Make and Buy: Outsourcing, Vertical Integration, and Cost Reduction*

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Abstract

Globalization reshaped supply chains and the boundaries of firms in favor of outsourcing. Now, even vertically integrated firms procure substantially from external suppliers. To study procurement and the structure of firms in this reshaped economy, we analyze a model in which integration grants a downstream customer the *option* to source internally. Integration is advantageous because it allows the customer sometimes to avoid paying markups, but disadvantageous because it discourages investments in cost reduction by independent suppliers. The investment-discouragement effect more likely outweighs the markup-avoidance effect if the upstream market is more competitive, as is so in a more global economy.

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1 Introduction

A dramatic transformation of American manufacturing occurred at the end of the twentieth century, away from in-house production toward outsourcing (Whitford, 2005). By the 1990's, outsourcing was widespread, to the point that even vertically integrated firms relied heavily on independent suppliers (Atalay, Hortaçsu, and Syverson, 2014). This transformation increasingly went hand in hand with offshoring. Make-and-buy strategies, whereby firms procure inputs both internally and externally, often from foreign sources, became prevalent (Magyari, 2017).

Accounting for a make-and-buy sourcing strategy requires embedding vertical integration in a multilateral supply setting. To do so, we consider a model that gives an integrated firm the option to source internally when this is more cost effective than sourcing from an independent supplier. Our model assumes that potential input suppliers invest unobservable effort to develop and propose acceptable cost-reducing designs to meet an input requirement of a customer, and, as a result of this design effort, the supplier gains private information about its cost of producing the input.

Automobile manufacturing exemplifies such a procurement environment. Discussing automobile manufacturing, Calzolari, Felli, Koenen, Spagnolo, and Stahl (2015) describes a dichotomy between production and design consistent with our model:[1]

> In series production, suppliers work with existing blueprints and completely designed (or existing) tools to produce the part in question. The product and services can be clearly specified through contracts, determining in detail, for example, acceptable failure rates and delivery conditions. None of this is possible in the model-specific development phase. While the desired functionality of a part can be described, highly complex interfaces with other parts (often under development simultaneously) cannot be specified *ex ante.* Blueprints for the part do not exist at the beginning of the design phase; indeed they are the outcome of such a phase.

Magyari (2017) reports a frequency of make-and-buy procurement among transport equipment manufacturers above 70%.

Our analysis reveals the following benefits and costs of vertical integration. On the one hand, there are rent-seeking and possible efficiency advantages of internal sourcing from avoiding a markup paid to an independent supplier. This "markup avoidance effect" shifts rents away from the lowest-cost independent supplier by distorting the sourcing decision in favor of internal supply, but may also improve efficiency because the project is pursued whenever its value exceeds the cost of internal sourcing. On the other hand, vertical integration has a disadvantageous "discouragement effect" on the investments of

[1]Calzolari et al. (2015) presents a model that supposes each supplier's investment in effort at the design stage ultimately is observed by the customer, and consequently can be enforced by a relational contract, and furthermore can be compensated by the combination of a fixed payment and the expected information rents earned at the production stage. In contrast, our model supposes that design effort is unobserved and rewarded entirely by information rents.

the independent suppliers.[2] Because the procurement process is tilted in favor of internal sourcing, independent suppliers are less inclined to make cost-reducing investments. Furthermore, while the integrated firm compensates for the discouragement effect by increasing its own *ex ante* investment, and thereby might improve the minimum cost distribution, this investment reallocation is socially and privately costly. The customer optimally divests its internal supply division to commit to a level playing field if the costs of investment discouragement outweigh the benefits of markup avoidance.

Our analysis also suggests that greater upstream competition disfavors vertical integration. A parametric version of the model with exponential cost distributions and quadratic effort costs yields two comparative static variants of this hypothesis. First, an increase in the number of symmetric upstream suppliers reduces the rents of the independent sector, making the markup avoidance benefit of vertical integration less compelling. Second, holding constant the number of suppliers, less cost uncertainty reduces *ex post* supplier heterogeneity, similarly squeezing markups and reducing rents. That more outsourcing opportunities encourages vertical divestiture is broadly consistent with hand-in-hand trends toward outsourcing and offshoring. That divestiture is more attractive in a less uncertain environment is consistent with the idea that vertical divestiture occurs in mature industries for which the prospects for dramatic cost reduction are scant.

Our theory builds on previous literature while differing in significant ways. Vertical integration in our model effectively establishes a preferred supplier with a right of first refusal, that is, a supplier who tenders a bid after all independent suppliers have submitted their bids. The allocative distortions from a preferred supplier are similar to those in the first-price auction model of Burguet and Perry (2009). However, due to endogenous investments in cost reduction in our model, in equilibrium the preferred supplier has a more favorable cost distribution than the independent suppliers.[3]

This preferred supplier interpretation relates to an older industrial organization literature that views vertical integration as a response to vertical externalities. This literature, surveyed by Perry (1989), has different strands. For example, backward vertical integration might be motivated by a downstream firm's incentive to avoid paying above-cost input prices. In the double markups strand, vertical integration of successive monopolies improves efficiency by reducing the final price to the single monopoly level. In the variable proportions strand, a non-integrated firm inefficiently substitutes away from a monopoly-provided input, and vertical integration corrects this input distortion. In our

[2]A further manifestation of the discouragement effect might be that suppliers exit if fixed costs are avoidable, which we abstract away from by assuming a fixed number of suppliers.

[3]Burguet and Perry (2009) assumes fixed identical cost distributions. Lee (2008), Thomas (2011), and Burguet and Perry (2014) study the right of first refusal (or vertical merger) in cases of two suppliers with exogenous asymmetric cost distributions, whereas we endogenize the asymmetry with unobservable investments. Riordan and Sappington (1989), Bag (1997) and Che, Iossa, and Rey (2015) study favoritism in auction design to incentivize unobservable investments by suppliers, assuming, contrary to our model, that the buyer can commit to a procurement mechanism prior to investments. Arozamena and Cantillon (2004) study procurement auctions preceded by observable investments, and Tan (1992) compares first-price and second-price procurement auctions preceded by unobservable cost-reducing investments. Neither consider preferred providers or vertical integration.

model, while alternative suppliers offer substitute inputs, there is no input distortion under non-integration because upstream market power is symmetric. Vertical integration, on the other hand, creates an input distortion by favoring a less efficient source.

Our emphasis on multilateral supply relationships is also reminiscent of Bolton and Whinston (1993).[4] The Bolton-Whinston model, in the same spirit as Grossman and Hart (1986), assumes an efficient bargaining process under complete information to allocate scarce supplies.[5] Vertical integration creates an "outside option" of the bargaining process that for given investments only influences the division of rents. In contrast, our model features incomplete information about costs, and, for given investments, vertical integration affects the sourcing decision as well as the division of rents. In the Bolton-Whinston model, the integrated downstream firm overinvests to create a more powerful outside option when bargaining with independent customers, but the *ex post* allocation decision is efficient conditional on investments. In contrast, in our model the rent-seeking advantage of vertical integration leads to *ex post* sourcing distortions, which in turn distorts *ex ante* investments relative to the first best.[6] Consequently, the two models give rise to starkly different conclusions. For the case that corresponds to the unit-demand model featured in our setup, Bolton and Whinston (1993) finds that non-integration is never an equilibrium market structure although it always is socially optimal. In contrast, in our model non-integration need not be socially optimal and can be an equilibrium outcome regardless of whether it is, depending on the competitiveness of the upstream market.[7]

The rise of outsourcing and the trend toward offshoring give renewed salience to the puzzle of selective intervention posed by Williamson (1985).[8] If the vertically integrated firm simply replicated the way it produced before integrating, the profit of the integrated entity would just equal the joint profit of the two independent firms. However, it can do strictly better because it can now avoid paying the markup for procuring from outside

[4]See Farrell and Katz (2000) for a related model of integration in systems markets. Integration also occurs in a multilateral setting in the models of Riordan (1998) and Loertscher and Reisinger (2014), which focus on possible foreclosure effects of vertical integration.

[5]The property rights literature based on the Grossman-Hart model typically views vertical integration as a "make or buy" decision in a bilateral setting, focusing on how agency problems inside an integrated firm compare with contracting problems across separate firms; see Hart (1995). Hart and Moore (1990) provides a multi-lateral generalization of the Grossman-Hart setup, but like Grossman and Hart (1986) and Bolton and Whinston (1993) maintains the assumption that bargaining is efficient.

[6]It is tempting to interpret our model as demonstrating a variant of the Grossman-Hart logic such that vertical integration *creates* a hold-up problem for independent suppliers; see Allain, Chambolle and Rey (2016) for a recent model along these lines. This interpretation, however, ignores that our model features private information about costs and inefficient sourcing after integration, with different implications for how vertical integration influences investment incentives compared to the complete-information efficient-sourcing framework of Grossman and Hart (1986).

[7]See Proposition 5.2 in Bolton and Whinston (1993), where $\lambda = 1$ corresponds to our unit demand case, and Footnote 18 below.

[8]Crémer (2010, p.44) summarizes the puzzle as follows: "Simplifying to the extreme, [Williamson] asked the following question: it seems that nothing would prevent the owner of a firm from purchasing one of its suppliers and then to tell the managers of what have now become two units of the same firm to behave as if the merger had not taken place. This would prove that, at its worse, vertical integration is never worse than vertical disintegration, which is clearly counterfactual."

suppliers whenever the cost of internal supply is below the lowest bid of the outside suppliers. In this sense, the vertically integrated firm's flexibility to change its behavior after integration is to its short-run benefit. But it raises the question why vertical integration would not always be profitable in our model. The answer is that, because the integrated firm favors internal sourcing, the independent suppliers incentives to invest in cost reduction are diminished. We show that this investment discouragement effect can be strong enough to offset the benefits from vertical integration.

Lastly, the multilateral setting at the heart of our model suggests a formalization of Stigler (1951)'s interpretation of Adam Smith's dictum that "the division of labor is limited by the extent of the market." In our setup, if the extent of the market, measured by the number of suppliers, is small, there is a strong incentive for the customer to integrate vertically, and to source internally only when profitable. As the extent of market increases, the incentive for internal sourcing diminishes, and the division of labor, measured by the frequency of outsourcing, increases.

The remainder of our paper is organized as follows. Section 2 lays out the model. Section 3 provides the equilibrium analysis. Extensions are briefly discussed in Section 4. Section 5 concludes. The Online Appendix contains the proofs and background for the extensions.

2 The Model

There is one downstream firm, called the customer, who demands a fixed requirement of a specialized input for a project. We assume that the customer's demand is inelastic. More precisely, we suppose the customer has a willingness to pay v, and consider the limit as v goes to infinity. This formulation captures, in the extreme, the idea that the value of the downstream good is large relative to the likely cost of the input. In Online Appendix A, we extend the model to allow for elastic demand.

There are n upstream firms, called suppliers, capable of providing different versions of the required input, indexed $i = 1, .., n$. Supplier i makes a non-contractible and unobservable investment in designing the input by exerting effort x_i. The cost of effort is given by the function $\Psi(x_i)$, satisfying $\Psi(0) = 0$, $\Psi'(0) = 0$, $\Psi'(x_i) > 0$ for $x_i > 0$ and $\Psi''(x_i) > 0$. We also use the notation $\psi(x) \equiv \Psi'(x)$. *Ex ante*, that is, prior to the investment in effort, a supplier's cost of producing the input is uncertain. *Ex post*, that is, after the investment, every supplier privately observes his cost realization:

$$c_i = \frac{1}{\mu} y_i - x_i + \beta, \tag{1}$$

where $\mu > 0$ and β are constants and supplier type y_i is the realization of a random variable with cumulative distribution function F with a smooth probability density function, defined on a support $[\underline{y}, \overline{y}]$, whose mean and variance are normalized to 1. Thus, more effort shifts the mean of the supplier's cost distribution downward, and we assume that supplier types y_i are independently distributed.[9]

[9]This is the same as in the Laffont and Tirole (1993) model of procurement, except that the realized cost is the private information of the supplier.

The initial market structure is such that the customer and supplier 1 are vertically integrated. The common owner has the option of offering the supply unit for sale to an outside bidder. The outsider has no private information and is willing to pay any price that allows him to break even.

If such a divestiture occurs, the subsequent market structure consists of the non-integrated customer and n independent suppliers. If no divestiture occurs, the market structure consists of the customer who is vertically integrated with supplier 1 and $n-1$ independent suppliers. Knowing the market structure, all suppliers make non-contractible investments in cost reduction. Once costs are realized, all independent suppliers submit a bid, and the customer sources from the cheapest supplier, which, under vertical integration, may mean sourcing from supplier 1 at cost.

Summarizing, we analyze the following three-stage game:

Stage 1: The common owner of the customer and supplier 1 has the option to make a take-it-or-leave-it offer to an outsider to acquire supplier 1. If the common owner exercises this option and the outsider accepts, then supplier 1 becomes an independent supplier and the vertical market structure is non-integration. Otherwise, it is integration.

Stage 2: Suppliers $i = 1, .., n$ know the vertical market structure and simultaneously make non-negative investments x_i, incurring costs $\Psi(x_i)$. Each supplier i makes his investment prior to the realization of his type y_i. Once y_i is realized, the cost c_i is determined according to (1).

Stage 3: Each supplier knows his own cost but not the costs of the other suppliers. The customer solicits bids from the suppliers in a reverse auction. Under non-integration, each supplier bids a price b_i. The bids $\mathbf{b} = (b_1, .., b_n)$ are simultaneous. The customer selects the low-bid supplier. Under integration, the $n-1$ independent suppliers $i = 2, .., n$ simultaneously each submit a bid b_i. The customer sources internally if $c_1 \leq \min\{\mathbf{b}_{-1}\}$, and purchases from the low-bid independent supplier if $\min\{\mathbf{b}_{-1}\} < c_1$, where $\mathbf{b}_{-1}$ is the collection of bids of all suppliers other than 1.

The three-stage game is solved backward. We first solve for the Bayesian Nash equilibrium of Stage 3, and then solve for the Nash equilibrium of the Stage 2 subgame. Finally, we analyze the incentives to divest for the common owner by comparing expected equilibrium payoffs for alternative market structures in Stage 1. Thus, the solution concept is subgame perfect Bayesian Nash equilibrium. Moreover, we restrict attention to equilibria that are symmetric in the independent suppliers' investments.

To illustrate and sharpen our results, we will repeatedly use the *exponential-quadratic specification*, according to which $F(y) = 1 - e^{-y}$ with support $[0, \infty)$, and $\Psi(x) = ax^2/2$. In Online Appendix A, we study alternative type distributions to show that our findings based on this exponential-quadratic specification are robust.

3 Equilibrium Analysis

We begin the equilibrium analysis with Stage 3. Let $G(c_i+x_i) \equiv F(\mu(c_i+x_i-\beta))$ denote the distribution of i's cost given investment x_i. Observe that the support of $G(c_i+x_i)$ is $[\underline{c}_i, \overline{c}_i]$ with $\underline{c}_i = \underline{y}/\mu - x_i + \beta$ and $\overline{c}_i = \overline{y}/\mu - x_i + \beta$.[10] Denote by

$$L(c;\mathbf{x}) = 1 - \prod_{i=1}^{n}(1 - G(c+x_i)) \tag{2}$$

the distribution of the lowest of n independent draws from the distributions $G(c+x_i)$ when the vector of investments is $\mathbf{x} = (x_1, .., x_n)$. Abusing notation, when $x_i = x$ for all $i = 1, .., n$, we denote this distribution by $L(c+x;n) = 1-(1-G(c+x))^n$, whose density we denote by

$$l(c+x;n) \equiv n(1-G(c+x))^{n-1}g(c+x) = n(1-L(c+x;n-1)g(c+x), \tag{3}$$

where $g(c+x)$ is the density function for c given x. When $F(y)$ is the exponential distribution, we have $G(c+x) = 1-e^{-\mu(c+x-\beta)}$ and $L(c+x;n) = 1-e^{-\mu n(c+x-\beta)}$ with $l(c+x;n) = \mu n e^{-\mu n(c+x-\beta)}$.

3.1 Bidding

The equilibrium bidding function $b_N^*(c;x,n)$ under *non-integration (N)* when all n independent suppliers invest the same amount x is well known from auction theory. The auction being a first-price procurement auction, $b_N^*(c;x,n)$ is equal to the expected value of the lowest cost of any of the $n-1$ competitors, conditional on this cost being larger than c. That is,

$$b_N^*(c;x,n) = c + \frac{\int_c^\infty (1-L(y+x;n-1))dy}{1-L(c+x;n-1)}, \tag{4}$$

where "star" refers to equilibrium. If $F(y)$ is exponential, we write $b_N^*(c;n) = c + \frac{1}{\mu(n-1)}$ because the bid is a constant markup independent of investment.

Since we confine attention to symmetric equilibrium, the focus on symmetric investments x for the equilibrium bidding function is without loss of generality: supplier i's deviation to some $x_i \neq x$ will not be observed by any of its competitors, and any bidder i's equilibrium bid does not depend on its own distribution, only on its own cost realization. Consequently, if i deviates to some $x_i < x$, it will optimally bid according to $b_N^*(c_i;x,n)$ for any possible cost realization. On the other hand, if $x_i > x$, i's optimal bid is $b_N^*(\beta - x;x,n)$ for all $c_i \in [\beta - x_i, \beta - x]$ and $b_N^*(c_i;x,n)$ for all $c_i > \beta - x$. The following lemma summarizes.

Lemma 1 *Given an equilibrium with symmetric investments x under non-integration, the optimal bid of supplier i is*

$$b_N(c_i;x,n) = \begin{cases} b_N^*(c_i;x,n) & \textit{if} \quad c_i \geq \beta - x \\ b_N^*(\beta - x;x,n) & \textit{otherwise} \end{cases}.$$

[10] Consequently, $G(c_i) = 0$ for $c_i < \underline{c}_i$ and $G(c_i) = 1$ for $c_i > \overline{c}_i$.

Next, we turn to the analysis of bidding under *integration (I)*. Let x_1 be the equilibrium investment level of the integrated supplier and x_2 be the symmetric investment level of all independent suppliers. The equilibrium bidding function $b_I^*(c;\mathbf{x},n)$ of the independent suppliers with $\mathbf{x}=(x_1,x_2)$ is then such that

$$c=\arg\max_z\left\{[b_I^*(z;\mathbf{x},n)-c]\,[1-G(b_I^*(z;\mathbf{x},n)+x_1)][1-G(z+x_2)]^{n-2}\right\}. \qquad (5)$$

For $n=2$, the maximization problem simplifies and (5) yields the optimal monopoly pricing condition for a seller with cost c facing a buyer who draws his willingness to pay c_1 from the distribution $G(c_1+x_1)$. For $n>2$, a necessary condition for the equilibrium bidding function is that it solves the differential equation associated with the first-order condition derived from (5) and a zero-markup boundary condition:[11]

$$b_I^*(c;\mathbf{x},n)=c+\frac{\int_c^{\infty}(1-L(z+x_2;n-2))(1-G(b_I^*(z;\mathbf{x},n)+x_1))dz}{(1-L(c+x_2;n-2))(1-G(b_I^*(c;\mathbf{x},n)+x_1))}. \qquad (6)$$

Vertical integration effectively establishes a preferred supplier as in Burguet and Perry (2009). The cost distribution of the integrated supplier in our model differs from the one of the independent suppliers, which contrasts with Burguet and Perry, who assume identical distributions. Whether or not there is a unique, increasing solution to (6) is difficult to say in general. For the exponential case, the unique solution is the same as under non-integration: $b_I^*(c;n)=c+\frac{1}{\mu(n-1)}$.

The bidding function $b_I^*(c_i;\mathbf{x},n)$ is useful for analyzing deviations from a candidate equilibrium in which independent suppliers invest symmetrically. For cost draws $c_i\geq\beta-x_2$, the optimal bid is $b_I^*(c_i;\mathbf{x},n)$ irrespective of whether i deviated at the investment stage. Cost draws $c_i<\beta-x_2$ can only occur if $x_i>x_2$ for $i\neq 1$. For cost draws close to but below $\beta-x_2$, a supplier who deviated at the investment stage submits the bid $b_I^*(\beta-x_2;\mathbf{x},n)$, which guarantees that i never loses to an independent supplier. For even smaller costs, supplier i competes only against the integrated supplier by bidding the maximum of $b_I^*(c_i;\mathbf{x},2)$ and $\beta-x_1$. This is summarized in the following lemma.

Lemma 2 *Under vertical integration, if x_1 and x_2 are the equilibrium investment levels, the optimal bid of supplier i for $i=2,..,n$ with cost c_i, denoted $b_I(c_i;\mathbf{x},n)$, is given as*

$$b_I(c_i;\mathbf{x},n)=\begin{cases} b_I^*(c_i;\mathbf{x},n) & \text{if}\quad c_i\geq\beta-x_2\\ \max\{\beta-x_1,\min\{b_I^*(\beta-x_2;\mathbf{x},n),b_I^*(c_i;\mathbf{x},2)\}\} & \text{otherwise}\end{cases}.$$

[11] To see this, note that the derivative of $b_I^*(c;\mathbf{x},n)$ defined in (6) with respect to c satisfies the first-order conditions associated with problem (5). The function $b_I^*(c;\mathbf{x},n)$ is then the solution to this differential equation with the boundary condition $\lim_{c\to\overline{c}}b_I^*(c;\mathbf{x},n)-c=0$. If the upper bound of the support of F, and hence of G, is bounded, the equilibrium bidding function $b_I^*(c;\mathbf{x},n)$ has to exhibit a markup of 0 at the upper bound of costs $\overline{c}$, i.e. $b_I^*(\overline{c};\mathbf{x},n)-\overline{c}=0$, for otherwise some type would have an incentive to deviate: If $b_I^*(\overline{c};\mathbf{x},n)-\overline{c}$ were positive, a supplier of type $\overline{c}$ would benefit from bidding something less than $b_I^*(\overline{c};\mathbf{x},n)$ and larger than $\overline{c}$ because this would allow it to win with positive probability whereas by bidding $b_I^*(\overline{c};\mathbf{x},n)$ it would win with probability 0; conversely, if $b_I^*(\overline{c};\mathbf{x},n)-\overline{c}$ were negative, there would be types close to $\overline{c}$ who would benefit from bidding $b_I^*(\overline{c};\mathbf{x},n)$, thereby getting an expected payoff of 0 whereas their expected equilibrium payoff would be negative. If the upper bound of support is infinite, markups must similarly vanish in the limit, i.e. $\lim_{c\to\infty}b_I^*(c;\mathbf{x},n)-c=0$, which can be established treating the infinite upper bound of support as the limit of $\overline{c}$ going to infinity.

3.2 Investment

We now turn to the equilibrium investments. Under *non-integration*, the expected profit at Stage 2 of supplier i when investing x_i while each of the $n-1$ competitors invests x, anticipating that he will bid according to $b_N(c_i; x, n)$, is

$$\begin{aligned}\Pi_N(x_i, x) &= \int_{\beta-\min\{x,x_i\}}^{\infty} [b_N^*(c; x, n) - c][1 - L(c + x; n-1)]dG(c + x_i) \\ &+ \int_{\beta-\max\{x,x_i\}}^{\beta-x} [b_N^*(\beta - x; x, n) - c]dG(c + x_i) - \Psi(x_i).\end{aligned}$$

The function $\Pi_N(x_i, x)$ is continuously differentiable at $x_i = x$. Using (4) to substitute for the markup $b_N^*(c; x, n) - c$ and integrating by parts, one sees that for $x_i \leq x$

$$\Pi_N(x_i, x) = \int_{\beta-x_i}^{\infty} (1 - L(c + x; n-1))G(c + x_i)dc - \Psi(x_i).$$

At the necessary first-order condition, evaluated at $x_i = x^*$, we thus have

$$\int_{\beta-x^*}^{\infty} (1 - L(c + x^*; n-1))g(c + x^*)dc = \psi(x^*). \tag{7}$$

Because the integrand is $l(c + x^*; n)/n$, the left-hand side of (7) integrates to $1/n$, that is, a supplier equates its market share to its marginal cost of investment. As $\psi(x)$ is increasing, $x^* = \psi^{-1}(1/n)$ is the unique solution to (7), and we have the following result:

Proposition 1 *(a) In a symmetric equilibrium under non-integration, each supplier invests $x^* = \psi^{-1}(1/n)$. (b) There exists a unique symmetric equilibrium if $\Pi_N(x_i, x^*)$ is quasi-concave in x_i.*

The expected procurement cost in equilibrium under non-integration is

$$PC_N^* := PC_N(x^*) = \int_{\beta-x^*}^{\infty} b_N^*(c; x^*, n)dL(c + x^*; n).$$

The expected profit of a supplier in a symmetric equilibrium with investments x^* is

$$\Pi_N^* := \Pi_N(x^*, x^*).$$

The next result characterizes symmetric equilibrium for the exponential-quadratic model.

Corollary 1 *In the exponential-quadratic model, a unique symmetric equilibrium exists under non-integration if and only if $\mu < \frac{n}{n-1}a$. In this equilibrium, each supplier invests $x^* = \frac{1}{an}$ and nets an expected profit of*

$$\Pi_N^* = \frac{1}{\mu n(n-1)} - \frac{1}{2an^2},$$

while the expected procurement cost of the customer is

$$PC_N^* = \beta - \frac{1}{an} + \frac{1}{\mu n} + \frac{1}{\mu(n-1)}.$$

The procurement cost PC_N^* and the suppliers' equilibrium profit Π_N^* both decrease in n.

We next turn to the equilibrium analysis when the customer is *vertically integrated* with supplier 1. The integrated firm's problem is to choose its investment x_1 to minimize the sum of expected procurement costs and investment, denoted $PC_I(x_1, x_2)$, anticipating that each of the $n-1$ independent suppliers invests x_2 and bids according to $b_I^*(c; \mathbf{x}, n)$, and that it will source externally if and only if the lowest bid of the independent suppliers is below its own cost realization c_1. Because the non-integrated suppliers play identical strategies, we refer to such an equilibrium as a *symmetric* equilibrium. The expected procurement cost $PC_I(x_1, x_2)$ under the additional assumption $x_1 \geq x_2 - \varepsilon$ with $\varepsilon > 0$ sufficiently small is[12]

$$\Psi(x_1) + \int_{\beta - x_1}^{\infty} c dG(c + x_1) - \int_{\beta - x_2}^{\infty} \int_{b_I^*(y; \mathbf{x}, n)}^{\infty} [c - b_I^*(y; n)] \, dG(c + x_1) dL(y + x_2; n-1). \quad (8)$$

Letting

$$s(x_1, x_2) := \frac{1}{(n-1)} \int_{\beta - x_2}^{\infty} (1 - G(b_I^*(c; \mathbf{x}, n) + x_1)) dL(c + x_2; n-1)$$

denote the market share of an independent supplier in a symmetric equilibrium, we can write the integrated supplier's first-order condition compactly as

$$1 - (n-1)s(x_1, x_2) = \psi(x_1). \quad (9)$$

The expected profit of independent supplier i who invests x_i, given investments x_1 by the integrated supplier and investments x_2 by independent rivals, is denoted $\Pi_I(x_i, x_2, x_1)$ and equal to

$$\begin{aligned} &\int_{\beta - \min\{x_2, x_i\}}^{\infty} [b_I(c; \mathbf{x}, n) - c](1 - L(c + x_2; n-2))(1 - G(b_I(c; \mathbf{x}, n) + x_1)) dG(c + x_i) \\ + &\int_{\beta - \max\{x_2, x_i\}}^{\beta - x_2} [b_I(c; \mathbf{x}, n) - c](1 - G(b_I(c; \mathbf{x}, n) + x_1)) dG(c + x_i) - \Psi(x_i). \end{aligned}$$

The function $\Pi_I(x_i, x_2, x_1)$ is continuously differentiable at $x_i = x_2$. Plugging in the equilibrium expression for the markup $b_I(c; \mathbf{x}, n) - c$ based on (6) and proceeding similarly to before, one can write the equilibrium first-order condition for an independent supplier compactly as

$$s(x_1, x_2) = \psi(x_2). \quad (10)$$

Note that, because $b_I^*(c; \mathbf{x}, n) > c$ for any $c < \overline{c}$, we have $s(x, x) < 1/n$.[13]

We now impose assumptions on the payoff functions $PC_I(x_1, x_2)$ and $\Pi_I(x_i, x_2, x_1)$ and the market share function $s(x_1, x_2)$ that ensure the existence of a unique equilibrium:

[12] The expression for $PC_I(x_1, x_2)$ for the case $b_I^*(\beta - x_2; \mathbf{x}, n) < \beta - x_1$, which requires $x_2 > x_1$, is provided in the proof of Corollary 3 in the Online Appendix (see equation (28)).

[13] To see this, it suffices to show that, $x_1 = x_2 = x$ would imply $s(x, x) = 1/n$ if $b_I^*(c; \mathbf{x}, n) = c$ were the case. The result then follows from the fact that $1 - G(b_I^*(c; \mathbf{x}, n) + x) < 1 - G(c + x)$ for $b_I^*(c; \mathbf{x}, n) > c$.

(i) $PC_I(x_1, x_2)$ is strictly quasi-convex in x_1 and $\Pi_I(x_i, x_2, x_1)$ is strictly quasi-concave in x_i over relevant ranges,[14]

(ii) $s_1(x_1, x_2) < 0 < s_2(x_1, x_2)$, where subscripts denote partial derivatives,

(iii) $s(\psi^{-1}(1), 0) > 0$ and $s(0, \psi^{-1}(1/(n-1))) < 1/(n-1)s$.

Assumption (i) ensures unique best responses. Assumption (ii) non-controversially states that investment increases market share.[15] Assumption (iii) rules out corner solutions.

Denote by (x_1^*, x_2^*) a solution to (9) and (10), and let $PC_I^* := PC_I(x_1^*, x_2^*)$ and $\Pi_I^* := \Pi_I(x_2^*, x_2^*, x_1^*)$.

Proposition 2 *(a) In a symmetric equilibrium under vertical integration, the integrated supplier invests x_1^* and each independent supplier invests x_2^* with (x_1^*, x_2^*) satisfying (9) and (10). (b) Furthermore, under assumptions (i), (ii) and (iii), a unique symmetric equilibrium exists and satisfies $x_1^* > x^* > x_2^*$.*

Part (b) formalizes the investment discouragement effect. The intuition for it is straightforward. Since a supplier's marginal return to cost reduction is equal to its market share, the integrated (an independent) supplier invests more (less) compared to non-integration. Furthermore, equations (9) and (10) imply

$$\psi(x_1) + (n-1)\psi(x_2) = 1. \tag{11}$$

That is, the marginal costs of investment add up to 1, which is the sum of the market shares of all suppliers. The same is also true under non-integration: $n\psi(x^*) = 1$. Using Jensen's inequality, one can show that this implies:

Corollary 2 *Aggregate effort in a symmetric equilibrium under vertical integration with investments x_1^* and x_2^* determined by (9) and (10) is the same, higher or lower than aggregate effort in a symmetric equilibrium under non-integration with x^* given by (7) if, respectively, $\psi''(x) = 0$, $\psi''(x) < 0$ or $\psi''(x) > 0$ for all $x \geq 0$.*

In particular, for quadratic costs of investments, aggregate equilibrium investment does not vary with the market structure. That is, $nx^* = x_1 + (n-1)x_2 = 1/a$.

[14] The relevant range for $PC_I(x_1, x_2)$ is for all $(x_1, x_2) \in [0, \psi^{-1}(1)] \times [0, \psi^{-1}(1/(n-1))]$ while the relevant range for $\Pi_I(x_i, x_2, x_1)$ is for all $(x_i, x_1, x_2) \in [0, \psi^{-1}(1)] \times [0, \psi^{-1}(1)] \times [0, \psi^{-1}(1/(n-1))]$. These are the relevant ranges because they constitute lower and upper bounds for suppliers' investments. To see why $\psi^{-1}(1/(n-1))$ and $\psi^{-1}(1)$ are upper bounds for the investments of an independent and the integrated supplier, respectively, notice that if the market share of the integrated (any independent) supplier were 0, by equating marginal cost of investment with market share, any independent (the integrated) supplier would invest $\psi^{-1}(1/(n-1))$ $(\psi^{-1}(1))$. Because market shares are non-negative, it follows that these are upper bounds.

[15] Indeed, one may wonder whether it is implied by the assumptions of how investments affect distributions. The answer is not obvious because there is an additional effect via the bidding function $b_I^*(c; \mathbf{x}, n)$.

The exponential-quadratic model is a useful and tractable specification. For exponentially distributed costs, we have

$$s(x_1, x_2) = \frac{1}{n} e^{-\mu(x_1 - x_2) - \frac{1}{n-1}},$$

which satisfies assumption (ii). Moreover, for any $x_1 < \infty$, $s(x_1, x_2) > 0$ and for any $x_2 < \infty$, $s(x_1, x_2) < 1/n$, implying that assumption (iii) is also satisfied for any convex cost of effort function. Notice also that $s(x_1, x_2)$ depends only on the difference $\Delta := x_1 - x_2$ in investments. Letting $s(\Delta) := s(x_1, x_2)|_{x_1 = x_2 + \Delta} = \frac{1}{n} e^{-\mu\Delta - \frac{1}{n-1}}$, the first-order conditions (9) and (10) become $1 - (n-1)s(\Delta) = \psi(x_1)$ and $s(\Delta) = \psi(x_2)$. Inverting and taking the difference yields

$$\psi^{-1}(1 - (n-1)s(\Delta)) - \psi^{-1}(s(\Delta)) = \Delta, \tag{12}$$

which implicitly defines Δ with $x_1 = \psi^{-1}(1-(n-1)s(\Delta))$ and $x_2 = \psi^{-1}(s(\Delta))$. Further, if Ψ is quadratic, we have $\psi(x) = ax$, and equation (12) simplifies to

$$\frac{1}{a}(1 - ns(\Delta)) = \Delta. \tag{13}$$

This equation has a unique positive solution, which we denote $\Delta(n, \mu)$.[16]

Corollary 3 *In the exponential-quadratic model, a symmetric equilibrium under integration exists if a symmetric equilibrium exists under non-integration. Investments are*

$$x_1^* = \frac{1}{an} + \frac{n-1}{n}\Delta(n, \mu) \quad \textit{and} \quad x_2^* = \frac{1}{an} - \frac{1}{n}\Delta(n, \mu), \tag{14}$$

with $\Delta(n, \mu)$ defined by (13). The expected procurement cost of the integrated firm is

$$PC_I^* = \beta + \frac{a - \mu}{\mu} x_1^* + \frac{a}{2}(x_1^*)^2$$

while the expected of profit of an independent supplier is

$$\Pi_I^* = \frac{1}{\mu(n-1)} a x_2^* - \frac{a}{2}(x_2^*)^2.$$

[16]That $\Delta = 0$ is not a solution follows from the fact that $s(0) < 1/n$. The right-hand side of (13) being linear and the left-hand side being increasing and concave in Δ then establishes the result. We omit the dependence of $\Delta(n, \mu)$ on a because $\Delta(n, \mu)$ is invariant with respect to the normalization $\hat{\mu} := \mu/a$. To see this this, let $\hat{\Delta} := a\Delta$. Equation (13), evaluated at μ and $a = 1$, reads $1 - e^{-\mu\Delta - \frac{1}{n-1}} = \Delta$ while evaluated at $\hat{\mu}$ it is $1 - e^{-\hat{\mu}\hat{\Delta} - \frac{1}{n-1}} = \hat{\Delta}$. Thus, the solutions will only vary to the extent that $\hat{\mu}$ differs from μ.

3.3 Divestiture

We now turn to Stage 1 of our three-stage game. The integrated firm is better off divesting its supply unit if its total production cost under integration, PC_I^*, is larger than the procurement cost under non-integration, PC_N^*, less the price Π_N^* the firm obtains from a competitive outside bidder for its supply unit. Consequently:

Proposition 3 *In Stage 1, the integrated firm divests if and only if*

$$PC_I^* + \Pi_N^* - PC_N^* > 0. \tag{15}$$

The proof is simple (and omitted). Evaluating whether or not (15) is satisfied is not. However, substituting the expressions in Corollaries 1 and 3 into (15) and simplifying yields the following result for the exponential-quadratic specification.

Corollary 4 *In the exponential-quadratic model, if a symmetric equilibrium exists under non-integration, then the common owner divests if and only if*

$$\Phi(n, \mu, a) := \frac{a}{2}\left(\frac{n-1}{n}\right)^2 (\Delta(n, \mu))^2 + \frac{n-1}{n}\left(\frac{a-\mu}{\mu} + \frac{1}{n}\right)\Delta(n, \mu) - \frac{1}{\mu n} > 0. \tag{16}$$

Figure 1 illustrates Corollary 4 for the normalization $a = 1$.[17] It shows that the benefits of divestiture increase with n when vertical integration is the more profitable organizational structure, that is, when $\Phi(n, \mu, a) < 0$. Moreover, the benefits from divestiture stay positive once they are positive. Divestiture also becomes more attractive as μ increases. This is intuitive because higher μ means a lower variance and therefore less rents accruing to independent suppliers. Finally, for $\mu \leq 1/2$, vertical integration dominates divestiture for any n.

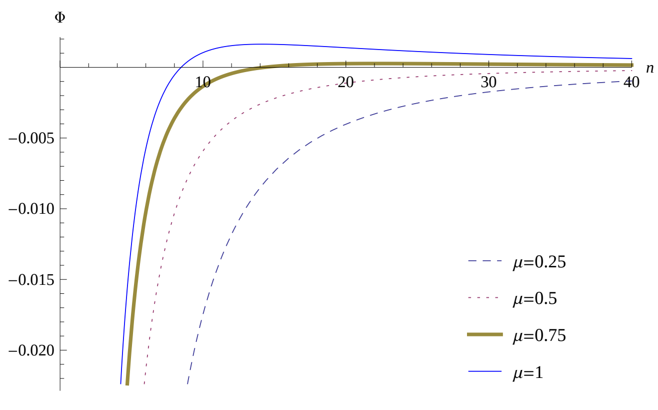

Figure 1: $\Phi(n, \mu, 1)$ for $\mu \in \{0.25, 0.5, 0.75, 1\}$ as a function of n.

To appreciate this result, it is important to understand the powerful advantages of vertical integration. With quadratic effort cost, aggregate investment is the same

[17]Recall from Footnote 16 that (13) can be solved for $a\Delta$ as a function of n and μ/a. This implies that $a\Phi(n, \mu/a, 1) = \Phi(n, \mu, a)$.

under non-integration and integration (Corollary 2). Furthermore, since the exponential distribution has a constant hazard rate, the distribution of minimum production cost is more favorable under vertical integration. The support of the minimum cost distribution is the union of the supports of the cost distributions of the integrated and independent suppliers, and the distribution depends only on aggregate investment on the support of an independent firm. Because the higher additional investment of the integrated firm shifts its support downward, however, the minimum cost distribution shifts to the left. On top of the cost advantages of vertical integration, the integrated firm distorts procurement in favor of internal sourcing, thereby avoiding always paying a markup and further reducing its procurement cost compared to non-integration.

From this perspective, the downside to vertical integration might seem more modest. Because the cost of effort is convex, the total effort cost increases as the same total investment is redistributed from independent suppliers to the integrated supplier. In other words, even though the vertically integrated firm fully compensates for the investment discouragement of the independent suppliers, it does so at a higher cost. Corollary 4 shows that the higher total investment cost can be enough to substantially offset and even outweigh the benefits of vertical integration.

Interestingly, divestiture may not be a subgame perfect equilibrium outcome even when the symmetric equilibrium under non-integration is *socially optimal* in the sense of minimizing the expected total cost of production under efficient sourcing plus the cost of investment. These total costs are given by

$$TC(\mathbf{x}) = \int_{\beta-\max\{x_i\}}^{\infty} c dL(c;\mathbf{x}) + \sum_{i=1}^{n} \Psi(x_i).$$

Proposition 4 *If $TC(\mathbf{x})$ is quasiconvex, then symmetric investments $x_i = x^*$ for all $i = 1,..,n$ with $x^* = \psi^{-1}(1/n)$ are socially optimal.*

Corollary 5 *In the exponential-quadratic model, the symmetric investments $x^* = 1/(an)$ are socially optimal if $\mu \leq a$.*

Because sourcing is efficient in the subgame under non-integration, the socially optimal investments are always an equilibrium outcome similarly to Rogerson (1992). However, for the full game, the socially optimal market structure need not be an equilibrium outcome. For example, divestiture is always socially optimal for the parameter configuration in Figure 1 according to Corollary 5. However, as the figure shows, divestiture is not always an equilibrium outcome.[18]

[18] In this model, a symmetric equilibrium under non-integration exists for all $\mu < na/(n-1)$ whereas symmetric investments are socially optimal if and only if $\mu \leq a$ (the "only if" is shown at the end of the proof of Corollary 5 in the Online Appendix). Taken together, this implies that for $a < \mu < na/(n-1)$ the symmetric equilibrium under non-integration is not the planner's first-best solution. Furthermore, there must also exist an asymmetric investment equilibrium under non-integration that is socially optimal. Whether the planner prefers vertical integration to the symmetric equilibrium under non-integration depends on intricate details because the sourcing distortion under vertical integration is not socially optimal. One can show numerically that, for small n, the planner prefers vertical integration to symmetric non-integration for $a < \mu < na/(n-1)$.

Alternatively, we could have stipulated that the initial market structure is non-integration and asked under what conditions the buyer acquires a supply unit in equilibrium. The Online Appendix analyzes an acquisition game in which integration is an equilibrium outcome when condition (15) fails. In this sense, the equilibrium market structure is robust to initial conditions.

4 Extensions

Our model and results extend in a number of relevant ways, as we show in detail in the Online Appendix. Here we outline the various extensions.

4.1 Alternative Cost Distributions

We address robustness of our results with respect to alternative cost distributions. Assume first that given investment x, the cost distribution is the *uniform distribution* $G(c+x) = c - (\beta - x)$ for $c \in [\beta - x, 1 + \beta - x]$. For $n = 2$, this is another special case for which there is a closed form solution for the equilibrium bidding function for the independent supplier. Let x_1 be the integrated supplier's investment and $x_2 \leq x_1$ the independent supplier's investment. Then, upon a cost realization $c \in [\beta - x_1, 1 + \beta - x_1]$, the independent supplier submits the bid

$$b_I(c; 2) = \frac{c + 1 + \beta - x_1}{2}$$

and an arbitrary bid $b > 1 + \beta - x_1$ for $c > 1 + \beta - x_1$. For $n = 2$ and assuming quadratic investment costs $\Psi(x) = ax^2/2$, vertical integration reduces procurement costs relative to the symmetric equilibrium under non-integration, which exists whenever $a \geq 1$. A numerical analysis of the uniform case for larger values of n requires nesting a numerical solution for the bidding function, which has no closed form solution under vertical integration because the cost distributions differ. The comparative statics with respect to n are similar to the ones for the exponential-quadratic model.

Another model that permits closed form solutions for the equilibrium bidding function with integration is the *fixed-support exponential model*, in which, for an arbitrary investment x, the distribution of costs c with support $[\beta, \infty)$ is $G(c; x) = 1 - e^{-\mu x(c-\beta)}$, where $\mu > 0$ and $\beta \geq 0$ are parameters. In this case, investment shifts the scale parameter μx rather than the location parameter β, thereby shifting both the mean, $\frac{1}{\mu x} - x\beta$, and the standard deviation, $\frac{1}{\mu x}$, of the cost distribution. The fixed support exponential cost distribution function has an appealing interpretation: greater design effort reduces the frequency of high cost outcomes. As a consequence, investments affect the bidding of independent suppliers, whose bidding function is cost plus one divided by aggregate investment of the other suppliers. As we show in Online Appendix A, the main insights are the same as those based on the exponential-quadratic model in which investments shift the lower bound of the exponential distribution. This extension also demonstrates that the results do not hinge on the assumptions embedded in (1).[19]

[19]In terms of the primitives of the model, this corresponds to replacing equation (1) by $c_i = y/(\mu x_i) + \beta$

4.2 Elastic Demand

While inelastic demand is a useful simplifying assumption that illuminates the main tradeoffs between non-integration and integration, it is of course more realistic for the buyer to abandon the project entirely if costs are prohibitively high. For the exponential-quadratic model, it is straightforward to generalize the analysis to allow for a downward sloping demand curve by assuming that the buyer's value is also drawn from an exponential distribution. The comparative statics remain essentially the same as in the model with inelastic demand (and infinite buyer value). The analysis of elastic demand also permits a richer welfare analysis. The Online Appendix shows that, from a social welfare perspective, divestitures are too infrequent.

4.3 Reserve Prices

A simple first-price auction captures commercial negotiations that require minimal commitments. Suppliers make offers and the customer accepts the best offer. If the required input were sufficiently standardized, so that acceptable designs were contractible, then the customer plausibly could exercise monopsony power by committing to a reserve price. For the case of inelastic demand, a positive reserve price is suboptimal under non-integration, because the risk of failing to procure the input is disastrous. A reserve price is valuable under vertical integration, however, because the monopsonist is able to fall back on internal sourcing if independent suppliers cannot beat the reserve price. Thus, the ability to set a credible reserve price option appears to favor vertical integration under inelastic demand. Nevertheless, a similar benefit-cost trade-off emerges, albeit with more stringent conditions for the superiority of non-integration. Numerical analysis for the elastic demand case in Online Appendix A shows that divestitures occur not frequently enough from the social planner's perspective.[20]

5 Conclusion

We develop a "make and buy" theory of vertical integration according to which vertical integration creates the opportunity, but not the necessity, to source inputs internally. This is consistent with the documented prevalence of outsourcing in American manufacturing even by vertically integrated firms. Our theory features a key tradeoff

and assuming that $F(y_i)$ has support $[0, \infty)$.

[20]We can also extend our model to allow for second-price auctions, although, as Burguet and Perry (2009, p. 284) observe, "a second-price auction is not an appropriate model for a market when the buyer has no ability to design and commit to rules of trade". For a second-price auction without a reserve, vertical integration has no effect on the joint surplus of the customer and the integrated supplier, as observed by Bikhchandani, Lippmann and Reade (2005) in the context of preferred suppliers. Consequently, vertical integration does not affect investments. Furthermore, a second-price auction with an optimally chosen reserve price has the same outcomes as a first-price auction. Because equilibrium bidding seems straightforward under a second-price auction, one might think that a modeling approach based on second-price auctions has computational advantages. However, because typically the optimal reserve cannot be expressed in closed form, one still needs to compute expected profits in equilibrium numerically, so that the gains in tractability are illusory.

between markup avoidance and investment discouragement. Upstream suppliers make relationship-specific investments in cost reduction before bidding to supply an input requirement to a downstream customer. Since neither the investment nor the cost realization are observable, independent suppliers exercise some degree of market power and bid above costs in the procurement. Vertical integration enables the customer sometimes to avoid the markup by sourcing internally, keeping investments fixed, and discourages independent suppliers' investments.

This fundamental tradeoff between markup avoidance and investment discouragement is central to our model. It raises the question: under what conditions is divestiture more attractive than vertical integration? Parametric specifications of our model demonstrate that divestiture becomes more profitable as the upstream market becomes more competitive. In this way, our model helps explain a trend toward non-integration in a global economy, resonating with Stigler's (1951) idea that vertical integration becomes less attractive as upstream industries grow mature.

An important direction for further research is to explore how repeated interaction alters the tradeoff between markup avoidance and investment discouragement. Another promising research direction is to allow a separation of ownership and control by interpreting the investment cost function to include the cost to a risk-neutral owner of inducing a risk-averse manager to undertake a given level of effort, as in Grossman and Hart (1983).[21] Finally, it would be interesting to embed the present setup with a single customer into a larger market environment. For example, if upstream firms supply other customers who benefit from cost-reducing investments, vertical integration might raise rivals' costs, as in Ordover, Saloner, and Salop (1992). Furthermore, acquisition of multiple suppliers – a possibility we ignored – presumably would reduce upstream horizontal competition, negatively impacting other downstream markets, and thus raising antitrust concerns.

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[21]Calzolari et al. (2015) study a relational contracting model assuming investments are noncontractible but ultimately observable. Relaxing this assumption by allowing for imperfect observability, and possibly better observability (or stronger relational incentives) inside the firm, is one possibility for future research.

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Online Appendix

A Extensions

In this section, we detail the various extensions discussed in Sections 3 and A.

A.1 Alternative Cost Distributions

Uniform Model We first consider the model with uniformly distributed costs, that is, for investment x the costs are distributed according to $G(c+x) = c - (\beta - x)$ for $c \in [\beta - x, 1 + \beta - x]$, and assume $\Psi(x) = ax^2/2$. For $n > 2$, this requires solving numerically for the equilibrium bidding under integration as mentioned in the main text.

Figure 2 plots the benefits from non-integration minus the payoff from vertical integration, $\Phi(n)$, as a function of n for $a = 1.75$.

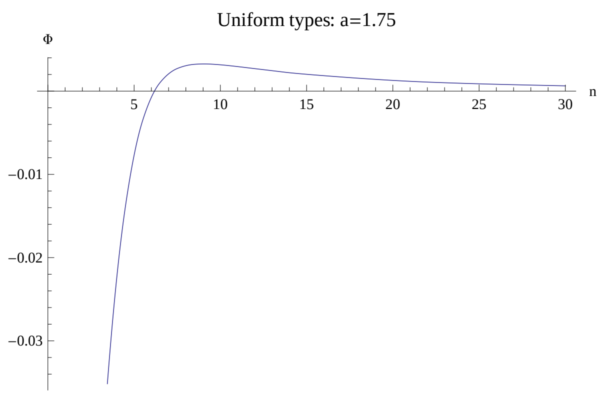

Figure 2: $\Phi(n)$ for Uniformly Distributed Costs.

An intuitive conjecture is that vertical integration has the advantage of squeezing (rather than just avoiding) markups. Analysis of the exponential case has already shown this intuition is not correct in general.[22] For the uniform case, equilibrium bid markups indeed decrease with vertical integration seemingly in line with the intuition. However, closer analysis reveals that the reason for this is the effect of vertical structure on equilibrium investments because, keeping investments fixed, vertical integration does not affect equilibrium bidding.[23] Figure 3 depicts the equilibrium bids given equilibrium investments.

[22] For the case of a fixed cost distribution with a convex decreasing inverse hazard rate, Burguet and Perry (2009) argue that a right of first refusal granted to a preferred supplier is profitable in part because it causes independent suppliers to bid more aggressively. The exponential cost distribution is a limiting case, in which the hazard rate is constant and the bid distribution does not change with vertical integration, consistent with a more basic markup avoidance motive for granting a right of first refusal.

[23] To see this, notice that in a standard first-price procurement auction with n bidders and costs independently drawn from the uniform distribution with support $[\underline{c}, \overline{c}]$ the equilibrium bidding function

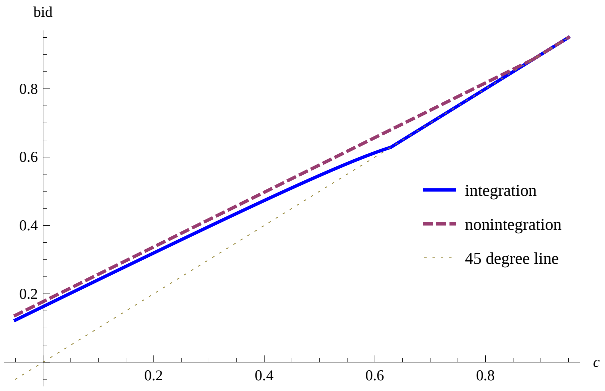

Figure 3: Equilibrium bidding with uniformly distributed costs.

Fixed-Support Exponential Model In the fixed-support exponential model, the distribution of the costs c given investment x is $G(c;x) = 1 - e^{-\mu x(c-\beta)}$. We assume quadratic costs of effort and set $a = 1 = \mu$. This is without loss of generality by appropriately choosing units of measurement for c and x. We also set $\beta = 0$ to simplify derivations.

Equilibrium bids by independent suppliers are again a constant markup on cost. The difference from the baseline model is that the markups depend endogenously on investments. In the case of non-integration the bid function is

$$b_N(c;n) = c + \frac{1}{(n-1)x_N},$$

where x_N is the symmetric investment of n independent suppliers. In the case of vertical integration, the bid function is

$$b_I(c;n) = c + \frac{1}{x_1 + (n-2)x_2},$$

where x_1 is the investment of the integrated supplier and x_2 the symmetric investment of the $n-1$ independent suppliers.

Equilibrium investments are derived from first-order conditions as before. In a symmetric equilibrium of the non-integrated environment, each of the suppliers invests an amount equal to 1 over the cube root on n^2, that is, $x_N = \frac{1}{\sqrt[3]{n^2}}$. For the integrated

is $\beta(c) = \overline{c}/n + (n-1)c/n$. With one integrated supplier whose bid is equal to his realized cost c_1 and $n-2$ competing independent suppliers who all bid according to $\beta_I(c) = \alpha_0 + \alpha_1 c$, satisfying the boundary condition $\beta_I(\overline{c}) = \overline{c}$ (which implies $\alpha_0 = \overline{c}(1-\alpha_1)$), the optimal bid of a representative independent bidder i, b_i, solves the problem of maximizing $(1/\alpha_1)^{n-2}\left(1/(\overline{c}-\underline{c})\right)^{n-1}(\overline{c}-b_i)^{n-1}(b_i - c_i)$, yielding $b_i = \overline{c}/n + (n-1)c_i/n$. The second-order condition is readily seen to be satisfied. This invariance is due to the linearity of equilibrium bidding strategies with uniformly distributed costs on identical supports. It reflects the fact that the equilibrium bidding strategy $\beta(c)$ is the best response to any collection of linear bidding strategies of the form $b_i(c) = \alpha_{0,i} + \alpha_{1,i}c$ that satisfy the boundary condition, i.e. $\alpha_{i,0} = \overline{c}(1-\alpha_{i,1})$. The integrated supplier has a particularly simple linear bidding strategy with $\alpha_{1,0} = 0$ and $\alpha_{1,1} = 1$.

environment, let $z = \frac{x_2}{x_1}$. The symmetric best response investments can be written as functions of z, $x_1 = x_1(z)$ and $x_2 = x_2(z)$, respectively. Equilibrium investments are then given by $x_1 = x_1(z(n))$ and $x_2 = x_2(z(n))$, where $z(n)$ is the unique fixed point to the equation

$$z = \frac{x_2(z)}{x_1(z)}.$$

For a given $z > 0$, the integrated supplier optimally invests

$$x_1(z) = \sqrt[3]{1 - \frac{(n-1)z[3 + z(2z-6) + 2n(4 + (n-3)z)]e^{-\frac{1}{1+(n-2)z}}}{[1 + (n-2)z][1 + (n-1)z]^2}}$$

and the independent suppliers symmetrically invest

$$x_2(z) = \sqrt[3]{\frac{z^2 e^{-\frac{1}{1+(n-2)z}}}{[1 + (n-1)z]^2}}.$$

Dividing $x_2(z)$ by $x_1(z)$ and simplifying yields the fixed point

$$z = \sqrt[3]{\frac{z^2}{[1 + (n-1)z]^2 e^{\frac{1}{1+(n-2)z}} - \frac{(n-1)\{3z + z^2[(2z-6) + 4n + n(n-3)z]\}}{1+(n-2)z}}}.$$

A simple graphical analysis shows that $z(n)$ is increasing in n.

Under non-integration, the equilibrium (expected) procurement cost of the buyer as a function of symmetric supplier investments x_N is

$$PC_N = \int_0^\infty b_N(c; n) dG(c; nx_N) = \frac{2n-1}{n(n-1)x_N}$$

and the (expected) profit of a supplier is

$$\Pi_N = \int_0^\infty [b_N(c; n) - c][1 - G(c; (n-1)x_N)] dG(c; x_N) - \frac{1}{2}x_N^2 = \frac{1}{n(n-1)x_N} - \frac{1}{2}x_N^2.$$

Substituting $x_N(n)$ into these expressions yields equilibrium values of procurement cost and profits as functions of the number of suppliers

$$PC_N(n) = \frac{2n-1}{(n-1)\sqrt[3]{n}} \quad \text{and} \quad \Pi_N(n) = \frac{n+1}{2n(n-1)\sqrt[3]{n}}.$$

Procurement cost under vertical integration can be expressed as a function of x_1 and z:

$$\begin{aligned} PC_I &= \int_0^{x_1} c dG(c; x_1) + \frac{1}{2}x_1^2 \\ &- \int_{\frac{1}{x_1+(n-2)x_2}}^{\infty} \int_0^{c_1 - \frac{1}{x_1+(n-2)x_2}} [c - b_I(c; n)] dG(c; (n-1)x_1 z) dG(c_1; x_1) \\ &= \frac{1}{x_1} + \frac{1}{2}x_1^2 - \frac{(n-1)ze^{-\frac{1}{1+(n-2)z}}}{1 + (n-1)z}. \end{aligned}$$

Substituting $x_1 = x_1(z(n))$ and $z = z(n)$ yields procurement cost $PC_I(n)$ as a function of n. Since $z(n)$ lacks a closed form solution, so does $PC_I(n)$.

Divestiture is more profitable than vertical integration if

$$\Phi(n) \equiv PC_I(n) + \Pi_N(n) - PC_N(n)$$

is positive. Figure 4 shows that $\Phi(n) < 0$ if and only if $n < 10$. Thus, as in the baseline model, non-integration and a complete reliance on outsourcing is more profitable than vertical integration if the upstream market is sufficiently competitive.

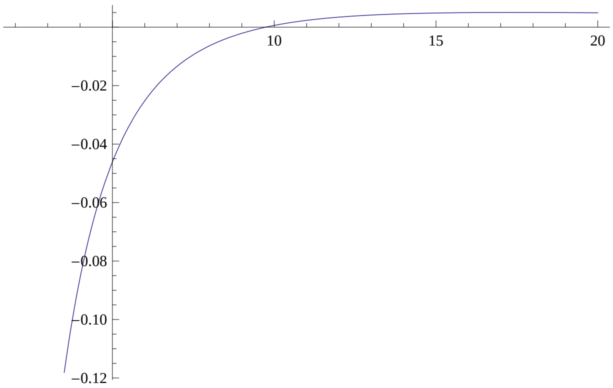

Figure 4: The benefit from divestiture, $\Phi(n)$ for the fixed-support exponential model.

It is also interesting to compare the independent bid functions under integration and non-integration. The difference in markups is

$$\Delta b(n) = \frac{1}{x_1(z(n)) + (n-2)x_2(z(n))} - \frac{1}{(n-1)x_N(n)}.$$

Figure 5 shows that $\Delta b(n) < 0$ if and only if $n < 6$. That is, the equilibrium markup is lower under vertical integration if and only if upstream competition is limited. Surprisingly, vertical integration fails to reduce markups for more competitive upstream market structures. The reason is an additional negative consequence of the investment discouragement effect: reduced investment by independent suppliers increases cost heterogeneity, causing the independent firms to bid more aggressively.

Furthermore, it can be shown that in this case vertical integration always decreases total investment, i.e. $x_1(z(n)) + (n-1)x_2(z(n)) < nx_N(n)$.

A.2 Elastic Demand

Setup To model elastic demand, we assume that the customer (or buyer, indicated with subscript B) has value v for the input, drawn from an exponential probability distribution $G_B(v) = 1 - e^{-\lambda(v-\alpha)}$ with support $[\alpha, \infty)$. The mean of the exponential distribution is $\alpha + \frac{1}{\lambda}$ and can be interpreted to indicate the expected profitability of the downstream market. The variance, which is $\frac{1}{\lambda^2}$, can be interpreted to indicate uncertainty

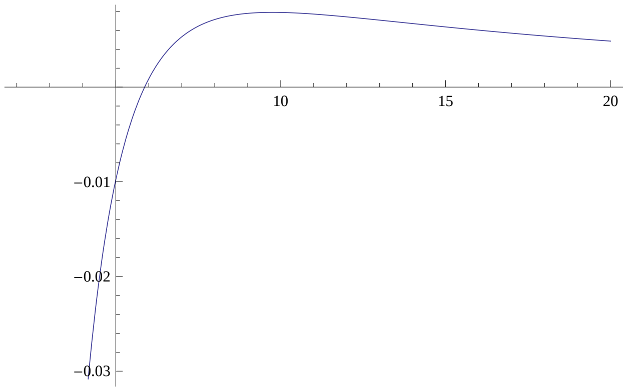

Figure 5: The function $\Delta b(n)$.

about product differentiation. This model converges to the inelastic case as $\lambda \to 0$. The customer learns the realization of v before making the purchase (or production) decision.

Under vertical integration, the investment x_1 in cost reduction is made before the customer learns the realized v. Independent suppliers know G_B but not v. All other assumptions regarding timing are as in Section 2. The cost of exerting effort x is $\frac{a}{2}x^2$ and given investment x_i supplier i's cost is drawn from the exponential distribution $1 - e^{-\mu(c+x_i-\beta)}$ with support $[\beta - x_i, \infty)$ for all $i = 1, .., n$ and with $\mu \leq a$. To simplify the equilibrium analysis, we impose the parameter restriction

$$\beta - \alpha \geq \frac{\mu}{a(\lambda + n\mu)} - \frac{1}{\lambda + (n-1)\mu}, \tag{17}$$

which makes sure that under non-integration (and therefore also under integration) the lowest equilibrium bid is always larger than the lowest possible draw of v. Observe that the right-hand side in (17) is negative, so that $\beta \geq \alpha$ is sufficient for the condition.[24]

Bidding As in the inelastic demand case, the bidding function is the same with or without vertical integration. The bidding function with elastic demand is denoted by $b_E(c; n)$ and given by

$$b_E(c; n) = c + \frac{1}{\lambda + \mu(n-1)} \tag{18}$$

for all $c \geq \beta - \frac{\mu}{a(\lambda+n\mu)}$ as shown next.

[24]To see where (17) comes from, notice that supplier i's expected profit when investing $x_i \leq x$ while all rivals invest x and when all suppliers bid according to (18) is $\int_{\beta-x_i}^{\infty}(1/(\lambda + (n-1)\mu))\mu e^{-\lambda(c+1/(\lambda+(n-1)\mu)-\alpha)-\mu(n-1)(c+x-\beta)-\mu(c+x_i-\beta)}dc - ax_i^2/2$. The first-order condition at $x_i = x$ is

$$\frac{\mu}{\lambda + \mu n} e^{\lambda\left(x+\alpha-\beta-\frac{1}{\lambda+(n-1)\mu}\right)} = ax.$$

If $x + \alpha - \beta - 1/(\lambda + (n-1)\mu) < 0$, the first-order condition implies $x < \frac{\mu}{a(\lambda+n\mu)}$. Plugging this back into the preceding inequality gives (17).

We begin with *non-integration*. Given symmetric investments x, a symmetric equilibrium bidding strategy $b(c)$ is such that

$$c = \arg\max_z \left\{ [b(z) - c]\,[1 - G_B(b(z))]\,[1 - G(c+x)]^{n-1} \right\}.$$

For G_B and G exponential, a representative supplier's problem becomes

$$\max_z (b(z) - c) e^{-\lambda(b(z)-\alpha)-\mu(n-1)(z+x-\beta)}.$$

Taking the derivative with respect to z and imposing the boundary condition $\lim_{c\to\infty}(b(c)-c)/c = 0$ yields the unique solution

$$b(c) = c + \frac{1}{\lambda + (n-1)\mu},$$

as claimed.

With *integration*, G_B and G exponential and $x_1 \geq x_2$, a representative non-integrated supplier's problem is

$$\max_z (b(z) - c) e^{-\lambda(b(z)-\alpha)-\mu(b(z)+x_1-\beta)-\mu(n-2)(z+x_2-\beta)}.$$

Taking the derivative with respect to z yields the first-order condition

$$b'(c) - [(\lambda + \mu)b'(c) + (n-2)\mu][b(c) - c] = 0.$$

Imposing the boundary condition $\lim_{c\to\infty}(b(c) - c)/c = 0$ then gives the unique solution

$$b(c) = c + \frac{1}{\lambda + (n-1)\mu},$$

which is the same as b_E defined in (18).

Profits Consider first non-integration when the symmetric investments of the independent suppliers are x. The profit $\Pi^B_{EN}(x)$ accruing to the buyer is

$$\Pi^B_{EN}(x) = n \int_{b_E(\beta-x;n)}^{\infty} \int_{\beta-x}^{y(v)} [v - b_E(c;n)][1 - G(c+x)]^{n-1} dG(c+x) dG_B(v),$$

where $y(v) = v - \frac{1}{\lambda+\mu(n-1)}$ denotes the inverse of the bidding function $b_E(c;n)$ with respect to c.

The expected profit $\Pi_{EN}(x_i, x)$ of an independent supplier under non-integration who invests x_i while each of the other suppliers is expected to invest x with $x_i \leq x$ is[25]

$$\Pi_{EN}(x_i, x) = \int_{b_E(\beta-x_i;n)}^{\infty} \int_{\beta-x_i}^{y(v)} [b_E(c;n) - c][1 - G(c+x)]^{n-1} dG(c+x_i) dG_B(v) - \frac{a}{2}x_i^2.$$

[25] For $x_i = x + \varepsilon$ with $\varepsilon > 0$ small, the expected profit function has a different functional form. However, the profit function $\Pi_{EN}(x_i, x)$ is continuously differentiable at $x_i = x$.

With integration, the buyer's profit is

$$\begin{aligned}
\Pi^B_{EI}(x_1,x_2) &= \int_\alpha^\infty \int_{\beta-x_1}^{\max\{v,\beta-x_1\}} [v-c_1]dG(c_1+x_1)dG_B(v) \\
&+ \int_{\beta-x_1}^\infty (1-G_B(c_1)) \int_{\beta-x_2}^{\max\{y(c_1),\beta-x_2\}} [c_1-b_E(c_2;n)]dL(c_2+x_2;n-1)dG(c_1+x_1) \\
&+ \int_\alpha^\infty (1-G(v+x_1)) \int_{\beta-x_2}^{\max\{y(v),\beta-x_2\}} [v-b_E(c_2;n)]dL(c_2+x_2;n-1)dG_B(v) - \frac{a}{2}x_1^2.
\end{aligned}$$

This profit is computed by deriving the expected profit from internal sourcing, which is done in the first line in the above expression, by then adding the cost savings from sourcing from the independent supplier with the lowest bid, which is captured in the second line, and by finally adding in the third line the expansion effect of external sourcing that arises whenever $c_1 > v$ and $b_E(\min\{c_j\}) < v$ with $j \neq 1$.

Given its own investment x_i, investments $x_2 \geq x_i$ by all other non-integrated suppliers and x_1 by the integrated supplier, the expected profit $\Pi_{EI}(x_i, x_1, x_2)$ of an independent supplier under vertical integration is

$$\Pi_{EI}(x_i,x_1,x_2) = -\tfrac{a}{2}x_i^2 +$$

$$\int_{\beta-x_i}^\infty [b_E(c;n)-c] \quad [1-G_B(b_E(c;n))][1-G(b_E(c;n)+x_1)][1-G(c+x_2)]^{n-2}dG(c+x_i).$$

Equilibrium Investments Under non-integration, the necessary first-order conditions for the symmetric equilibrium investment x is

$$x = \frac{1}{a}\frac{\mu}{\lambda+n\mu}e^{-\lambda\left[\frac{1}{\lambda+(n-1)\mu}+\beta-\alpha-x\right]}. \tag{19}$$

With vertical integration, the vertically integrated supplier invests x_1 and all $n-1$ independent suppliers invest x_2 satisfying

$$x_1 = x_2 + \frac{1}{a}\frac{\mu}{\lambda+\mu}e^{-\mu(x_1-x_2)}\left[e^{\mu(\beta-\alpha-x_2)} - e^{-\lambda(\beta-\alpha-x_2)-\frac{\lambda+\mu}{\lambda+(n-1)\mu}}\right] \tag{20}$$

and

$$x_2 = \frac{1}{a}\frac{\mu}{\lambda+n\mu}e^{-\lambda(\beta-\alpha-x_2)-\mu(x_1-x_2)-\frac{\lambda+\mu}{\lambda+(n-1)\mu}} \tag{21}$$

according to the necessary first-order conditions for equilibrium. We proceed by presuming that these conditions are also sufficient. (For the parameters underlying Figure 6 this can be verified numerically.)

Profitability of Non-Integration Evaluating (19), (20) and (21) numerically we can determine the buyer's and the independent suppliers' equilibrium profits under non-integration and vertical integration. Denoting these equilibrium payoffs with an asterisk, the analogue for the case of elastic demand to the function $\Phi(n,\mu)$ defined in (16) is

$$\Phi_E(n,\mu,\alpha,\lambda,\beta) := \Pi^{B*}_{EN} + \Pi^*_{EN} - \Pi^{B*}_{EI}.$$

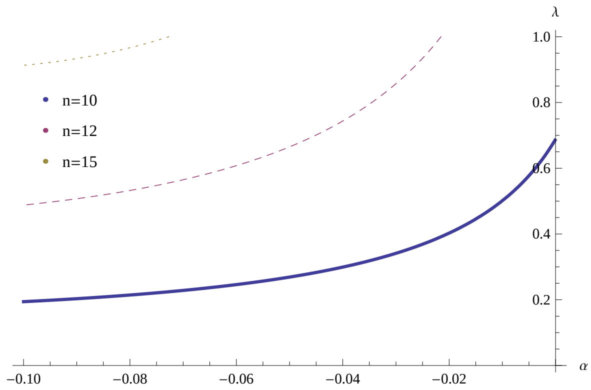

Figure 6: $\Phi_E(n, \mu, \alpha, \lambda) = 0$ for selected parameters.

Figure 6 contains contour sets of $\Phi_E(n, \mu, \alpha, \lambda) = 0$ for different values of n in (α, λ)-space with $\mu = 1$ and $\beta = 0$. Non-integration is profitable for a given n for values of α and λ below the corresponding curve.

Social Welfare Effects In the model with inelastic demand, non-integration is always socially optimal because it minimizes the sum of expected production and investment costs although it is not always an equilibrium outcome. In contrast, with elastic demand vertical integration has an additional, socially beneficial effect because it increases the market demand by inducing production for realizations of costs and values for which there is no production under non-integration, (and because it decreases the lowest cost of production by increasing investment by the integrated supplier).

The numerical analysis for the shifting support exponential model with elastic demand, displayed in Figure 7, reveals that vertical integration is better than non-integration when n is small. As before Φ is the private benefit from divestiture while ΔW is the difference between social welfare under divestiture and under vertical integration. The figure plots Φ and ΔW for $\beta = 0$ and $a = 1$. The figure illustrates a substantial range of upstream market structures for which vertical integration is privately optimal but socially inefficient.

A.3 Reserve Prices

We perform the analysis of the effect of reserve prices within the exponential-quadratic model with inelastic demand, setting $a = 1$. Suppose that the vertically integrated customer commits to a reserve price r after learning the cost of internal supply c_1. Given the symmetric equilibrium investment of independent firms x_2, the optimal reserve price satisfies

$$c_1 = r + \frac{G(r + x_2)}{g(r + x_2)} \equiv \Gamma_{x_2}(r)$$

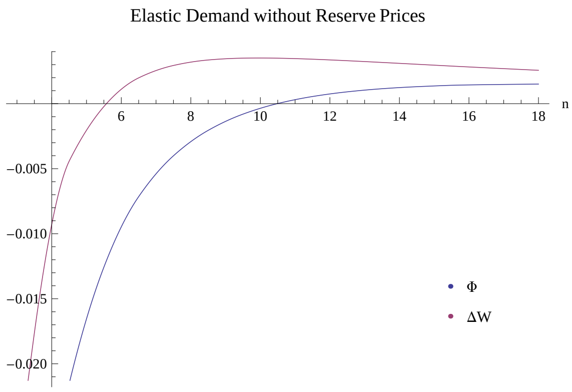

Figure 7: Φ and ΔW as functions of n without reserves.

while the symmetric bidding function $b(c, r)$ depends on the reserve price r according to[26]

$$b(c, r) = c + \frac{1}{\mu(n-1)}\left[1 - e^{-\mu(n-1)(r-c)}\right]$$

where we drop its dependence on n for notational ease.

In equilibrium, the vertically integrated firm chooses its own investment x_1 to minimize expected procurement cost given x_2, and each independent supplier invests to maximize expected profit given x_1 and x_2. The optimal reserve given $c_1 \geq \beta - x_2$ then satisfies

$$r(c_1) := \Gamma_{x_2}^{-1}(c_1). \tag{22}$$

Total equilibrium procurement cost (net of investment cost) is equal to the expected cost of internal supply, denoted $E_{x_1}[c_1] = \beta - x_1 + \frac{1}{\mu}$, minus the expected cost savings from sourcing externally:

$$E_{x_1}[c_1] - \int_{\beta-x_2}^{\infty}\int_{\beta-x_2}^{r(c_1)} [c_1 - b(c, r(c_1))]dL(c + x_2; n-1)dG(c_1 + x_1). \tag{23}$$

Assuming $x_1 > x_2$, the expected profit of a representative independent firm choosing x in the neighborhood of x_2 is equal to the expected value of the markup times the probability of winning the auction:

$$\int_{\beta-x_2}^{\infty}\int_{\beta-x}^{r(c_1)} [b(c, r(c_1)) - c][1 - L(c + x_2; n-2)]dG(c + x)dG(c_1 + x_1)$$

In equilibrium each independent supplier chooses $x = x_2$. We compute the equilibrium investments levels (x_1, x_2) solving the necessary first-order conditions, presuming the appropriate second-order conditions are satisfied.

[26]In the exponential case, the virtual cost function $\Gamma_{x_2}(r)$ is strictly increasing in r for given x_2, and therefore invertible. We denote its inverse by $\Gamma_{x_2}^{-1}(c_1)$. The bid function $b(c, r)$ solves the usual necessary differential equation for optimal bidding with the boundary condition $b(r, r) = r$.

The condition for non-integration to be preferred to vertical integration is similar to before. Figure 8 graphs Φ as a function of n for $\mu = 1$ and compares it to the case without reserves, depicted also in Figure 1. The curve is shifted to the right compared to the base model in which there is no reserve price. Although an optimal reserve price does lower procurement costs under vertical integration, non-integration nevertheless is preferred for n sufficiently large.

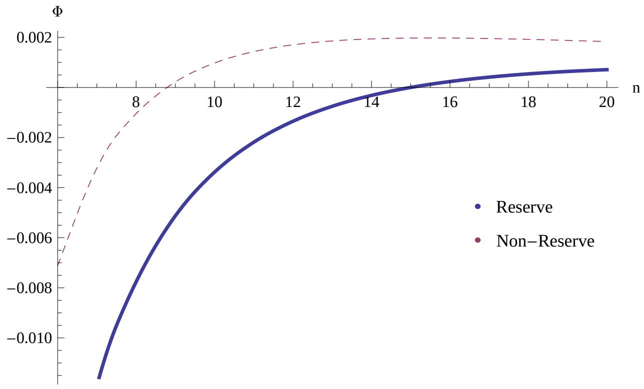

Figure 8: The function Φ with and without reserves for $\mu = 1$.

Elastic Demand with Reserve The analysis with elastic demand can also be extended to account for optimal reserves. Under non-integration, the optimal reserve is $r(v)$, where the function $r(.)$ is defined in (22). With vertical integration, the optimal reserve will be given by the same function $r(.)$, evaluated at $\hat{v} := \min\{c_1, v\}$. Because of continuity, it is intuitive that, with elastic demand and optimal reserves, non-integration will be profitable in the neighborhood of the parameter region for which it is profitable with perfectly inelastic demand and a reserve, that is, for values of λ close to zero. This intuition is corroborated by numerical analysis. Figure 9 plots the buyer's gain from non-integration with reserves, denoted Φ_{ER}, and her gain from non-integration without reserves, Φ_E, as a function of λ for $n = 16$ and $\alpha = \beta = 0$.

Figure 10 plots the social welfare effects of and the private incentives for divestiture for elastic demand when the customer can set a reserve price. Comparing Figure 7 to Figure 10 reveals that the ability to set a reserve hardly matters for the social welfare effects but increases the private benefits from vertical integration, thereby increasing the range in which vertical integration is an equilibrium outcome but not socially desirable.

A.4 Acquisition Game

This subsection considers an acquisition game when the initial market structure is non-integration. The game proceeds as follows.

In Stage 1, the customer makes sequential take-it-or-leave-it offers t_i to the independent suppliers $i = 1, .., n$. The sequence in which offers are made is pre-determined but since suppliers are symmetric *ex ante* this is arbitrary. Without loss of generality, we

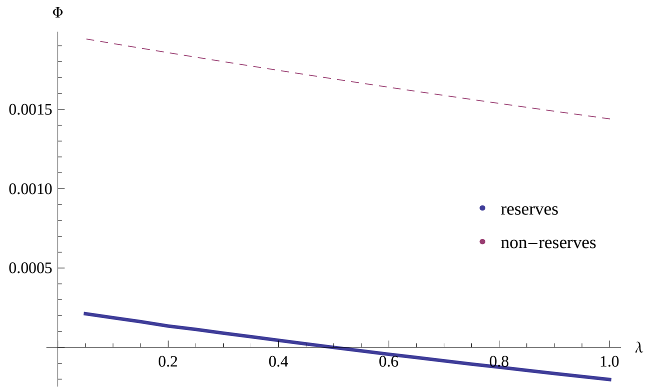

Figure 9: Φ_{ER} and Φ_E as functions of λ.

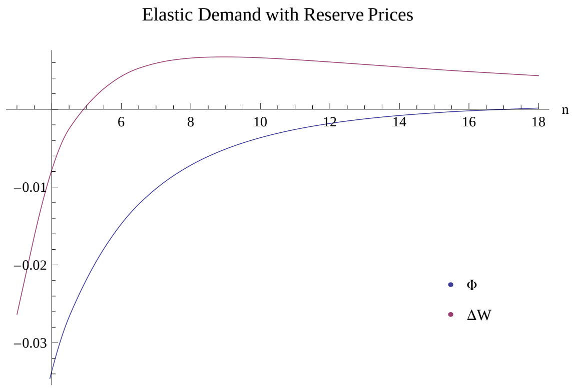

Figure 10: Φ and ΔW as functions of n with reserves.

assume that supplier i receives the i-th offer. If i accepts, the acquisition stage (i.e. Stage 1) ends and the Stage 2 subgame with vertical integration analyzed above ensues. If firm $i < n$ rejects, the customer makes the offer t_{i+1} to firm $i+1$. If supplier n receives an offer but rejects it, the Stage 2 subgame with non-integration analyzed above ensues.

The equilibrium behavior in Stage 1 is readily determined. Suppose first that $\Phi(n, \mu) < 0$. That is, vertical integration is jointly profitable. Then the subgame perfect equilibrium offers are $t_i = \Pi_I^*$ for $i < n$ and $t_n = \Pi_N^*$. On and off the equilibrium path, these offers are accepted. Notice that in order for supplier n to accept the offer he receives, he must be offered $t_n \geq \Pi_N^*$ because the alternative to his accepting is that the game with the non-integrated market structure ensues, in which case he nets Π_N^*. Anticipating that the last supplier would accept the offer if and only if he is offered Π_N^*, the alternative for any supplier $i < n$ when rejecting is that the ensuing market structure will be non-integration if $\Phi < 0$ and integration, with i as an independent supplier netting Π_I^* otherwise. Therefore, it suffices to offer $t_i = \Pi_I^*$ to i with $i = 1, .., n-1$, provided $t_n = \Pi_N^*$. But as the latter is only a credible threat if $\Phi(n, \mu) \leq 0$, it follows that vertical integration is more profitable than the necessary (and sufficient) condition for it to be an equilibrium outcome suggests: $\Phi(n, \mu) \leq 0$ must be the case for integration to occur on the equilibrium path, but if $\Phi(n,\mu) \leq 0$, the profit of integration to the customer is actually strictly larger than $-\Phi(n,\mu)$ because she has to pay less than Π_N^* on the equilibrium path.

Lastly, if $\Phi(n,\mu) > 0$, vertical integration is not jointly profitable and the customer will only make offers that will be rejected (e.g. $t_i \leq 0$ for all i would be a sequence of such offers).

B Proofs

Proof of Corollary 1: The necessary conditions have been derived in the main text. We are thus left to verify the conditions under which the second-order conditions for an equilibrium are satisfied. For $x_i \geq x$, the first derivative of $\Pi_N(x_i, x)$ with respect to x_i is

$$\frac{\partial \Pi_N(x_i, x)}{\partial x_i} = 1 - \frac{n-1}{n} e^{-\mu(x-x_i)} - ax_i.$$

Observe that this partial is decreasing in x_i. It is thus largest at $x_i = x$. Evaluated at $x_i = x$, the second right-hand partial is

$$\frac{\partial^2 \Pi_N(x_i, x)}{\partial x_i^2}|_{x_i=x} = \mu\frac{n-1}{n} - a.$$

For $x_i < x$, the first partial of $\Pi_N(x_i, x)$ with respect to x_i is

$$\frac{\partial \Pi_N(x_i, x)}{\partial x_i} = \frac{1}{n} e^{-\mu(n-1)(x-x_i)} - ax_i. \tag{24}$$

Because $\frac{1}{n}e^{-\mu(n-1)(x_i-x)}$ is increasing and convex in x_i while ax_i is linear and increasing in x_i, there is a unique solution $x_i \in [0,\infty)$ of (24). Evaluated at $x_i = x$, one obtains $x_i = x = 1/(an)$.

In turn, evaluated at $x_i = x$, the second left-hand partial is

$$\frac{\partial^2 \Pi_N(x_i, x)}{\partial x_i^2}|_{x_i=x} = \mu\frac{n-1}{n} - a.$$

Thus, the profit function is quasiconcave and the second-order condition is satisfied if and only if $\frac{\mu}{a} < \frac{n}{n-1}$.

To see that PC_N^* decreases in n, observe that

$$\frac{\partial PC_N^*}{\partial n} = \frac{(\mu - a)(n-1)^2 - an^2}{\mu a n^2 (n-1)^2},$$

which is negative if and only if $\frac{\mu}{a} < 1 + \frac{n^2}{(n-1)^2}$. The derivative of Π_N^* with respect to n is

$$\frac{\partial \Pi_N^*}{\partial n} = \frac{\mu(n-1)^2 - an(2n-1)}{\mu a n^3 (n-1)^2},$$

which has the same sign as $\mu(n-1) - an\left(1 + \frac{n}{n-1}\right)$. This is negative if and only if $\frac{\mu}{a} < \frac{n}{n-1}\left(1 + \frac{n}{n-1}\right)$. Both inequalities are satisfied if $\mu < a\frac{n}{n-1}$. ■

Proof of Proposition 2: Part (a): Equations (9) and (10) are the necessary first-order conditions as shown in the text.

Part (b): Denote by $x_2^1(x_1)$ and $x_2^2(x_1)$, respectively, the solutions to (9) and (10) in x_2. Invoking the implicit function theorem, we have

$$\frac{dx_2^2(x_1)}{dx_1} = -\frac{s_1(x_1, x_2^2(x_1))}{s_2(x_1, x_2^2(x_1)) - \psi'(x_2^2(x_1))} < 0, \tag{25}$$

where the inequality holds because strict quasi-concavity implies that the second-order conditions are strictly satisfied, that is, $(s_2 - \psi') < 0$, and

$$\frac{dx_2^1(x_1)}{dx_1} = -\frac{s_1(x_1, x_2^1(x_1)) + \frac{1}{n-1}\psi'(x_1)}{s_2(x_1, x_2^1(x_1))} < 0, \tag{26}$$

where the inequality holds because of strict quasi-convexity (i.e. $(n-1)s_1 + \psi' > 0$) and because of assumption (ii), i.e. $s_2 > 0$.

Assume that that there is a point of intersection of $x_2^1(x_1)$ and $x_2^2(x_1)$, that is, there is at least one value of x_1, denoted x_1', such that $x_2^1(x_1') = x_2^2(x_1')$. Under assumptions (i) and (ii), we have

$$\frac{dx_2^1(x_1)}{dx_1}|_{x_1=x_1'} < \frac{dx_2^2(x_1)}{dx_1}|_{x_1=x_1'} < 0, \tag{27}$$

which proves uniqueness of such a point of intersection. Next, we establish that such a point exists, is an equilibrium, and satisfies $x_1' = x_1^* > x_2^* = x_2^2(x_1')$.

Let $\overline{x}$ be the smallest number such that $1 - (n-1)s(\overline{x}, \overline{x}) = \psi(\overline{x})$ and let $\hat{x}$ be the smallest number such that $s(\hat{x}, \hat{x}) = \psi(\hat{x})$. Because $s(x, x) < 1/n$ as noted, it follows that $\overline{x} > \psi^{-1}(1/n) > \hat{x}$. This implies that $x_2^2(\hat{x}) = \hat{x} < \overline{x} = x_2^1(\overline{x})$.

Next, let $\overline{x}_1$ be such that $1 - (n-1)s(\overline{x}_1, 0) = \psi(\overline{x}_1)$. Notice that $\overline{x}_1 > \overline{x}$. Because of assumptions (ii) and (iii), we know that $\overline{x}_1 < \psi^{-1}(1)$. Therefore, $s(\overline{x}_1, 0) > 0$. Consequently, $x_2^2(\overline{x}_1) > 0$. Lastly, let $\tilde{x}_1$ be such that $s(\tilde{x}_1, 0) = 0$. Notice that $\tilde{x}_1$ may be infinity. Because $s_1 < 0$, $\tilde{x}_1 > \overline{x}_1$ follows.

Taken together we have thus shown that $x_2^2(x_1)$ is a continuously decreasing function in x_1 on $[\hat{x}, \tilde{x}_1]$ satisfying $x_2^2(\overline{x}_1) > 0$ and $x_2^2(\overline{x}) < x_2^2(\tilde{x}) < x_2^1(\overline{x})$. Moreover, on $[\overline{x}, \overline{x}_1]$, $x_2^1(x_1)$ is a continuous function satisfying $x_2^1(\overline{x}) > x_2^2(\overline{x})$ and $x_2^1(\overline{x}_1) = 0 < x_2^2(\overline{x}_1)$. Thus, the functions $x_2^1(x_1)$ and $x_2^2(x_1)$ have a point of intersection on $[\overline{x}, \overline{x}_1]$.

Quasi-concavity and quasi-convexity imply that this point of intersection is an equilibrium. For all $x_1 \in (\hat{x}, \overline{x}_1]$, we have $x_2^2(x_1) < x_1$, which proves that $x_1^* := x_1' > x_2^* := x_2^2(x_1')$. Finally, $x_1^* > x^*$ and $x^* > x_2^*$ then follows from the first-order condition under non-integration, $1/n = \psi(x^*)$, and $s(x_1^*, x_2^*) < 1/n$, which holds because $s(x, x) < 1/n$, $x_1^* > x_2^*$ as just shown, and $s_1 < 0 < s_2$ by assumption (ii). ■

Proof of Corollary 2: Under non-integration, equilibrium effort is given by $\psi(x^*) = \frac{1}{n}$. On the other hand, rewriting the consolidated equilibrium condition with vertical integration, (11), as $\frac{n-1}{n}\psi(x_2)+\frac{1}{n}\psi(x_1)=\frac{1}{n}$, it follows from Jensen's inequality that $(n-1)x_2+x_1=nx^*$ if $\psi''=0$ and $(n-1)x_2+x_1>nx^*$ if $\psi''<0$ and $(n-1)x_2+x_1<nx^*$ if $\psi''>0$. ■

Proof of Corollary 3: The arguments in the main text imply that PC_I^* and Π_I^* are the equilibrium payoffs of the integrated firm and the independent suppliers with Δ given by (13) and x_1 and x_2 given by (14).

Having already argued in the main text why assumptions (ii) and (iii) are satisfied, we are thus left to show that assumption (i) is satisfied.

We begin by establishing *quasi-concavity* of $\Pi_I(x_i,x_1,x_2)$.

Case 1: $x_i<x_2$ We first look at a downward deviation $x_i<x_2$ by a non-integrated supplier. The first and second partials of $\Pi_I(x_i,x_1,x_2)$ with respect to x_i are

$$\frac{\partial\Pi_I(x_i,x_1,x_2)}{\partial x_i}=\frac{1}{n}e^{-\mu\Delta-\frac{1}{n-1}+\mu(n-1)(x_i-x_2)}-ax_i$$

and

$$\frac{\partial^2\Pi_I(x_i,x_1,x_2)}{\partial x_i^2}=\frac{\mu(n-1)}{n}e^{-\mu\Delta-\frac{1}{n-1}+\mu(n-1)(x_i-x_2)}-a.$$

The profit function is thus concave on $[0,x_2]$ if and only if $\frac{\mu(n-1)}{n}e^{-\mu\Delta-\frac{1}{n-1}+\mu(n-1)(x_i-x_2)}-a\leq 0$. As the term $\frac{\mu(n-1)}{n}e^{-\mu\Delta-\frac{1}{n-1}+\mu(n-1)(x_i-x_2)}$ increases in x_i, this second-order condition is thus satisfied if and only if

$$\frac{\mu}{a}\leq\frac{n}{(n-1)(1-a\Delta)},$$

where $1-a\Delta=e^{-\mu\Delta-\frac{1}{n-1}}$ has been used. Since $a\Delta<1$, this second-order condition is always satisfied if the necessary condition for a symmetric equilibrium under non-integration holds.

Let $\hat{\hat{x}}=x_2+\frac{n-2}{\mu(n-1)}$.

Case 2: $x_i\in[x_2,\hat{\hat{x}}]$ Next we consider deviations by i such that $c_i\in\left[\beta-x_2-\frac{1}{\mu}\frac{n-2}{n-1},\beta-x_2\right]$ occur with positive probability, and no lower c_i can occur. From Lemma 2 we know that for cost realizations in this interval, the optimal bid by i will be the constant bid $\beta-x_2+\frac{1}{\mu(n-1)}$.

For $x_i\in[x_2,\hat{\hat{x}}]$ the profit function for the deviating supplier i is

$$\begin{aligned}\Pi_I(x_i,x_1,x_2) &= \frac{1}{n-1}\int_{\beta-x_2}^{\infty}e^{-\mu[n(c_i-\beta)+x_1+(n-2)x_2+x_i+\frac{1}{\mu(n-1)}]}dc_i\\ &+\int_{\beta-x_i}^{\beta-x_2}\mu\left(\beta-x_2+\frac{1}{\mu(n-1)}-c_i\right)e^{-\mu\Delta-\frac{1}{n-1}-\mu(c_i+x_i-\beta)}dc_i-\frac{a}{2}x_i^2\\ &= e^{-\mu\Delta-\frac{1}{n-1}}\left[x_i-x_2-\frac{n-2}{\mu(n-1)}+e^{-\mu(x_i-x_2)}\frac{n-1}{\mu n}\right]-\frac{a}{2}x_i^2.\end{aligned}$$

The first and second partial derivatives are

$$\begin{aligned}\frac{\partial \Pi_I(x_i,x_1,x_2)}{\partial x_i} &= e^{-\mu\Delta-\frac{1}{n-1}}\left[1-\frac{n-1}{n}e^{-\mu(x_i-x_2)}\right]-ax_i\\ \frac{\partial^2 \Pi_I(x_i,x_1,x_2)}{\partial x_i^2} &= e^{-\mu\Delta-\frac{1}{n-1}}\left[\mu\frac{n-1}{n}e^{-\mu(x_i-x_2)}\right]-a.\end{aligned}$$

Therefore, on $[x_2,\hat{\hat{x}}]$, the deviator's profit function is concave in x_i, and maximized at $x_i = x_2$ if and only if

$$\frac{\mu}{a} < \frac{n}{(n-1)(1-a\Delta)},$$

which is the same condition derived in Case 1.
Case 3: $x_i \in [\hat{\hat{x}}, x_1+\frac{1}{\mu}]$ We next consider investments $x_i \in [\hat{\hat{x}}, x_1+\frac{1}{\mu}]$. The expected profit of the deviating supplier is

$$\begin{aligned}\Pi_I(x_i,x_1,x_2) &= \frac{1}{n-1}\int_{\beta-x_2}^{\infty} e^{-\mu[n(c_i-\beta)+x_1+(n-2)x_2+x_i+\frac{1}{\mu(n-1)}]}dc_i\\ &+ \int_{\beta-x_2-\frac{n-2}{\mu(n-1)}}^{\beta-x_2} \mu\left(\beta-x_2+\frac{1}{\mu(n-1)}-c_i\right)e^{-\mu\Delta-\frac{1}{n-1}-\mu(c_i+x_i-\beta)}dc_i\\ &+ \int_{\beta-x_i}^{\beta-x_2-\frac{n-2}{\mu(n-1)}} e^{-\mu[2(c-\beta)+x_1+x_i]-1}dc_i-\frac{a}{2}x_i^2\\ &= e^{-\mu\Delta-\frac{1}{n-1}-\mu(x_i-x_2)}\frac{1}{\mu}\frac{n-1}{n}-\frac{1}{2\mu}e^{-\mu\Delta-\frac{1}{n-1}}\left[e^{-\mu(x_i-x_2)+\frac{n-2}{n-1}}-e^{\mu(x_i-x_2)-\frac{n-2}{n-1}}\right]-\frac{a}{2}x_i^2\end{aligned}$$

as the profit function for a deviating independent supplier choosing investment $x_i \in [\hat{\hat{x}}, x_1+\frac{1}{\mu}]$. Using the facts that $1-a\Delta = e^{-\mu\Delta-\frac{1}{n-1}}$ and $x_2 = \frac{1}{an}(1-a\Delta)$ and defining $y := \mu(x_i-x_2)-\frac{n-2}{n-1}$, we can express the deviator's profit equivalently as

$$\hat{\Pi}_I(y,x_1,x_2) = \frac{1-a\Delta}{\mu}\left[e^{-y-\frac{n-2}{n-1}}\frac{n-1}{n}+\frac{1}{2}\left[e^{y}-e^{-y}\right]\right]-\frac{a}{2}\left(\frac{1}{\mu}\left[y+\frac{n-2}{n-1}\right]+\frac{1}{an}(1-a\Delta)\right)^2,$$

for $y \in [0, \mu\Delta+\frac{1}{n-1}]$.

We are now going to show that $\hat{\Pi}_I(y,x_1,x_2)$ is decreasing and concave in y for all $y \in [0,\mu\Delta+\frac{1}{n-1}]$. We do so by first establishing that $\frac{\partial \hat{\Pi}_I(y,x_1,x_2)}{\partial y}|_{y=0} < 0$. Second, we show that the third derivative with respect to y is positive. This implies that the second derivative is largest over this interval at $y = \mu\Delta+\frac{1}{n-1}$. The final step in the argument is then to show that $\frac{\partial^2 \hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}|_{y=\mu\Delta+\frac{1}{n-1}} < 0$, which then implies that $\hat{\Pi}_I(y,x_1,x_2)$ is concave over the interval in question.
Step 1:

$$\frac{\partial \hat{\Pi}_I(y,x_1,x_2)}{\partial y} = \frac{1-a\Delta}{\mu}\left[-e^{-y-\frac{n-2}{n-1}}\frac{n-1}{n}+\frac{1}{2}\left(e^{y}+e^{-y}\right)\right]-\frac{a}{\mu}\left[\frac{1}{\mu}(y+\frac{n-2}{n-1})+\frac{1}{an}(1-a\Delta)\right].$$

At $y = 0$, we get

$$\frac{\partial \hat{\Pi}_I(y, x_1, x_2)}{\partial y}|_{y=0} = \frac{1}{\mu}\left\{\frac{n-1}{n}(1-a\Delta)\left[1 - e^{\frac{n-2}{n-1}}\right] - \frac{a}{\mu}\frac{n-2}{n-1}\right\}.$$

Since $(1 - a\Delta) < 1$ and $\frac{a}{\mu} \geq \frac{n-1}{n}$ under the necessary and sufficient condition for the existence of a symmetric equilibrium under non-integration, we have

$$\frac{\partial \hat{\Pi}_I(y, x_1, x_2)}{\partial y}|_{y=0} < \frac{1}{\mu}\frac{n-1}{n}\left\{1 - e^{\frac{n-2}{n-1}} - \frac{n-2}{n-1}\right\}.$$

The term in brackets is decreasing in n and equal to 0 at $n = 2$. Thus, $\frac{\partial \hat{\Pi}_I(y,x_1,x_2)}{\partial y}|_{y=0} < 0$ holds for all n.
Step 2: Differentiating further we get

$$\begin{aligned}\frac{\partial^2 \hat{\Pi}_I(y, x_1, x_2)}{\partial y^2} &= \frac{1-a\Delta}{\mu}\left[e^{-y-\frac{n-2}{n-1}}\frac{n-1}{n} + \frac{1}{2}\left(e^y - e^{-y}\right)\right] - \frac{a}{\mu^2} \\ &= \frac{1-a\Delta}{\mu}\left[\frac{1}{2}e^y + \left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}} - \frac{1}{2}\right)e^{-y}\right] - \frac{a}{\mu^2},\end{aligned}$$

where $\frac{n-1}{n}e^{-\frac{n-2}{n-1}} - \frac{1}{2} \leq 0$ for all $n \geq 2$ with strict inequality for $n > 2$ (at $n = 2$, it is equal to 0; differentiating with respect to n yields $-\frac{e^{-\frac{n-2}{n-1}}}{n^2(n-1)}$, which is negative), and

$$\frac{\partial^3 \hat{\Pi}_I(y, x_1, x_2)}{\partial y^3} = \frac{1-a\Delta}{\mu}\left[\frac{1}{2}e^y - \left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}} - \frac{1}{2}\right)e^{-y}\right] > 0.$$

Thus, $\frac{\partial^2 \hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}$ is an increasing function of y and hence largest at $y = \mu\Delta + \frac{1}{n-1}$.

Step 3: Evaluating $\frac{\partial^2 \hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}$ at $y = \mu\Delta + \frac{1}{n-1}$ one gets

$$\frac{\partial^2 \hat{\Pi}_I(y, x_1, x_2)}{\partial y^2}|_{y=\mu\Delta+\frac{1}{n-1}} = \frac{1-a\Delta}{\mu}\left[\frac{1}{2}e^{\mu\Delta+\frac{1}{n-1}} + \left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}} - \frac{1}{2}\right)e^{-\mu\Delta-\frac{1}{n-1}}\right] - \frac{a}{\mu^2}.$$

Replacing $e^{-\mu\Delta-\frac{1}{n-1}}$ by $1 - a\Delta$ and $e^{\mu\Delta+\frac{1}{n-1}}$ by $\frac{1}{1-a\Delta}$ and collecting terms yields

$$\frac{\partial^2 \hat{\Pi}_I(y, x_1, x_2)}{\partial y^2}|_{y=\mu\Delta+\frac{1}{n-1}} = \frac{\mu - 2a}{2\mu^2} + \left(\frac{n-1}{n}e^{-\frac{n-2}{n-1}} - \frac{1}{2}\right)\frac{(1-a\Delta)^2}{\mu}.$$

As just noticed the last expression is not positive. Therefore, $\frac{\partial^2 \hat{\Pi}_I(y,x_1,x_2)}{\partial y^2}|_{y=\mu\Delta+\frac{1}{n-1}} < 0$ if $\frac{\mu}{a} < 2$, which is certainly the case if $\frac{\mu}{a} < \frac{n}{n-1}$, which is the necessary and sufficient condition for the existence of a symmetric equilibrium under non-integration.

Case 4: $x_i > x_1 + \frac{1}{\mu}$ Finally, consider investments $x_i > x_1 + \frac{1}{\mu}$. For such investments, the expected profit of a deviating non-integrated supplier is

$$
\begin{aligned}
\Pi_I(x_i, x_1, x_2) &= \frac{1}{n-1}\int_{\beta-x_2}^{\infty} e^{-\mu[n(c_i-\beta)+x_1+(n-2)x_2+x_i+\frac{1}{\mu(n-1)}]}dc_i \\
&+ \int_{\beta-x_2-\frac{n-2}{\mu(n-1)}}^{\beta-x_2} \mu\left(\beta - x_2 + \frac{1}{\mu(n-1)} - c_i\right) e^{-\mu\Delta-\frac{1}{n-1}-\mu(c_i+x_i-\beta)}dc_i \\
&+ \int_{\beta-x_1-\frac{1}{\mu}}^{\beta-x_2-\frac{n-2}{\mu(n-1)}} e^{-\mu[2(c-\beta)+x_1+x_i]-1}dc_i \\
&+ \int_{\beta-x_i}^{\beta-x_1-\frac{1}{\mu}} (\beta - x_1 - c_i)\mu e^{-\mu(c_i+x_i-\beta}dc_i - \frac{a}{2}x_i^2 \\
&= \frac{1}{\mu}\frac{n-1}{n}e^{-\mu\Delta-\frac{1}{n-1}-\mu(x_i-x_2)} \\
&+ \frac{1}{2\mu}e^{-\mu(x_i-x_i)+1}\left[1 - e^{-2(\mu\Delta+\frac{1}{n-1})}\right] \\
&+ x_i - x_1 - \frac{1}{\mu} - \frac{a}{2}x_i^2.
\end{aligned}
$$

The key observation is that the terms in the third to last and second to last lines decrease in x_i. The derivative of the last line with respect to x_i is $1 - ax_i$. Since $x_i \geq x_1 + \frac{1}{\mu} \geq \frac{1}{an} + \frac{1}{\mu}$, we have

$$
1 - ax_i \leq \frac{n-1}{n} - \frac{a}{\mu} \leq 0,
$$

where the inequality follows because it is equivalent to $\frac{\mu}{a} \leq \frac{n}{n-1}$.

We now turn to establishing *quasi-convexity* of $PC_I(x_1, x_2)$ over the relevant range.

For the exponential model with $x_1 \geq x_2 - \frac{1}{\mu(n-1)}$, $PC_I(x_1, x_2)$ is as defined in (8) because $b_I^*(\beta - x_2) = \beta - x_2 + \frac{1}{\mu(n-1)} \geq \beta - x_1$ under this condition. With quadratic costs of effort, we have for any $x_1 \geq x_2 - \frac{1}{\mu(n-1)}$,

$$
\frac{\partial PC_I(x_1, x_2)}{\partial x_1} = -(1 - (n-1)s(x_1, x_2) - ax_1).
$$

Noticing that $s_1(x_1, x_2) = -\mu s(x_1, x_2)$, the second-order condition is

$$
\frac{\partial^2 PC_I(x_1, x_2)}{\partial x_1^2} = -(n-1)\mu s(x_1, x_2) + a \geq 0.
$$

Observe that for all $x_1 \geq x_2 - \frac{1}{\mu(n-1)}$, $s(x_1, x_2) \leq \frac{1}{n}$, with equality only if $x_1 = x_2 - \frac{1}{\mu(n-1)}$. Thus, $-(n-1)\mu s(x_1, x_2) + a \geq a - (n-1)\mu/n \geq 0$, where the last inequality holds because of the assumption $\frac{\mu}{a} \leq \frac{n}{n-1}$. Thus, on $[x_2 - \frac{1}{\mu(n-1)}, \infty)$, $PC_I(x_1, x_2)$ is convex in x_1.

For $x_1 \leq x_2 - \frac{1}{\mu(n-1)}$, the procurement cost of the integrated supplier can be written as

$$PC_I(x_1,x_2) = \Psi(x_1) + \int_{\beta-x_2}^{\infty} b_I^*(c;\mathbf{x},n)dL(c+x_2;n-1) \tag{28}$$
$$+ \int_{\beta-x_1}^{\infty}\int_{b_I^{-1}(c)}^{\infty}(c-b_I^*(y;\mathbf{x},n))dL(y+x_2;n-1)dG(c+x_1),$$

where $b_I^{-1}(c)$ is the inverse of $b_I^*(y;\mathbf{x},n)$, i.e. $b_I^*(b_I^{-1}(c);\mathbf{x},n)=c$ (for example, for the exponential $b_I^{-1}(c)=c-\frac{1}{\mu(n-1)}$). Here the first line captures cost of effort plus the cost of always procuring the good from the independent suppliers. The second line represents the cost savings from avoiding the markup by producing internally. Observe that the integral in the first line does not depend on x_1 if x_1 is a deviation from equilibrium (only the equilibrium level of x_1 affects $b_I^*(c;\mathbf{x},n)$ with $\mathbf{x}=(x_1,x_2)$). Therefore, for the purpose of cost minimization, it can be treated as a constant, denoted K.

Making use of the exponential-quadratic assumptions, we obtain

$$PC_I(x_1,x_2) = ax_1^2/2 + K - \frac{1}{\mu(n-1)}\frac{e^{\mu(n-1)(x_1-x_2)+1}}{n},$$

whose derivatives are

$$\frac{\partial PC_I(x_1,x_2)}{\partial x_1} = -\frac{e^{\mu(n-1)(x_1-x_2)+1}}{n} + ax_1$$

and

$$\frac{\partial^2 PC_I(x_1,x_2)}{\partial x_1^2} = -\mu(n-1)\frac{e^{\mu(n-1)(x_1-x_2)+1}}{n} + a.$$

Thus, the function is (twice) continuously differentiable.

Notice also that because $x_2 \le \frac{1}{a(n-1)}$, $x_1 \le x_2 - \frac{1}{\mu(n-1)}$ is not possible if $\mu \le a$ as $\mu \le a$ implies $\frac{1}{a(n-1)} - \frac{1}{\mu(n-1)} \le 0$, which would thus require $x_1 \le 0$. In the following analysis, we can thus assume $1 \le \frac{\mu}{a} \le \frac{n}{n-1}$.

For a fixed $x_2 > 0$, let $\underline{x}_1(x_2)$ denote the smallest non-negative value of x_1 such that $\frac{\partial PC_I(x_1,x_2)}{\partial x_1} = 0$, that is, $\underline{x}_1(x_2)$ is such that:

$$\frac{e^{\mu(\underline{x}_1(x_2)-x_2)+1}}{n} = a\underline{x}_1(x_2).$$

(If no such value exists, we set $\underline{x}_1(x_2)=\infty$). Because $h(x_1) := \frac{e^{\mu(x_1-x_2)+1}}{n}$ is increasing and convex in x_1, satisfying $\frac{e^{\mu(-x_2)+1}}{n} > 0$, while ax_1 is increasing linearly in x_1 and equal to 0 at $x_1=0$, it follows that $\underline{x}_1(x_2) > 0$ and $\partial PC_I(x_1,x_2)/\partial x_1 < 0$ for all $x_1 < \underline{x}_1(x_2)$.

We are now going to show that $\underline{x}_1(x_2) > x_2 - \frac{1}{\mu(n-1)}$. This then completes the proof of quasiconvexity. Observe that $h\left(x_2 - \frac{1}{\mu(n-1)}\right) = \frac{1}{n}$. This is larger than $a\left(x_2 - \frac{1}{\mu(n-1)}\right)$ if and only if

$$\frac{1}{n} > ax_2 - \frac{a}{\mu(n-1)}.$$

Because $x_2 \leq \frac{1}{a(n-1)}$, the right-hand side is not more than $\frac{1}{n-1}\left(1-\frac{a}{\mu}\right)$ and because $\frac{\mu}{a} \leq \frac{n}{n-1}$, we have in turn

$$\frac{1}{n-1}\left(1-\frac{a}{\mu}\right) \leq \frac{1}{n-1}\frac{1}{n}.$$

But this is less than $\frac{1}{n}$, which thereby completes the proof. ■

Proof of Corollary 4: Inserting the expressions obtained in Corollaries 1 and 3, one obtains

$$\beta + \frac{a-\mu}{\mu}x_1 + \frac{a}{2}x_1^2 + \frac{1}{n}\left[\frac{1}{\mu(n-1)} - \frac{1}{2an}\right]$$

for $PC_I^* + \Pi_N^*$. As $PC_N^* = \beta - \frac{1}{an} + \frac{1}{\mu}\frac{2n-1}{n(n-1)}$, vertical divestiture is thus jointly profitable if and only if

$$\beta + \frac{a-\mu}{\mu}x_1 + \frac{a}{2}x_1^2 + \frac{1}{n}\left[\frac{1}{\mu(n-1)} - \frac{1}{2an}\right] > \beta - \frac{1}{an} + \frac{1}{\mu}\frac{2n-1}{n(n-1)},$$

which is equivalent to the inequality in the corollary. ■

Proof of Proposition 4: The proof uses symmetry and quasiconvexity of the function $TC(\mathbf{x})$.

The function $TC(\mathbf{x})$ is *symmetric* in the sense that, for $x_i = x$ and $x_j = x'$ with $i \neq j$, we have

$$TC(x, x', \mathbf{x}_{-i-j}) = TC(x', x, \mathbf{x}_{-i-j}),$$

where $\mathbf{x}_{-i-j} = (x_k)_{k\neq i,j}$.

The rest of the proof is by contradiction. That is, suppose to the contrary that $\min_{\mathbf{x}} TC(\mathbf{x}) = TC(\hat{\mathbf{x}})$, where $\hat{\mathbf{x}}$ is not symmetric, i.e. $\hat{x}_i \neq \hat{x}_j$ for some $i \neq j$, and that there is no symmetric investment, denoted $\mathbf{x}^S$ such that $TC(\mathbf{x}^S) = \min_{\mathbf{x}} TC(\mathbf{x})$. Without loss of generality, let $i = 1$ and $j = 2$. Let $\hat{\hat{\mathbf{x}}} = (\hat{x}_2, \hat{x}_1, \hat{x}_3, .., \hat{x}_n)$. That is, $\hat{\hat{\mathbf{x}}}$ is a permutation of $\hat{\mathbf{x}}$. By symmetry, we have

$$TC(\hat{\hat{\mathbf{x}}}) = TC(\hat{\mathbf{x}}).$$

But by quasiconvexity, we have, for any $t \in (0, 1)$,

$$TC(t\hat{\hat{\mathbf{x}}} + (1-t)\hat{\mathbf{x}}) \leq TC(\hat{\mathbf{x}}),$$

which is a contradiction to the hypothesis that TC is minimized at $\hat{\mathbf{x}}$ and not at a symmetric investment $\mathbf{x}^S$.

The last part of the statement follows by noting that at symmetry, i.e. $x_i = x$ for all i, total cost, denoted $TC_S(x)$ is

$$TC_S(x) = \int_{\beta-x}^{\infty} cl(c+x;n)dc + n\Psi(x),$$

Noting $\partial l(c+x;n)/\partial x = \partial l(c+x;n)/\partial c$, we can write the derivative $TC'_S(x)$ using integration by parts as

$$TC'_S(x) = -\underbrace{\int_{\beta-x}^{\infty} l(c+x;n)dc}_{=1} + n\psi(x).$$

Setting $TC'_S(x) = 0$, we thus get $x = \psi^{-1}(1/n)$. Moreover, $TC''_S(x) = n\psi'(x) > 0$, so this is indeed a minimum. ■

Proof of Corollary 5: We first show that $TC(\mathbf{x})$ is quasiconvex if $\mu \le a$ by showing that there is a unique solution to the system of first-order conditions. Second, we show that for $\mu > a$, the symmetric solution to the first-order conditions is not socially optimal. Although this is not required to prove the corollary, we state it here because we referred to this result in the text.

Substituting the expressions for the exponential case gives us the following expression for the expected production cost:

$$EC(\mathbf{x}) = \mu \sum_{j=1}^{n} je^{-\mu X_j} \int_{\beta-x_j}^{\beta-x_{j+1}} ce^{-j\mu(c-\beta)}dc,$$

where $X_j := \sum_{i=1}^{j} x_i$, $x_{n+1} := -\infty$, and $TC(\mathbf{x}) = EC(\mathbf{x}) + \sum_i \Psi(x_i)$. Letting

$$S_j := e^{-\mu(X_j - jx_j)}\left[\beta - x_j + \frac{1}{j\mu} - \left(\beta - x_{j+1} + \frac{1}{j\mu}\right)e^{-j\mu(x_j - x_{j+1})}\right],$$

it then follows that

$$\frac{\partial EC(\mathbf{x})}{\partial x_j} = \mu e^{-\mu(X_j - jx_j)}(\beta - x_j) - \mu \sum_{i=j}^{n} S_i$$

for all $j = 1, .., n$ and

$$\frac{\partial EC(\mathbf{x})}{\partial x_j} - \frac{\partial EC(\mathbf{x})}{\partial x_{j+1}} = -\frac{1}{j}e^{-\mu(X_j - jx_j)}(-1 + e^{-\mu(x_j - x_{j+1})})$$

for all $j < n$.

Finally,

$$\frac{\partial S_n}{\partial x_n} = \mu(n-1)e^{-\mu(X_n - nx_n)}\left(\beta - x_n + \frac{1}{n\mu}\right)$$

and the derivative of $EC(\mathbf{x})$ with respect to x_n is

$$\frac{\partial EC(\mathbf{x})}{\partial x_n} = \frac{\partial S_n}{\partial x_n} + \frac{\partial S_{n-1}}{\partial x_n} = -\frac{1}{n}e^{-\mu(X_n - nx_n)}.$$

Observe that

$$\partial TC(\mathbf{x})/\partial x_i = \partial EC(\mathbf{x})/\partial x_i + \psi(x_i).$$

Using the first-order condition $\partial TC(\mathbf{x})/\partial x_n = 0$, we obtain the boundary condition

$$\frac{1}{n}e^{-\mu(X_n - nx_n)} = ax_n. \tag{29}$$

Subtracting $\frac{\partial TC(\mathbf{x})}{\partial x_i}$ from $\frac{\partial TC(\mathbf{x})}{\partial x_{i+1}}$ and simplifying yields for $i = 1, .., n-2$ with $n > 2$ a system of first-order difference equations

$$\frac{1}{i}e^{-\mu X_i}\left[e^{i\mu x_{i+1}} - e^{i\mu x_i}\right] = a(x_{i+1} - x_i) \tag{30}$$

with the boundary condition (29) and the constraints $x_i \geq x_{i+1}$. Notice that the symmetric solution $x_i = \frac{1}{an}$ for all $i = 1, .., n$ is always a solution of this system. We are now going to show that for $a \geq \mu$ it is the unique solution.

Notice first that the right-hand side of (30) is linear in x_{i+1} with slope a. The left-hand side of (30) is increasing and convex in x_{i+1} with slope μ at symmetry. Fix then an arbitrary x_1. Provided $\mu \leq a$, $x_2 = x_1$ is the unique solution to (30). Iterating the argument, we get that $x_i = x_1$ is the unique solution to (30) for all $i = 1, .., n-1$. Notice then that the left-hand side of (29) is convex and increasing in x_n with slope $\mu\frac{n-1}{n}$ at symmetry. Since $\mu \leq a$ implies $\mu\frac{n-1}{n} < a$, where a is the slope of the right-hand side of (29), it follows that $x_n = x_1$ is the unique solution to (29). But at symmetry, (29) implies $x_n = \frac{1}{an}$. Thus, for $\mu \leq a$, $x_i = \frac{1}{an}$ for all $i = 1, .., n$ is the unique solution. This completes the proof of the claim in to corollary.

The remainder of the proof shows that the symmetric solution is not a minimizer of $TC(\mathbf{x})$ if $\mu > a$ by showing that $\mathbf{x} = (x_1, x_2, ..., x_2)$ with $x_1 > x_2$ optimally chosen does strictly better. The second own and cross partial of $TC(x_1, x_2)$ are

$$\begin{aligned}
\frac{\partial^2 TC(x_1, x_2)}{\partial x_1^2} &= a - \mu\frac{n-1}{n}e^{-\mu(x_1 - x_2)} = \frac{\partial^2 TC(x_1, x_2)}{\partial x_2^2} \\
\frac{\partial^2 TC(x_1, x_2)}{\partial x_1 \partial x_2} &= \mu\frac{n-1}{n}e^{-\mu(x_1 - x_2)}.
\end{aligned}$$

At $x_1 = x_2$, the Hessian matrix $\mathbf{H}$ can be shown to positive semi-definite if and only if $\mu \leq a$. This can be shown by noting that the product $\mathbf{z} \cdot \mathbf{H} \cdot \mathbf{z}$ with $\mathbf{z} = (z_1, z_2) \neq \mathbf{0}$ is quasiconvex (quasiconcave) in z_2 if $\mu \leq an$ ($\mu > an$) and minimized (maximized) at $z_2 = -z_1/(n-1)$. Evaluated at $z_2 = -z_1/(n-1)$, we have

$$\mathbf{z} \cdot \mathbf{H} \cdot \mathbf{z} = \frac{anz_1^2(a - \mu)}{an - \mu}.$$

For $a < \mu < an$, $\mathbf{z} \cdot \mathbf{H} \cdot \mathbf{z} > 0$, which proves that $\mathbf{H}$ is positive semi-definite. Because of our restriction $\mu \leq \frac{n}{n-1}a$, we know that $\mu < an$. (For completeness, if $\mu > an$, we know that $\mathbf{z} \cdot \mathbf{H} \cdot \mathbf{z}$ is maximized at $z_2 = -z_1/(n-1)$ and negative at this point. Consequently, $\mathbf{z} \cdot \mathbf{H} \cdot \mathbf{z} < 0$ for all $\mathbf{z} \neq \mathbf{0}$ if $\mu > an$.) ■