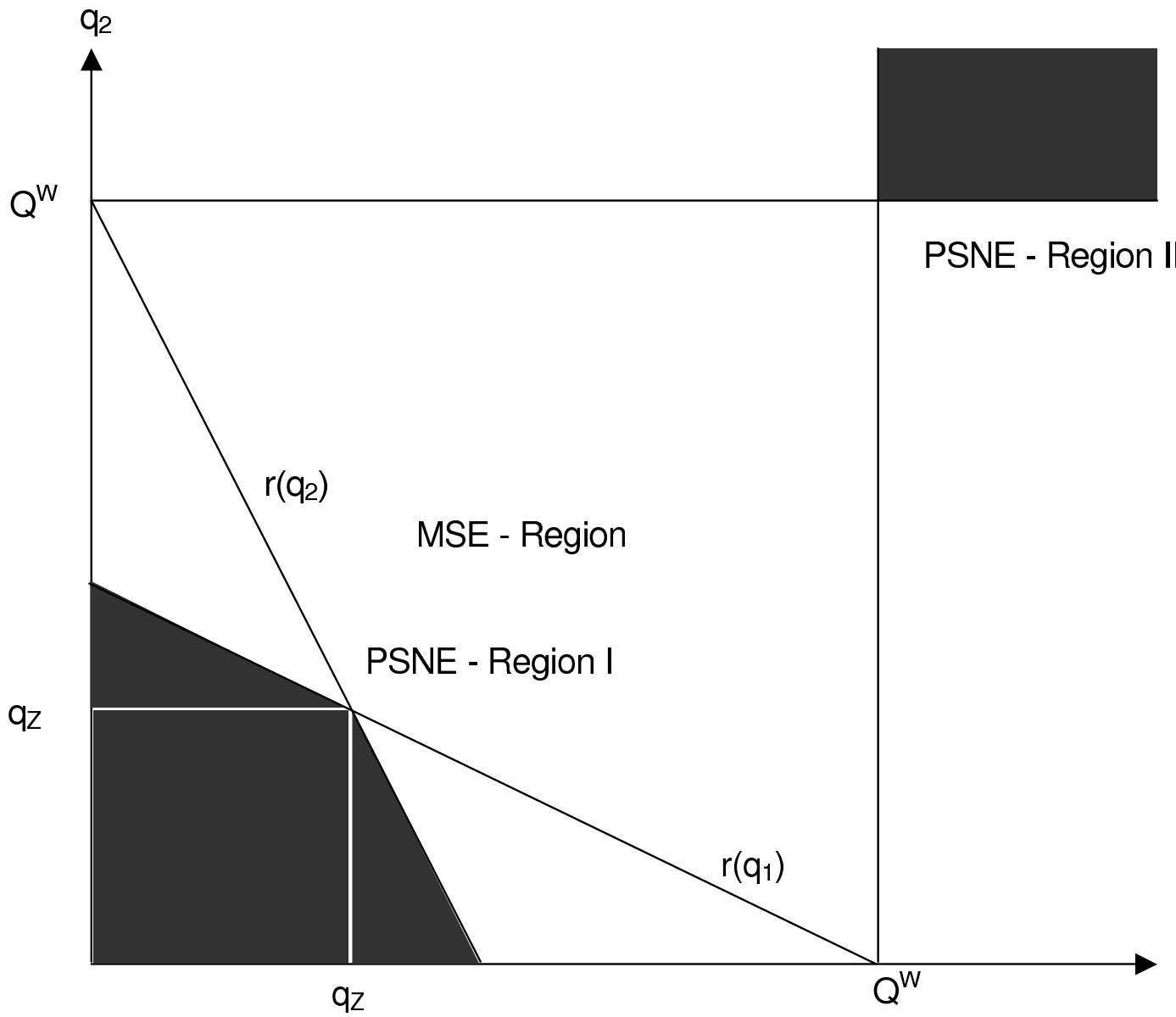


Figure 2: Cournot Reaction Functions and Cournot Equilibrium.

There is a fairly clear intuition for this result. Smaller firms are more aggressive in the bid price subgame because they have more to lose from low bid prices and, by the same token, more to win from high bid prices. To see this, note that when a small firm is overbid by a large firm, the small firm faces the threat of not buying anything because, ultimately, the large firm may take the whole market. This problem does not exist for the largest firm, say  $i$ , because its profit is positive even if it is overbid by all small firms, who cannot take the entire market: Even when setting  $\underline{b} \equiv B(r(\bar{q}_{-i}) + \bar{q}_{-i})$ ,  $i$  buys a positive quantity. In contrast, there is no guarantee that any of its competitors gets to buy anything when being overbid while setting prices close to  $\underline{b}$  as  $\bar{q}_i$  might be larger than  $S(\underline{b}) = r(\bar{q}_{-i}) + \bar{q}_{-i}$ . As a consequence of this vulnerability from low prices, smaller firms are more aggressive, i.e., are willing to overbid higher prices than larger firms. As large firms incur the cost of high bid and low ask prices on larger capacities, they have a greater dislike for high bid prices. Consequently, all small firms can earn more than their Stackelberg follower profit. In the terminology of Fudenberg and Tirole [1986], large firms are thus fat cats while small firms are lean and hungry.

### III.(iv) *Equilibrium of the Full Game*

Lemma 4 says that the profit of the largest market maker, say  $i$ , in the mixed strategy region is equal to its Stackelberg follower profit, given the aggregate capacity of all other market makers. If all other market makers set Cournot capacities, the Stackelberg follower profit equals the Cournot profit. Hence, if all others set Cournot capacities, then unilateral deviation does not pay even if capacity is costless. Consequently, Cournot is the outcome of a subgame perfect equilibrium, whence the first part of the following proposition follows. The proof of the second part is in the Appendix.

*Proposition 1.* The Cournot outcome is a subgame perfect equilibrium outcome. If capacity is costly, Cournot is the unique Nash equilibrium outcome with pure strategies at the capacity stage.

A few comments are in order. First, the result shows that previously reported findings from product market models with capacity constraints carry over to models of market making. The only substantial additional assumption we made is that demand be price elastic for  $Q \leq Q^W$ , a relaxation of which we will analyze below. The main difference between the two types of models resides in the pricing subgames where equilibria are in mixed strategies. Second, with more than two firms or market makers there are multiple subgame perfect equilibria if capacity is costless. In these equilibria, at least three firms set  $\bar{q}_i \geq Q^W$ . Clearly, unilateral deviation by one of these firms to a smaller capacity

will not change the subsequent equilibrium outcome of the pricing subgames. This is in contrast to duopoly models, where such a deviation pays. Consequently, with two firms such equilibria are not subgame perfect.

### III.(v) *Fixed Cost and Entry*

The number of active market makers has so far been taken as exogenous. However, assuming that market entry is possible at a positive fixed cost  $f$  and that entry takes place simultaneously and prior to capacity setting, the number of active market makers can be determined endogenously. Since there is a negative relationship between the number of active market makers and profits, the relationship between the equilibrium number of active market makers and fixed cost  $f$  will also be negative.

Two comments are in order. First, if physical capacity is a necessary condition for firms to be able to engage in price competition, legal and other barriers to build capacity will have exactly the same effect as barriers to entry. Second, the assumption that intermediaries simultaneously enter the market and then simultaneously set capacities is likely to matter here. In a product market competition model, Allen, Deneckere, Faith and Kovenock [2000] show that the equilibrium outcome is typically different from Cournot when firms enter sequentially and build capacity as they arrive.

## IV. EXTENSIONS

In this section, we deal in turn with three extensions of the model, forward contracts, inelastic demand, and simultaneous ask and bid price setting.

### IV.(i) *Forward Contracts*

In some markets, intermediaries first sell forward contracts to buyers and then go to producers to obtain the stocks required to fulfill the obligations arising from the forward contracts sold. For example, intermediaries using forward contracts sell imported goods to retailers and agricultural produce to food processing firms. Therefore, it is important to see whether our model is robust to the introduction of forward contracts.

Forward contracts are modelled as a reversal of stages 2 and 3. The modified time structure is as follows. In stage 1, market makers simultaneously set capacities. In stage 2, each firm  $i$  sells forward contracts to consumers at a uniform ask price  $a_i$ , entitling each consumer with the right to get one unit of the good at that price. Each firm  $i$  is allowed to ration consumers if the demand it faces exceeds its physical capacity  $\bar{q}_i$ .<sup>20</sup> In stage 3, after forward contracts are sold, market makers buy the amount of the good on

the input market necessary to fulfill the obligations arising from the volume of forward contracts sold. Following Stahl [1988, pp. 196-7], we assume that the penalty for default is sufficiently severe to deter any default in equilibrium. In addition, the legal proceedings are assumed to be lengthy and time consuming enough, so that consumers involved in it cannot buy from another market maker while the proceeding lasts. This rules out that a market maker can gain additional consumers by forcing another one into default. Note that these "no default in equilibrium" assumptions imply that whatever volume of forward contracts is sold, market makers will set the market clearing bid price on the input market. As before, we assume that rationing on the output market is efficient.

In Stahl's model without *ex ante* capacity constraints and with forward contracts, the equilibrium outcome is Walrasian regardless of whether demand is elastic or inelastic: Whenever a firm tries to sell forward contracts at a price above the Walrasian price, it will be undercut by a competitor. Consequently, the equilibrium condition that all firms net zero profits requires that all firms that trade a positive volume sell at the Walrasian price. Due to the severe default penalty, all firms fulfill their obligations to sell, and therefore must buy at the Walrasian price. In contrast when capacity constraints are set first, quite a different result obtains, which is a corollary to Proposition 1:

*Corollary 2.* When market makers sell forward contracts, Cournot actions are a subgame perfect equilibrium outcome. If capacities are costly, Cournot is the unique Nash equilibrium outcome with pure strategies at the capacity stage.

Two remarks are in order. First, the proof of uniqueness is completely analogous to the one for Proposition 1. Second, it is not necessary to assume that demand is price elastic for any  $Q \leq Q^W$  for the result to hold. With forward contracts and sufficiently severe punishment for default, bid prices will always be market clearing, be the equilibrium on the output market (i.e., in stage 2) in pure or in mixed strategies.

#### IV.(ii) *Inelastic Demand in the Basic Model*

When demand is inelastic for some quantity smaller than the Walrasian one, Stahl [1988] observes for the timing of Section II that absent *ex ante* capacity constraints the unique subgame perfect equilibrium outcome is non-Walrasian because the winner on the input market throws away some of the quantity bought.<sup>21</sup> This motivates examining whether the same holds true in a model where capacity constraints are set *ex ante*.

We make the following assumptions. There are no forward contracts, and the timing is as in the basic model outlined in Section II. All previous actions are observed, and in case rationing occurs, the efficient rationing rule applies. As before, we assume  $A' < 0$ ,

$B(0) \geq 0$ ,  $A(0) - B(0) > C'(0)$  and  $B' > 0$ . In addition, we now assume that  $A(Q)$  is concave. These assumptions imply that the spread function  $Z(Q) \equiv A(Q) - B(Q)$  has the properties  $Z(Q) < A(Q)$ ,  $Z'(Q) < A'(Q)$  and  $Z''(Q) \leq 0$ . The main modification is that the ask price elasticity of demand at the Walrasian price  $p^W$  is now assumed to be greater than minus one.

Consider first Stahl's model with inelastic demand and two firms. Assume that the winner on the input market in case of tied bids is determined by flipping a fair coin and takes over the whole supply.<sup>22</sup> As either bidder can gain by slightly overbidding its competitor as long as they expect positive profits, an equilibrium condition is that both make zero profits. Since the monopoly revenue on the output market exceeds the revenue at the Walrasian price, it follows that both must set a bid price above the Walrasian price on the input market. Consequently, quantity bought by the winner will be larger than  $Q^W$ , whereas the quantity sold on the output market will be the monopoly quantity, which is smaller than  $Q^W$ . Hence, there is waste in equilibrium.

In our model, inelastic demand may give rise to a region of capacity constraints where firms will randomize on the input market taking into account that they will, eventually, also randomize on the output market. As the equilibrium of such a model has not yet been determined, a definite and conclusive answer cannot be given. Nevertheless, it can be shown that small deviations from Cournot capacity do not pay if all other firms set Cournot capacities. Denote by  $\Pi_j^*(\bar{q}_j, \bar{\mathbf{q}}_{-j})$  the expected profit of  $j$  when setting the capacity  $\bar{q}_j$  while all others set  $\bar{\mathbf{q}}_{-j}$ , given that in subsequent subgames equilibrium play ensues, and denote by  $q_A$  the Cournot equilibrium quantity when all firms have zero costs and face the demand function  $A(\cdot)$ . Observe that  $q_A > q^C$ , where  $q^C$  is the Cournot equilibrium quantity for the spread  $Z(\cdot)$  when the cost function is  $C(\cdot)$ . The following holds:

*Proposition 2.* There is a  $\bar{x} > q_A$  such that if  $\bar{q}_i = q^C$  for all  $i \neq j$ , then

$$\Pi_j^*(q^C, \bar{\mathbf{q}}_{-j}) \geq \Pi_j^*(\bar{q}_j, \bar{\mathbf{q}}_{-j}),$$

for any  $\bar{q}_j \in (q^C, \bar{x}]$ , with a strict inequality if capacity is costly.

Let us briefly comment on this result. Deviations from the Cournot equilibrium capacity  $q^C$  to some  $\bar{q}_j \in (q^C, \bar{x}]$  can be called *small*. We already know that Cournot is the outcome of a Nash equilibrium. The import of the proposition is that it is also the outcome of a subgame perfect Nash equilibrium if only small deviations are considered or if, equivalently, the capacity choice is restricted to the set  $[0, \bar{x}]$ .



#### IV.(iii) *Simultaneous Ask and Bid Price Setting*

For completeness, we address the question what happens if market makers simultaneously set ask and bid prices. We assume that the demand function satisfies all of the assumptions made in Section II, except that it is not required to be price elastic at  $p^W$ . The time structure is as follows. In stage 1, market makers simultaneously set capacity constraints  $\bar{q}_i$ ,  $i = 1, \dots, n$ . In stage 2, having observed the capacity of all competitors, they simultaneously set a pair of ask and bid prices  $(a_i, b_i)$ ,  $i = 1, \dots, n$ . These prices are such that a market maker is obliged to buy up to its capacity and to pay  $b_i$  per unit supplied. The demand for a market maker can exceed its stock. In this case, the market maker is obliged to sell its stock at the ask price set. On both sides of the market, the efficient rationing rule applies. The following result obtains:

*Proposition 3.* When ask and bid prices are set simultaneously, Cournot actions are a subgame perfect equilibrium outcome, independent of whether demand is elastic or inelastic at  $p^W$ . If capacities are costly, the Cournot outcome is the unique Nash equilibrium outcome with pure strategies at the capacity stage.

## V. CONCLUSIONS

To the best of our knowledge, this paper is the first to analyze capacity constrained price competition and market making when capacities are set *ex ante*.<sup>23</sup> As the key results of Kreps and Scheinkman [1983] carry over to market making and to any number of market making firms, one conclusion from the present study is that these results are fairly robust in these respects. Another, perhaps more important lesson is that capacity constraints matter for models of market making. Consequently, the Bertrand-Walras outcome should not be taken for granted for market making, but should rather be seen as an exception that occurs only if costly capacity constraints are either not present or naturally larger than the Walrasian quantity.

An interesting avenue for further research is to study a setting where intermediaries face financial constraints instead of physical capacity constraints. A financial constraint  $m_i$  would be such that  $i$ 's expenditure  $b_i q_i$  on the input market cannot be larger than  $m_i$ . If  $m_i$  is observed by all competitors and if sellers only sell for cash, it is straightforward to establish that under efficient rationing the regions of pure strategy equilibria in the pricing subgames correspond to those in the model with physical capacity constraints. On the other hand, the behavior in the region where equilibria are in mixed strategies turns out to be quite different and awaits separate analysis.

## APPENDIX

*Proof of Lemma 1:* We first show that no non-degenerate mixed strategy equilibrium exists for  $Q \leq Q^W$ . Assume to the contrary that such an equilibrium exists and denote by  $\bar{a}$  the highest ask price in the support of prices over which firms randomize. Because the equilibrium is in non-degenerate mixed strategies by hypothesis,  $\bar{a} > A(Q)$  must hold. For reasons discussed in detail in the proof of Lemma 4, there will be a firm, say  $i$ , who is underbid with probability one when setting  $\bar{a}$ . Thus,  $\bar{a}$  must be the argument that maximizes  $a(D(a) - q_{-i})$  over  $a$ . This is equivalent to maximizing  $A(s + q_{-i})s$  over  $s$ , yielding  $s(q_{-i}) = \frac{A(s(q_{-i}) + q_{-i})}{-A'(s(q_{-i}) + q_{-i})}$  as the Cournot best response function of a firm with zero costs facing the inverse demand function  $A(\cdot)$ . Dropping arguments,  $s' = \frac{AA'' - (A')^2}{-AA'' + 2(A')^2}$  can be derived completely analogous to  $r'$  in (4). It is easy to check that  $s' > -1$  holds if  $-AA'' + 2(A')^2 > 0 \Leftrightarrow A'' < \frac{2(A')^2}{A}$ . Since for the inverse  $A' = \frac{1}{D'}$  and  $A'' = -\frac{D''}{(D')^3}$  holds, this is equivalent to our assumption  $D'' < -\frac{2D'}{a}$ . Thus,  $s(x) + x$  will be increasing in  $x$ , and consequently  $A(s(x) + x)$  decreases in  $x$ . Therefore,  $\bar{a} \equiv A(s(\bar{q}_{-i}) + \bar{q}_{-i}) \leq A(Q)$ , which contradicts the assumption that this is a non-degenerate mixed strategy equilibrium. Part (ii) now follows from the above and the argument in the text once (i) is shown. Part (i): Assume to the contrary that an equilibrium exists where the aggregate quantity exceeds  $Q^W$ . In any such equilibrium, the market maker who sets the lowest price on the input market while still buying a positive amount pays a bid price greater than  $B(Q^W)$ . However, for any aggregate stock  $Q > Q^W$ , the price any seller gets in the output market equilibrium will be less than  $A(Q^W) \equiv B(Q^W)$ . Either there is a pure strategy equilibrium with  $a_i = A(Q) < B(Q^W)$  or the equilibrium will be in mixed strategies where the range of prices over which firms randomize will not exceed  $a^M \leq B(Q^W)$ . Thus, each market maker who trades a positive amount will make negative profits, which cannot be an equilibrium given the possibility to make zero profits (e.g., by setting  $b_i = 0$ ).  $\square$

*Proof of Lemma 2: Existence.* Note first that  $\bar{q}_i \leq r(\bar{q}_{-i})$  for all  $i$  implies  $\bar{Q} < Q^W$  because  $r(Q^W) = 0$  and  $r(0) < Q^W$ . To see that  $b_i = B(\bar{Q})$  is an equilibrium for  $\bar{q}_i \leq r(\bar{q}_{-i})$ , suppose that all firms other than  $i$  set  $b_{-i} = B(\bar{Q})$ . The question then is when deviation from  $b_i = B(\bar{Q})$  pays for  $i$ . Bid prices above  $B(\bar{Q})$  being strictly dominated, only downward deviations need to be considered. When  $i$  sets  $b_i < B(\bar{Q})$ , it faces a residual supply of  $\max[S(b_i) - \bar{q}_{-i}, 0]$ . All other market makers setting  $b_{-i} = B(\bar{Q})$ ,  $S(b_i) - \bar{q}_{-i} < \bar{q}_i$  will hold. Therefore,  $\max[S(b_i) - \bar{q}_{-i}, 0]$  will be the quantity bought by  $i$  when underbidding. For  $S(b_i) - \bar{q}_{-i} > 0$  and  $b_{-i} = B(\bar{Q})$ , aggregate quantity bought will just be  $S(b_i)$ . The unique equilibrium of the ask price setting game being

to set  $a_i = A(Q)$ , the equilibrium price on the output market is a direct function of the smallest bid price for which residual supply is positive. If all others set  $b_{-i} = B(\bar{Q})$ , it is a function only of  $b_i$ . For  $b_i > B(\bar{q}_{-i}) \Leftrightarrow S(b_i) > \bar{q}_{-i}$ , then  $A(Q) = A(S(b_i))$ . Otherwise,  $A(Q) = A(\bar{q}_{-j})$ , but then the profit of  $i$  is zero independently of  $A(\cdot)$ . Given that its profit is positive when setting  $B(\bar{Q})$  whenever  $\bar{Q} < Q^W$ , the deviation  $b_i \leq B(\bar{q}_{-i})$  will not pay. Therefore, we can concentrate on  $b_i > B(\bar{q}_{-i})$ . In this case,  $i$ 's profit when deviating from  $B(\bar{Q})$  is  $\Pi_i(b_i, b_{-i}, \bar{q}_{-i}) = (A(S(b_i)) - b_i)(S(b_i) - \bar{q}_{-i})$ , which is what  $i$  maximizes over  $b_i$  when optimally deviating. Define  $x \equiv S(b_i) - \bar{q}_{-i}$ . Then,  $A(S(b_i)) = A(x + \bar{q}_{-i})$  and  $b_i = B(x + \bar{q}_{-i})$ . Maximizing  $\Pi_i(b_i, b_{-i}, \bar{q}_{-i})$  over  $b_i$  is thus equivalent to maximizing  $\Pi_i(x, \bar{q}_{-i}) = (A(x + \bar{q}_{-i}) - B(x + \bar{q}_{-i}))x = Z(x + \bar{q}_{-i})x$  over  $x$ , whence it becomes clear that the optimal deviation over  $b_i$  is equivalent to choosing the optimal quantity under Cournot competition with zero costs. The optimal  $x$  will thus be such that  $x = r(\bar{q}_{-i})$  implying that the optimal bid price  $b_i$  is equal to  $B(r(\bar{q}_{-i}) + \bar{q}_{-i})$ , the Cournot best response bid price.<sup>24</sup> Since the optimal deviation of  $i$  is to set  $b_i = B(r(\bar{q}_{-i}) + \bar{q}_{-i})$  when all others set higher prices, it is now clear that such a deviation does not pay when  $\bar{q}_i \leq r(\bar{q}_{-i})$  holds for all  $i$ . Prices above  $B(\bar{Q})$  being dominated, deviation does not pay whenever  $B(r(\bar{q}_{-i}) + \bar{q}_{-i}) \geq B(\bar{Q}) \Leftrightarrow \bar{q}_i \leq r(\bar{q}_{-i})$ . Thus, existence is shown.

*Uniqueness.* The argument establishing uniqueness is analogous to the one of the output market subgame. Bid prices above  $B(\bar{Q})$  being strictly dominated, the only alternative candidates for a pure strategy equilibrium are bid prices smaller than  $B(\bar{Q})$ . However, whenever a firm  $i$  sets a bid price  $b_i < B(\bar{Q})$ , at least one other firm, say  $j$ , will optimally set a price below  $B(\bar{Q})$  and above  $b_i$ , so that firm  $i$ 's profit would discontinuously increase by setting a slightly higher price than  $j$  does. Thus, there is no other equilibrium in pure strategies. Last, non-degenerate mixed strategy equilibria have to be ruled out. Assume to the contrary that such an equilibrium exists and denote by  $\underline{b}$  the lowest bid price in the support of bid prices over which firms randomize. Because the equilibrium is in non-degenerate mixed strategies by hypothesis,  $\underline{b} < B(\bar{Q})$  must hold. For reasons discussed in detail in the proof of Lemma 4, there will be a firm, say  $i$ , who is overbid with probability one when setting  $\underline{b}$ . Thus,  $\underline{b}$  must be the argument that maximizes  $(A(S(b)) - b)(S(b) - \bar{q}_{-i})$  over  $b$ , yielding  $\underline{b} = B(r(\bar{q}_{-i}) + \bar{q}_{-i}) \geq B(\bar{Q})$ , which contradicts the assumption that this is a non-degenerate mixed strategy equilibrium.  $\square$

*Proof of Lemma 3:* Given  $\bar{q}_{-i} \geq Q^W$  for all  $i$ , if all firms other than  $i$  set  $b_{-i} = B(Q^W)$ , there is no way  $i$  can increase its profit by setting a price other than  $B(Q^W)$ . For  $b_i \leq B(Q^W)$ ,  $i$ 's profit is zero, while for  $b_i > B(Q^W)$ ,  $i$ 's profit is negative. Thus,  $b_i = B(Q^W)$  for all  $i$  is an equilibrium.

Next, we show that the unique equilibrium outcome is that all firms make zero profits.

Let  $b_k$  be the lowest bid price set by any of the  $-i$  firms for which residual supply is positive, absent  $i$ 's bidding. If  $b_k < B(Q^W)$ , the best response of  $i$  will be to set a price lower than  $B(Q^W)$  but higher than  $b_k$ . (How much higher this price will be depends on the capacities and prices set by the other firms among  $i$ 's competitors, but is not material.) Since  $i$ 's price is below  $B(Q^W)$ ,  $k$  can increase its profit by slightly overbidding  $i$ 's price. This race to the top does, obviously, stop only as the lowest bid price for which the residual supply absent  $i$ 's price setting,  $b_k$ , equals  $B(Q^W)$ . Thus all firms that buy positive quantity must set  $B(Q^W)$ . Moreover, all of the firms that buy positive quantity make zero profit since the equilibrium ask price will be  $p^W$ . Trivially, firms that do not buy any quantity make zero profits.  $\square$

*Proof of Lemma 4:* The proof has three steps. First, it is shown that in equilibrium at most one firm sets the lowest bid price in the support over which firms randomize with positive probability. Therefore, there is a firm who is overbid with probability one when setting this price. In the second step, this fact is used to determine the expected equilibrium revenue of any such firm. Based on the indifference property of mixed strategy equilibria, the expected equilibrium revenue of such a firm equals the revenue it gets when setting the lowest price. Third, having determined this revenue, the firm that nets this revenue is determined.

Step 1: Let  $\Phi_h(b)$  be the equilibrium distribution function of firm  $h$ ,  $h = 1, \dots, n$ . Denote by  $\underline{b}$  the lowest price in the support of any firm. That is,  $\underline{b} \equiv \sup\{b \mid \Phi_h(b) = 0 \ \forall h\}$ . Let  $j$  be one of the firms whose support includes  $\underline{b}$ , i.e.,  $j$ 's strategy is such that  $\Phi_j(\underline{b} + \epsilon) > 0$  for  $\epsilon$  arbitrarily small but positive. At most one firm will set this price with positive probability. To see this, note first that  $\underline{b} < \min\{B(\bar{Q}), B(Q^W)\}$ . Otherwise, we would have  $\underline{b} \geq \min\{B(\bar{Q}), B(Q^W)\}$  implying that we are in a pure strategy Nash equilibrium since prices above  $\min\{B(\bar{Q}), B(Q^W)\}$  are strictly dominated. But downward deviation from  $\min\{B(\bar{Q}), B(Q^W)\}$  has been shown to pay for the largest firm, say  $k$ , because  $\bar{q}_k > r(\bar{q}_{-k})$ , implying that  $k$  must net more than  $Z(\min\{\bar{Q}, Q^W\})\bar{q}_k$  in equilibrium. This implies then that  $j$  buys less than  $\bar{q}_j$  when setting  $\underline{b}$ . Assume  $j$  sets  $\underline{b}$  with positive probability. Then, if another firm set  $\underline{b}$  with positive probability,  $j$  could strictly increase its expected profit by setting a slightly higher price. This leaves both aggregate quantity bought and thus the spread  $j$  gets almost unaffected, but it discontinuously increases the quantity traded by  $j$  and thus increases its expected profit.

Step 2: There is a firm who is overbid with probability one when setting  $\underline{b}$ . Either it sets  $\underline{b}$  with positive probability. Then, no other firm sets  $\underline{b}$  with positive probability, which implies that all other firms set higher prices with probability one. Or no firm sets  $\underline{b}$  with positive probability. Then, obviously any firm whose support includes  $\underline{b}$  will be

overbid with probability one when setting  $\underline{b}$ . Therefore,  $\underline{b}$  must maximize the profit for any such firm under the condition that this firm is overbid with probability one when setting  $\underline{b}$ . That is, for any firm, say  $j$ , who is overbid with probability one when setting  $\underline{b}$ ,  $\underline{b} = \arg \max_b (A(S(b)) - b)(S(b) - \bar{q}_{-j})$ . Otherwise,  $j$  could not be indifferent between setting  $\underline{b}$  and setting  $\arg \max_b (A(S(b)) - b)(S(b) - \bar{q}_{-j})$ , but would prefer the latter. It follows from Lemma 2 that maximizing  $(A(S(b)) - b)(S(b) - \bar{q}_{-j})$  over  $b$  is equivalent to maximizing  $Z(r + \bar{q}_{-j})r$  over  $r$ , which yields the Cournot reaction function  $r(\bar{q}_{-j})$ , implying  $\underline{b} = B(r(\bar{q}_{-j}) + \bar{q}_{-j})$ . Moreover, by the indifference property of mixed strategy equilibria, firm  $j$ 's expected equilibrium profit will be  $R(\bar{q}_{-j}) \equiv Z(r(\bar{q}_{-j}) + \bar{q}_{-j})r(\bar{q}_{-j})$ , which is the Stackelberg follower profit. There is thus a firm  $j$  earning  $R(\bar{q}_{-j})$ . The final thing to be shown is that it is (one of) the largest firm(s).

Step 3: The problem is more complicated than in Kreps and Scheinkman [1983] and no direct equivalent to their calculations in Lemma 5 (d) and (e) can be applied. So as to see that  $j$  is one of the largest firms, note first that for any  $j$  to be a candidate for setting  $\underline{b}$ ,  $\bar{q}_j > r(\bar{q}_{-j})$  is required for otherwise  $B(r(\bar{q}_{-j}) + \bar{q}_{-j}) \geq B(\min[\bar{Q}, Q^W])$ . Second, for any firm  $j$  who is among the candidates for earning revenue  $R(\bar{q}_{-j})$ , there is a bid price  $\bar{b}_j$  satisfying  $\underline{b} < \bar{b}_j < \min[B(\bar{Q}), B(Q^W)]$  such that  $j$  would never overbid  $\bar{b}_j$  if all other firms set  $\bar{b}_j$  with certainty. That is, for every firm  $j$  there is a 'security level bid price'  $\bar{b}_j$  implicitly defined by

$$(5) \quad Z(S(\bar{b}_j)) \equiv \frac{R(\bar{q}_{-j})}{\min\{S(\bar{b}_j), \bar{q}_j\}}.$$

Note that  $Z(S(\bar{b}_j)) > Z(\min\{\bar{Q}, Q^W\}) \Leftrightarrow \bar{b}_j < \min\{B(\bar{Q}), B(Q^W)\}$  follows because in the mixed strategy region  $R(\bar{q}_{-j}) > Z(\min\{\bar{Q}, Q^W\})\bar{q}_j$ . Next, define  $\bar{z}_i \equiv Z(S(\bar{b}_i))$  and bear in mind that  $\bar{z}_i > \bar{z}_j \Leftrightarrow \bar{b}_i < \bar{b}_j$ .

The crucial argument to be made is the following. So as to simplify the illustration, assume that  $\bar{b}_i < \min_{k \neq i} \{\bar{b}_k\}$ , so that  $i$  is the single firm with the lowest security level bid price. Then,  $i$  is the least aggressive firm and will earn  $R(\bar{q}_{-i})$  and all other firms will earn more than  $R(\bar{q}_{-k})$ ,  $k \neq i$ . The reason for this is that any  $k \neq i$  can set  $\bar{b}_i$  (or a slightly higher price) and be sure not to be overbid by  $i$ . But since  $\bar{b}_i < \bar{b}_k$ , any  $k \neq i$  earns more than  $R(\bar{q}_{-k})$ . So as to find out which firm  $i$  has the lowest  $\bar{b}_i$ , we have to determine the dependence of  $\bar{z}_i$  on  $\bar{q}_i$  and on  $\bar{q}_{-i}$ . Obviously, for  $\bar{q}_i > S(\bar{b}_i)$ ,  $\partial \bar{z}_i / \partial \bar{q}_i = 0$ . Otherwise, we have

$$(6) \quad \frac{\partial \bar{z}_i}{\partial \bar{q}_i} = -\frac{R(\bar{q}_{-i})}{\bar{q}_i^2} = -\frac{r(\bar{q}_{-i})Z(r(\bar{q}_{-i}) + \bar{q}_{-i})}{\bar{q}_i^2} < 0$$

$$(7) \quad \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}} = \frac{R'(\bar{q}_{-i})}{\bar{q}_i} = \frac{r(\bar{q}_{-i})Z'(r(\bar{q}_{-i}) + \bar{q}_{-i})}{\bar{q}_i} < 0.$$

The inequality in (6) follows immediately for  $R(\bar{q}_{-i}) > 0$ . As to the inequality in (7), drop arguments and note that

$$(8) \quad R'(\bar{q}_{-i}) = r'Z + rZ'(r' + 1) = r'(Z + rZ') + rZ' = rZ' < 0,$$

where the last equality is due to the fact that by definition of a reaction function the term in parentheses is zero.

For any two firms with  $\bar{q}_i = \bar{q}_h$ , we have  $\bar{z}_i = \bar{z}_h$ . The question is thus whether  $\bar{z}_i$  decreases more than  $\bar{z}_h$  when  $\bar{q}_i$  increases while  $\bar{q}_h$  is kept constant.<sup>25</sup> Put differently, the crucial question is whether  $\bar{z}_i$  decreases more in  $\bar{q}_i$  than in  $\bar{q}_{-i}$ . But for  $\bar{q}_i > r(\bar{q}_{-i})$  (which is a condition that must hold for one  $i$  for firms to be in the mixed strategy region in the first place), the inequality  $\frac{\partial \bar{z}_i}{\partial \bar{q}_i} \geq \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}}$  holds. To see that this is true, note first that quite trivially for  $r = 0$ ,  $\frac{\partial \bar{z}_i}{\partial \bar{q}_i} \geq \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}}$  holds with equality. Second, note that  $\bar{q}_i > r(\bar{q}_{-i})$  is equivalent to  $0 > \bar{q}_i Z' (r(\bar{q}_{-i}) + \bar{q}_{-i}) + Z (r(\bar{q}_{-i}) + \bar{q}_{-i}) \Leftrightarrow \frac{Z'}{\bar{q}_i} < -\frac{Z}{\bar{q}_i^2}$ . For  $r > 0$  (which holds at least for one  $i$  because  $\bar{q}_{-i} < Q^W$  for one  $i$ ) this implies  $-\frac{rZ}{\bar{q}_i^2} \equiv \frac{\partial \bar{z}_i}{\partial \bar{q}_i} > \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}} \equiv \frac{rZ'}{\bar{q}_i}$ . Since  $\frac{\partial \bar{z}_i}{\partial \bar{q}_i} > \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}}$  implies  $\frac{\partial \bar{b}_i}{\partial \bar{q}_i} < \frac{\partial \bar{b}_i}{\partial \bar{q}_{-i}}$ , it follows that  $\bar{b}_i < \bar{b}_j$  if and only if  $\bar{q}_i > \bar{q}_j$ . Applying the argument for any  $i$  and  $j$ , it follows that  $\bar{z}_i \in \max_k \{\bar{z}_k\}$  if and only if  $\bar{q}_i \in \max_k \{\bar{q}_k\}$ . Thus,  $i$  earns  $R(\bar{q}_{-i})$  if and only if  $\bar{q}_i \in \max_k \{\bar{q}_k\}$ .  $\square$

*Proof of Proposition 1:* Existence has been shown in the text. What remains to be proved is the uniqueness of the equilibrium outcome when capacity is costly. Recall that  $r_c(\cdot)$  denotes the Cournot best response function and let  $\bar{\mathbf{q}}^c$  denote the Cournot equilibrium capacities when the costs of capacity are  $C(\bar{q}_i)$  with  $C(0) = 0$ ,  $C'(0) > 0$  and  $C''(\bar{q}_i) \geq 0$  and consider an arbitrary tuple of capacities  $\bar{\mathbf{q}}' \equiv (\bar{q}'_1, \dots, \bar{q}'_n)$ . We are now going to show by contradiction that  $\bar{\mathbf{q}}'$  can be an equilibrium outcome only if  $\bar{q}'_i = \bar{q}^c$  for all  $i = 1, \dots, n$ .

*Preliminary.* As we aim to show that  $\bar{\mathbf{q}}^c$  is the unique Nash equilibrium outcome, no restrictions are imposed on off equilibrium behavior. In particular, if we assume that  $\bar{\mathbf{q}}'$  is an equilibrium outcome and let  $i$  contemplate deviating from  $\bar{q}'_i$ , all firms  $j \neq i$  can be assumed to subsequently take the actions that are worst for  $i$ . If then there is still a profitable deviation for  $i$ , the conclusion will be that  $\bar{\mathbf{q}}'$  is not an equilibrium outcome. Formally, all firms  $j \neq i$  choose  $(a_j, b_j)$  in order to minimize  $\Pi_i(a_i, b_i, a_{-i}, b_{-i})$ , i.e.,

$$\min_{(a_j, b_j) \forall j \neq i} \Pi_i(a_i, b_i, a_{-i}, b_{-i}),$$

whenever  $i$  deviates from the prescribed capacity  $\bar{q}'_i$ . Clearly, there is nothing worse  $i$ 's competitors can do than to slightly overbid  $i$  on the input market by setting  $b_j = b_i + \epsilon$  and underbid him on the output market by setting  $a_j = a_i - \epsilon$ , where  $\epsilon$  is a small positive number.<sup>26</sup> Following a deviation at the capacity stage,  $i$  can choose  $(a_i, b_i)$  so

as to maximize  $\Pi_i(a_i, b_i, a_i - \epsilon, b_i + \epsilon)$ . There is no use buying a quantity that exceeds the quantity sold. Thus,  $i$ 's problem can be written as

$$\max_{b_i} (A(S(b_i)) - b_i) (S(b_i) - \bar{q}'_{-i}),$$

where  $a_i = A(S(b_i))$  is the ask price at which the quantity  $S(b_i)$  clears. As noticed in the text, this is equivalent to choosing a  $q_i$  that maximizes  $Z(q_i + \bar{q}'_{-i})q_i$ , yielding  $r(\bar{q}'_{-i})$  as optimal  $q_i$  and  $R(\bar{q}'_{-i})$  as the expected revenue, provided  $r(\bar{q}'_{-i}) \leq \bar{q}_i$ . Otherwise, the optimal  $q_i$  is  $\bar{q}_i$  and expected revenue is  $Z(\bar{q}_i + \bar{q}'_{-i})\bar{q}_i$ .

The rest is straightforward. Assume contrariwise that  $\bar{\mathbf{q}}'$  is such that for at least one firm  $\bar{q}'_i \neq \bar{q}^c$  but that it is an equilibrium outcome. There are three cases. First, if  $\bar{\mathbf{q}}'$  satisfies  $\bar{q}'_j \leq r(\bar{q}'_{-j})$  for all  $j$  with strict inequality for one firm, at least one firm, say  $i$ , is off its best response  $r_c(\bar{q}'_{-i})$  because  $\bar{\mathbf{q}}' \neq \bar{\mathbf{q}}^C$ . Thus, the deviation to  $\bar{q}_i = r_c(\bar{q}'_{-i})$  would pay for  $i$  since  $Z(r_c(\bar{q}'_{-i}) + \bar{q}'_{-i})r_c(\bar{q}'_{-i}) - C(r_c(\bar{q}'_{-i})) > Z(\bar{q}_i + \bar{q}'_{-i})\bar{q}_i - C(\bar{q}_i)$  by the construction of  $r_c(\cdot)$ . Second, assume that  $\bar{\mathbf{q}}'$  is such that on the equilibrium path at least two firms set  $b_i = a_i = p^W$ , implying that  $\bar{q}_i \geq Q^W$  for at least two firms. Then, all firms net zero revenue in the alleged equilibrium. However, since capacity is costly, at least the two firms with  $\bar{q}_i \geq Q^W$  make negative profits, which is strictly less than what they would get by setting  $\bar{q}_i = 0$ . Third, assume that  $\bar{\mathbf{q}}'$  induces a mixed strategy equilibrium in the pricing subgames, i.e.,  $\bar{\mathbf{q}}'$  is such that  $\bar{q}'_i > r(\bar{q}'_{-i})$  for at least one (of the largest) firm(s). Setting  $\bar{q}'_i$ , firm  $i$ 's expected profits are  $R(\bar{q}'_{-i}) - C(\bar{q}'_i)$ , which clearly is smaller than  $R(\bar{q}'_{-i}) - C(r(\bar{q}'_{-i}))$ , which is what  $i$  would get by deviating to  $\bar{q}_i = r(\bar{q}'_{-i})$ . Thus,  $\bar{\mathbf{q}}'$  cannot be an equilibrium outcome.  $\square$

*Proof of Corollary 2:* Since on the input market all market makers set market clearing bid prices, we can directly begin with the output market equilibrium analysis. When setting  $a_i \geq A(\bar{Q})$  while all others set  $a_{-i} = A(\bar{Q})$ , aggregate quantity demanded is  $D(a_i) \leq \bar{Q}$ . Therefore, the market clearing bid price on the input market will be  $B(D(a_i))$  implying that the spread  $i$  earns on its quantity traded is  $a_i - B(D(a_i))$ . Since  $a_i \geq a_{-i} = A(\bar{Q})$ , quantity traded by  $i$  will be  $D(a_i) - \bar{q}_{-i} \leq \bar{q}_i$ . Given capacities  $\bar{Q} < Q^W$ , the profit for market maker  $i$  when setting  $a_i \geq A(\bar{Q})$  while all other market makers set the market clearing ask price  $a_{-i} = A(\bar{Q})$  is thus

$$(9) \quad \Pi_i(a_i, a_{-i}) = (a_i - B(D(a_i)))(D(a_i) - \bar{q}_{-i}).$$

Therefore, given capacities the maximization problem of  $i$  is equivalent to that of a market maker  $i$  who faces market clearing prices on both sides and trades quantity  $q_i \equiv D(a_i) - \bar{q}_{-i} \leq \bar{q}_i$  when all others trade aggregate quantity  $\bar{q}_{-i}$ . That is,  $i$ 's problem

is

$$(10) \quad \max_{q_i} \Pi_i(q_i, \bar{q}_{-i}) = (A(q_i + \bar{q}_{-i}) - B(q_i + \bar{q}_{-i}))q_i \equiv Z(q_i + \bar{q}_{-i})q_i,$$

from where it becomes apparent that this problem is identical to the one previously studied. In particular, the region of pure strategy equilibria (i.e., the region where all market makers set the capacity clearing ask price  $a_i = A(\bar{Q})$ ) will be given by capacities such that  $\bar{q}_i \leq r(\bar{q}_{-i})$  for all  $i$ . Moreover, it follows that in the region of capacities with mixed strategies, the largest firm, say  $k$ , will have an expected profit of  $R(\bar{q}_{-k}) \equiv r(\bar{q}_{-k})Z(r(\bar{q}_{-k}) + \bar{q}_{-k})$ .  $\square$

*Proof of Proposition 2:* Recall that  $r(x) \equiv \frac{Z(r(x)+x)}{-Z'(r(x)+x)}$  is the Cournot best response function for a firm facing the spread  $Z(\cdot)$  with zero marginal cost, where  $x$  is the quantity traded by all other firms, and that  $0 > r' > -1$ . Note also that  $r(0)$  is the quantity traded by a monopolistic market maker. Furthermore, for  $x \geq Q^W$ ,  $r(x) = 0$ . Denote by  $s(x) \equiv \frac{A(s(x)+x)}{-A'(s(x)+x)}$  the Cournot best response function of a firm with zero marginal cost facing the demand  $A(\cdot)$  when all other firms supply quantity  $x$ . Since  $A(\cdot)$  is weakly concave,  $0 > s' > -1$  follows. Note that  $s(0) \equiv Q^M < Q^W$  is the quantity sold by a monopoly facing zero costs. To see that  $s(x) > r(x)$  holds for any  $x \leq Q^W$ , assume to the contrary  $s(x) \leq r(x)$ . But this implies  $\frac{A(s(x)+x)}{-A'(s(x)+x)} \leq \frac{Z(r(x)+x)}{-Z'(r(x)+x)}$ , which is a contradiction since  $Z(Q) < A(Q)$ ,  $Z'(Q) < A'(Q) < 0$  and  $Z'' \leq 0$ . Moreover, the Cournot equilibrium quantities for firms facing the spread function  $Z(\cdot)$  and the ask price function  $A(\cdot)$  with zero marginal costs, respectively, are given by  $q_Z \equiv r((n-1)q_Z)$  and  $q_A \equiv s((n-1)q_A)$ . That these quantities are unique follows by noting that  $x < r((n-1)x)$  and  $x < s((n-1)x)$  for  $x = 0$ . Since both right-hand sides decrease in  $x$  while both left-hand sides increase in  $x$ , there is a unique fixed point. Finally to see that  $q_Z < q_A$ , plug  $q_Z$  into  $s(\cdot)$ . Since  $s(x) > r(x)$  holds,  $s((n-1)q_Z) > q_Z$  follows. Note also that  $s(0) < Q^W$  because demand is inelastic for some  $Q < Q^W$ .

Since setting market clearing prices is an equilibrium given  $\bar{q}_i \leq r(\bar{q}_{-i})$  for all  $i$ , which implies  $\bar{q}_i < s(\bar{q}_{-i})$ , it follows that Cournot is a Nash equilibrium outcome. To check whether Cournot is the outcome of a subgame perfect equilibrium, assume that every firm  $i \neq j$  sets the Cournot capacity  $\bar{q}_i = q^C$  and let  $j$  deviate. As before, only upward deviations need to be considered since for smaller capacities firms will set market clearing prices in the following subgames. A sufficient condition for the output market equilibrium to be in pure strategies is that  $\bar{q}_i \leq s(\bar{q}_{-i})$  for all  $i$ . If the constraint is satisfied for the largest firm, then it is satisfied for any other. Denote  $\bar{x} \equiv s((n-1)q_Z)$ , where it will be recalled that  $q_Z$  is the Cournot equilibrium quantity for the spread  $Z(\cdot)$  when costs are zero, satisfying  $q_Z \geq q^C$ . That is,  $\bar{x}$  is such that if the largest firm has



stocks not exceeding  $\bar{x}$ , while every other firm has  $q_Z$  or less in stock, then in the output market equilibrium all firms will set the market clearing price. Note that because  $s' < 0$  and  $q_Z < q_A$ ,  $\bar{x} > q_A$ . Given that prices are market clearing,  $q^C$  is a best response to all other firms' playing  $q^C$ , and given market clearing prices on the output market, it has been shown that deviation does not pay, since in the (possibly mixed strategy) equilibrium of the input market, the deviating firm will earn the Stackelberg follower profit. But for  $\bar{q}_j \leq \bar{x}$  and  $\bar{q}_i = q^C$  for all  $i \neq j$ ,  $\bar{q}_h \leq s(\bar{q}_{-h})$  for all  $h = 1, \dots, n$ . Therefore, the equilibrium ask price will be market clearing and deviation will not pay. If capacity is costly, costs of the excess capacity have to be borne, making expected equilibrium profits strictly smaller.  $\square$

*Proof of Proposition 3:* We first state and prove two lemmas:

*Lemma 5.* If  $\bar{q}_i \leq r(\bar{q}_{-i})$  for all  $i$ ,  $b_i = B(\bar{Q})$  and  $a_i = A(\bar{Q})$  is the unique equilibrium of the simultaneous price setting subgame. If  $\bar{q}_j > r(\bar{q}_{-j})$  for at least one  $j$ , there is no pure strategy equilibrium in the pricing subgame in which at least one firm nets a positive revenue.

*Proof:* If all  $j \neq i$  set market clearing prices, then the optimal deviation of  $i$  is  $a_i = A(r(\bar{q}_{-i}) + \bar{q}_{-i})$  and  $b_i = B(r(\bar{q}_{-i}) + \bar{q}_{-i})$ . For  $\bar{q}_i < r(\bar{q}_{-i})$ , though,  $A(r(\bar{q}_{-i}) + \bar{q}_{-i}) < A(\bar{Q})$  and  $B(r(\bar{q}_{-i}) + \bar{q}_{-i}) > B(\bar{Q})$ . These prices being dominated by setting the market clearing prices, deviation does not pay off. Uniqueness follows along the previous lines. The second part is proved by contradiction. That is, assume to the contrary that  $\bar{q}_j > r(\bar{q}_{-j})$  for at least one  $j$  holds and that at least one firm, say  $i$ , nets a positive revenue in this equilibrium. Denote by  $(a^*, b^*)$  the price pair set by  $i$  in this hypothesized equilibrium. For  $i$  to make positive profits at these prices,  $a^* > b^*$  must hold. Moreover,  $a^* > A(\bar{Q})$  and  $b^* < B(\bar{Q})$  must hold because bid and ask prices satisfying  $b > B(\bar{Q})$  and  $a < A(\bar{Q})$  are dominated by the capacity clearing prices and because deviation from the capacity clearing prices pays for  $j$ . But since aggregate capacity does not clear at  $(a^*, b^*)$ , at least one firm will get rationed if it sets  $(a^*, b^*)$ . Therefore, by slightly undercutting  $a^*$  and slightly overbidding  $b^*$  this firm could strictly increase its profit. But then,  $i$ 's setting  $(a^*, b^*)$  cannot be a best response. Hence,  $(a^*, b^*)$  cannot be set in a pure strategy equilibrium. Thus, there is no pure strategy equilibrium in which at least one firm nets positive revenue.  $\square$

If  $\bar{q}_j > r(\bar{q}_{-j})$  for at least one  $j$ , there is thus either a pure strategy equilibrium with zero revenue for all firms, or the equilibrium is in mixed strategies.<sup>27</sup>

*Lemma 6.* Let  $i$  be (one of) the largest firm(s) when capacities are such that there is no pure strategy equilibrium. Then  $i$ 's expected revenue in any mixed strategy equilibrium

is  $R(\bar{q}_{-i}) \equiv r(\bar{q}_{-i})Z(r(\bar{q}_{-i}) + \bar{q}_{-i})$ .

*Proof:* The proof is based on a series of claims. Let  $\bar{a}$  ( $\underline{b}$ ) denote the upper (lower) bound of the support of ask (bid) prices in the mixed strategy equilibrium.

Claim 1:  $\underline{b}$  and  $\bar{a}$  can be set at most by one firm with positive probability in equilibrium. Proof: Suppose not. Then, as in the proof of Lemma 4 either of these firms could strictly increase its expected profit by setting a slightly higher bid or slightly lower ask price with positive probability instead. This would increase its expected quantity traded while leaving the spread it earns largely unaffected, thereby increasing its expected profit.

Claim 2:  $\bar{a} = A(S(\underline{b}))$ . Proof: Since bid prices  $b > B(\bar{Q})$  are dominated, aggregate quantity bought will be weakly larger than  $S(\underline{b})$  implying that the market clearing ask price will be weakly smaller than  $A(S(\underline{b}))$ . Since by claim 1 there is a firm that is underbid with probability one when setting  $\bar{a}$ ,  $\bar{a}$  must maximize this firm's expected profit, conditional on being underbid with probability one on the output market. Since aggregate stock is weakly larger than  $S(\underline{b})$  and because demand is price elastic,  $\bar{a} \leq A(S(\underline{b}))$ . Now consider a firm who is overbid with probability one when setting  $\underline{b}$ . In this case, its quantity bought can be zero in case the capacities of all other firms are larger than  $S(\underline{b})$ . In this case, though, the ask price it sets is immaterial. In the other case, its quantity bought is positive, and  $A(S(\underline{b}))$  is the market clearing and profit maximizing for this firm. Thus,  $\bar{a} = A(S(\underline{b}))$ .

Claim 3: Let  $i$  be one of the firms who is overbid with probability one when setting  $\underline{b}$ . Then,  $\underline{b} = B(r(\bar{q}_{-i}) + \bar{q}_{-i})$  and  $\bar{a} = A(r(\bar{q}_{-i}) + \bar{q}_{-i})$  and  $i$ 's expected revenue is  $R(\bar{q}_{-i})$ . Proof: As soon as  $\underline{b} = B(r(\bar{q}_{-i}) + \bar{q}_{-i})$  is shown,  $\bar{a} = A(r(\bar{q}_{-i}) + \bar{q}_{-i})$  follows from claim 2 and the claim about expected revenue from the indifference property of mixed strategy equilibria. But  $\underline{b} = B(r(\bar{q}_{-i}) + \bar{q}_{-i})$  is the profit maximizing bid price of firm  $i$  when it faces a market clearing ask price. Therefore, the claim is proved.

Claim 4: Firm  $i$  is (one of) the largest firm(s). Proof: Any smaller firm, say  $j$ , could set  $\bar{b}_i$  (as defined in the proof of Lemma 4) and  $\bar{a}_i \equiv A(S(\bar{b}_i))$ , thereby making more profit than  $R(\bar{q}_{-j})$ , while  $i$  would make less profit by slightly overbidding  $b_j = \bar{b}_i$  on the input and slightly underbidding  $a_j = \bar{a}_i$  on the output market.  $\square$

Given that all other market makers set Cournot capacities, unilateral deviation does not pay off. If a firm sets a smaller capacity, equilibrium prices will be market clearing, and given market clearing prices, the best response is setting the Cournot capacity. When increasing capacity, the price setting equilibrium will eventually be in mixed strategies, in which case the largest firm earns the Stackelberg follower profit. Uniqueness for costly capacity follows from the fact that for any firm, conditional on being the largest firm, its best response given capacity  $x$  of other firms is uniquely given by  $r_c(x)$ .  $\square$

## Notes

<sup>1</sup>See, e.g., Stahl [1988], Gehrig [1993], Spulber [1996a], Fingleton [1997] and Rust and Hall [2003]. According to a conservative estimate by Spulber [1996b], market making and intermediation activities accounted for more than a quarter of U.S. GDP in 1994.

<sup>2</sup>Beside the differences in proof strategies, the only major result of Kreps and Scheinkman which we cannot show to hold for more than two market makers is that there are no equilibria with mixed strategies at the capacity stage. That said, there is no proof that such equilibria exist. A second qualification we should mention is that some of our results are valid under the provision that demand is price elastic at or above the Walrasian price. Clearly, this is an assumption Kreps and Scheinkman [1983] can dispense with.

<sup>3</sup>For example, if demand is inelastic at the Walrasian price in Stahl's model, then the equilibrium will be non-Walrasian; see Section IV below.

<sup>4</sup>The main difference between our model as well as the models of Stahl, Gehrig, Fingleton and Rust and Hall and the one of Spulber [1996a] is that in the former models, market makers set publicly observable prices, whereas the prices in the latter model are private information.

<sup>5</sup>The only models with product differentiated market makers we are aware of are Spulber [1999, Ch.3] and Loertscher [2007].

<sup>6</sup>The time structure of this section also corresponds to the one analyzed by Yannelle [1996] in her Game 2. The time structure with forward contracts we analyze in Section IV.(i) is analogous to her Game 1.

<sup>7</sup>To be precise, there are different types of capacity used by market makers. On the one hand, they need to have the capacity to store, transport and sell the good in order to be able to compete on the output market. On the other hand, they must also have the capacity to buy the good, residing, e.g., in the number of clerks or salesmen employed. In general, these different kinds of capacities may involve different costs. For our analysis to be exactly correct, it is required that market makers do not set different capacities at different levels, i.e., they do not set, say, a capacity to sell that exceeds their capacity to buy.

<sup>8</sup>The assumption of prohibitive production cost beyond capacity is quite standard in the literature on capacity constrained product market competition. An exception is Bocard and Wauthy [2000] who consider the possibility that the cost of production beyond capacity may be less than prohibitively large, though it is still larger than below capacity.

<sup>9</sup>This is equivalent to assuming  $A'' < 2(A')^2/A$  for  $Q < \hat{Q}$ . A sufficient condition for  $\varepsilon_a(a) \leq -1$  for any  $a \geq p^W$  is  $\varepsilon'_a(a) < 0$  in addition to  $\varepsilon_a(p^W) \leq -1$ . This is the case if  $D$  satisfies  $D'' < -D'/a + (D')^2/D$ . Under the stronger assumption  $D'' < -2D'/a$  that we make, the Cournot best response  $s$ , given inverse demand  $A$ , satisfies  $s' > -1$ , which is used below.

<sup>10</sup>So,  $a_{-i} \equiv (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$  and  $b_{-i} \equiv (b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_n)$ . If all firms other than  $i$  set the same prices, we will denote these as  $a_{-i}$  and  $b_{-i}$ , slightly abusing notation. In contrast,  $q_{-i} \equiv \sum_{j \neq i} q_j$  and  $\bar{q}_{-i} \equiv \sum_{j \neq i} \bar{q}_j$  denote sums. Boldface is used to denote tuples like  $\bar{\mathbf{q}}_{-i} \equiv (\bar{q}_1, \dots, \bar{q}_{i-1}, \bar{q}_{i+1}, \dots, \bar{q}_n)$ .

<sup>11</sup>Madden [1998] has shown for a large family of rationing schemes, including proportional and efficient rationing, that under elastic demand the unique equilibrium price is the market or capacity clearing price. Allen and Hellwig [1986] were the first to report the result for proportional rationing.

<sup>12</sup>See, e.g., Levitan and Shubik [1972], Kreps and Scheinkman [1983], Osborne and Pitchik [1986], Deneckere and Kovenock [1992,1996], Ghemawat [1997], Allen, Deneckere, Faith and Kovenock [2000], Bocard and Wauthy [2000] and Reynolds and Wilson [2000].

<sup>13</sup>For example, Graddy [1995, p.77] finds that in the Fulton fish market in New York City "quantity received by the dealer is readily observable by everyone" [see also Graddy, 2006].

<sup>14</sup>The retailer interpretation deserves two comments. First, as pointed out by one referee, in procurement retailers often use more complicated contracts than linear prices. In this case, our assumption of bid price setting cannot be taken literally but should be seen as a short-cut to competition in more complicated mechanisms. It should also be noted, though, that in the present model, where each producer has measure zero and sells at most one unit of a homogenous good, there is little scope for non-linear prices. Second, according to our model retailers buy and sell a homogenous good (composed of many smaller items such as tooth paste and beverages). Consequently, retailer capacity is given by the sale space, which is observable, and not by the stock of, say, tooth paste, which arguably is not.

<sup>15</sup>Note that this holds both for proportional and efficient rationing. Under proportional rationing, the residual demand function for  $i$  when all others set a non-market clearing price  $a_{-i}$  is  $D(a) \frac{D(a_{-i}) - q_{-i}}{D(a_{-i})}$  for  $a > a_{-i}$ . Obviously, the price elasticity of the residual demand function equals the elasticity of the demand function  $D(a)$ . Under the same conditions the residual demand function under efficient rationing is  $D(a) - q_{-i}$ , the ask price elasticity of which is  $D'(a) \frac{a}{D(a) - q_{-i}}$ , which is strictly smaller (i.e., greater in absolute terms) than the elasticity of the demand function  $D(a)$ .

<sup>16</sup>In this figure, linearity merely serves the purpose of simplification.

<sup>17</sup>Clearly, the strategy profile  $(\bar{q}_i = q^C, b_i = B(\bar{Q}), a_i = A(\bar{Q}))$  for all  $i$  is not subgame perfect: If, say, aggregate quantity bought  $Q$  is smaller than  $\bar{Q}$ ,  $a_i = A(\bar{Q})$  is dominated by  $a_i = A(Q)$ .

<sup>18</sup>A necessary condition for multiple equilibria is that in addition to  $\bar{q}_{-i} \geq Q^W$  for all  $i$ ,  $\bar{q}_{-i} - \bar{q}_j \geq Q^W$  holds for some  $j$  and  $i$ . In this case,  $j$  can set or randomize over any bid price  $b \leq B(Q^W)$ , provided the aggregate capacity of all firms other than  $j$  who set  $B(Q^W)$  is at least  $Q^W$ . Because  $\bar{q}_{-i} - \bar{q}_j \geq Q^W$ , firm  $i$  cannot gain by deviating from  $B(Q^W)$  if all other firms but  $j$  set  $B(Q^W)$ .

<sup>19</sup>Note also that under this assumption  $\bar{q}_{-i} < Q^W$  and  $\bar{q}_i > r(\bar{q}_{-i})$  holds for  $i$ , where  $i$  is (one of) the largest firm(s) and where  $i = k$  is possible.

<sup>20</sup>This assumption is appropriate if a firm's capacity is limited by, say, its sales force because then excess demand can simply not be served. As observed by one referee, it would otherwise be possible to support higher than capacity clearing ask prices in equilibrium.

<sup>21</sup>A similar observation is made by Yanelle [1996].

<sup>22</sup>Without this somewhat peculiar tie-breaking rule, the game has no equilibrium; see Stahl [1988, p.195].

<sup>23</sup>Ju, Linn and Zhu [2006] introduce oligopolistic competition between market makers in a model à la Rust and Hall [2003], but do not consider mixed strategies in prices.

<sup>24</sup>This price is the optimal price for a firm who is certain to be the lowest price bidder on the input market, which is a property that will be used in the proof of Lemma 4 below.

<sup>25</sup>For any  $k \neq i$ ,  $\frac{\partial \bar{q}_{-i}}{\partial \bar{q}_k} = 1$  implying  $\frac{\partial \bar{z}_i}{\partial \bar{q}_k} = \frac{\partial \bar{z}_i}{\partial \bar{q}_{-i}}$ .

<sup>26</sup>Of course, under our assumption of efficient rationing, whether they underbid and overbid slightly

or by a larger margin is immaterial.

<sup>27</sup>The existence of a mixed strategy equilibrium can be shown using the arguments by Dasgupta and Maskin [1986] in case there is no pure strategy equilibrium.

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