Market Structure and the Competitive Effects of Vertical Integration[*]

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February 12, 2014

Abstract

We analyze the competitive effects of backward vertical integration when firms exert market power upstream and compete in quantities downstream. Contrasting with previous literature, a small degree of vertical integration is always procompetitive because efficiency gains dominate foreclosure effects, and vertical integration even to full foreclosure can be procompetitive. Interestingly, vertical integration is more likely to be procompetitive if the industry is otherwise more concentrated. Extensions analyze welfare effects of integration and the incentives to integrate. Our analysis suggests that antitrust authorities should be wary of vertical integration when the integrating firm faces many competitors and should be permissive otherwise.

Keywords: Vertical Integration, Market Structure, Downstream Oligopsony, Competition Policy.

JEL-Classification: D43, L41, L42

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1   Introduction

The effects of vertical integration on consumer surplus and overall welfare are subjects of ongoing debate amongst economists, antitrust lawyers, and policy makers. In the industrial organization literature, substantial progress has been made in identifying pro- and anticompetitive effects of vertical integration. An important source of efficiency gains is the elimination of the monopsony distortion when a downstream firm exerts market power on the upstream market: With increasing marginal costs, purchasing additional units raises the market price on all infra-marginal units, which results in inefficiently low purchases because the monopsonist does not take the rents of infra-marginal suppliers into account. As vertical integration induces the buyer to internalize these positive externalities, efficiency increases.[1] In contrast, the ability of integrating parties to raise their rivals' costs has been recognized as a main factor fostering foreclosure, which typically harms consumers and lowers welfare.[2]

An open theoretical question of substantial practical relevance is how these effects depend on the underlying market structure. In particular, is vertical integration more likely to harm consumers when the industry consists of many competitors, or should antitrust authorities be more vigilant when the integrating firm's competitors are small in number and exert substantial market power?

To shed light on these questions we present a model that permits us to study the competitive effects of vertical integration as a function of the underlying market structure and of the degree of vertical integration, taking into account both efficiency gains and incentives to raise rivals' costs. The following is a sketch of our basic model, which builds on Riordan (1998). There are a number of non-integrated firms and one partly vertically integrated firm. All firms exert oligopolistic market power downstream, where they compete in quantities, and oligopsonistic market power upstream.[3] To produce the final good, firms need a fixed input, termed capacity, that is competitively offered on an upward sloping supply curve. The more capacity a firm purchases on the market, the lower is its marginal cost of producing the final good. The vertically integrated firm already owns some capacity at the time when upstream market transactions occur. This is referred to as its degree of vertical integration. It can be as

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[1]See Perry (1978) for a detailed analysis of the monopsony case. Similarly to monopsony, in case of monopoly at the input market, vertical integration eliminates the well-known double mark-up distortion.

[2]For recent surveys on the effects of vertical integration, see Church (2008), Rey and Tirole (2007) and Riordan (2008).

[3]In a working paper available at http://ssrn.com/abstract=2388260 we show that our results carry over the case of differentiated Bertrand competition downstream.

low as zero or so large that the integrated firm completely forecloses its rivals, or anything in between. The degree of vertical integration plays an important strategic role: The internal units are protected from the capacity price increase when the integrated firm purchases additional capacity. That is, a rise in the capacity price applies to a smaller number of infra-marginal units. Hence, an increase in the degree of vertical integration induces the integrated firm to behave more aggressively on the input market. Therefore, increases in the degree of vertical integration lead to increases in the market price of capacity, which raises the costs of the integrated firm's rivals and thus leads to (partial) foreclosure. As a higher degree of vertical integration induces the integrated firm to purchase more capacity, the firm will also produce more output. Thus, our model allows for efficiency gains from vertical integration. In what follows, we use the term "procompetitive" ("anticompetitive") effects to mean that consumer surplus increases (decreases) following increases in the degree of vertical integration. Similarly, we speak of "efficiency" effects of vertical integration when referring to its effects on total welfare.

Within this setup, we obtain the following results. First, vertical integration is more likely to be procompetitive (i) the *more* concentrated is the industry, that is, the fewer are the non-integrated rivals, and (ii) the smaller is the degree of integration.[4] Whereas result (ii) is probably as one would expect, result (i) may seem counterintuitive at first.[5] However, a clear intuition for this result will be provided below. It implies that, within the confines of our model, antitrust authorities should be more vigilant vis-à-vis vertical mergers when there is a *larger* number of rival firms in the industry. We also demonstrate that the effects from vertical integration on consumer surplus can be substantial even if the number of firms is large. Second, vertical integration is procompetitive under a fairly wide array of circumstances. In the extreme, even complete foreclosure of the non-integrated firms can enhance consumer surplus because the integrated firm expands its quantity by a large extent after integrating.[6] Third, we show that, as the number of competitors becomes large, vertical integration is anticompetitive irrespective of the degree of vertical integration. In the limit, our model thus yields Riordan's

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[4]When saying that something is more (less) likely we mean that it occurs for a larger (smaller) set in the parameter space.

[5]For example, Lafontaine and Slade (2007) note that most empirical studies on vertical integration are conducted for highly concentrated markets because evidence for foreclosure is thought most likely to be found there.

[6]This point is related to but distinct from Quirmbach's (1986) observation that consumer prices fall *after* vertical integration to monopoly is complete. Our result is that consumer prices can fall along all the way towards complete foreclosure.

(1998) powerful result that vertical integration by a dominant firm who faces a competitive fringe is always anticompetitive.[7] Fourth, even if it is procompetitive, vertical integration is not necessarily welfare increasing. Procompetitive but welfare reducing mergers are possible because vertical integration changes the cost structure in the industry. Fifth, endogenizing the degree of vertical integration in an extension, we find that the privately optimal degree of integration is smaller than the ones that maximize social welfare and consumer surplus when the number of non-integrated rivals is small but above those thresholds when the number is large. Therefore, explicit consideration of the incentives to integrate reinforces the message that antitrust authorities should be the more vigilant vis-à-vis vertical mergers the *larger* is the number of rivals.

Let us now develop the basic intuition for these results, starting with a few preliminaries. If a firm purchases more capacity, it faces lower production costs and produces a larger quantity. This implies that firms with higher capacities incur larger infra-marginal losses from a downstream price decrease. Therefore, a firm with a larger capacity utilizes its capacity less intensively, that is, it produces less output per unit of capacity — the monopoly effect. Conversely, firms with little market power have marginal costs that are approximately equal to price — much like fringe firms in the dominant firm model — and utilize their capacity intensively. By increasing the degree of vertical integration, the integrated firm purchases more capacity because its internal units are protected from the capacity price increase, that is, it partly internalizes the gain in rents to the infra-marginal units. This increases both the aggregate capacity level and the market clearing price for capacity. Therefore, rival firms purchase less capacity. That is, capacities are strategic substitutes. Vertical integration has the strongest negative effect on consumer surplus if rival firms have little to no market power. Operating already close to the point where marginal costs equal price, their only way to adapt is to decrease their output. In contrast, if rival firms exert market power themselves, the anticompetitive effect of reducing the capacity available to them will be partly offset because smaller capacities induce them to use capacity more intensively. In other words, market power of rival firms mitigates the anticompetitive foreclosure effect of vertical integration.

Based on these preliminary observations, rather intuitive explanations for our main results are now at hand. In an industry with a large number of competitors, the market power of each non-integrated firm is low. As noted above, the non-integrated firms therefore operate relatively

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[7]This means also that the dominant firm model provides a good approximation to nearby market structures.

efficiently. Vertical integration by one firm shifts production from the non-integrated firms to the integrated firm, which produces a lower quantity per unit of capacity. As a consequence, vertical integration is more likely to be anticompetitive when the industry consists of a large number of firms. This is also the reason why, in the limit as the number of firms grows large, our model encompasses the case with a dominant firm who faces a competitive fringe, in which vertical integration is always anticompetitive.

If a firm is already highly integrated at the outset, its capacity utilization is low. Further vertical integration then reallocates capacity from firms with high capacity utilization to a firm with low capacity utilization. Therefore, vertical integration is more likely to be anticompetitive the larger is the integrated firm's degree of vertical integration.

As vertical integration leads to larger capacity purchases of the integrating firm, aggregate capacity employed in the industry rises. This effect counteracts the effect that the integrated firm utilizes its capacity less intensively. If all firms have considerable market power, the effect of less intensive capacity utilization is small. Therefore, the dominating effect is that aggregate capacity increases, implying that even vertical integration to full foreclosure can be procompetitive.

In determining the effects of vertical integration on social welfare rather than on consumer surplus, one needs additionally to account for the costs of production. As vertical integration shifts capacity to the integrated firm that utilizes it less intensively, aggregate costs of production increase with vertical integration. This may render vertical integration welfare reducing even when it is consumer surplus enhancing.

Our article is most closely related to Riordan (1998), whose setup includes a dominant, partly integrated firm facing a competitive fringe. We extend this by allowing the integrated firm's rivals to exert market power as well. Riordan's model is a notable exception within the theoretical literature on vertical integration because it incorporates exercise of market power by a single firm in both markets whereas most of this literature is concerned with the trade-off between avoidance of double marginalization, that is, the exercise of market power by different firms, and foreclosure. For example, Hart and Tirole (1990), Ordover, Saloner, and Salop (1990) and Chen and Riordan (2007) are only concerned with foreclosure motives. In Salinger (1988), Choi and Yi (2000), Chen (2001) and Inderst and Valletti (2011a), the downstream market is comprised of an oligopoly and both effects are present but downstream firms have

no market power in the intermediate goods market.[8]

A different approach to vertical integration is developed by De Fontenay and Gans (2005),[9] in which there is efficient bilateral bargaining between pairs of upstream and downstream firms.[10] As Gans (2007) notes, the bargaining approach fits relatively well to an industry with few upstream and downstream firms, whereas in our model, the input is supplied competitively, which corresponds to general mass markets for inputs.

An article that, like ours, considers a competitive upstream industry is Esö, Nocke, and White (2010). They study a model in which competing downstream firms bid for scarce upstream capacity and show that if this capacity is sufficiently large, the asymmetric downstream market structure analyzed here and in Riordan (1998) emerges endogenously.[11]

As in most of the literature, we consider the case of one-shot interaction between firms. An important exception is the article by Nocke and White (2007),[12] who consider a dynamic model and show that vertical integration facilitates upstream collusion because it reduces the number of buyers for rival firms, which decreases their incentives to deviate from a collusive agreement.[13]

The remainder of the article is organized as follows. Section 2 lays out the model and Section 3 presents the equilibrium. In Section 4 we derive the competitive effects of vertical integration and show how these effects change with the number of firms in the industry. Section 5 analyzes the effects of vertical integration on social welfare. In Section 6 we study the incentives to acquire capacity, and Section 7 concludes. All proofs are in the appendix.

2   The Model

There are two types of firms, one (partially) vertically integrated firm, which we index by I and $N \geq 1$ non-integrated firms. A typical non-integrated firm is indexed by j. All firms

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[8]Hendricks and McAfee (2010) present a model, where upstream and downstream firms exert market power in the input market. However, in order to keep the model tractable, they assume that vertical integration does not change the downstream price and that the market structure consists of no vertical integration at the outset.

[9]This approach is used by Gans (2007) to derive concentration measures for vertical and horizontal mergers in an oligopolistic vertical market structure.

[10]For a related analysis of vertical integration with multilateral bargaining, see Bolton and Whinston (1993).

[11]Inderst and Valletti (2011b) consider a model with take-it-or-leave-it offers of an upstream firm but without vertical integration. They allow for one buyer to be larger than the others and show that this buyer obtains a favorable deal. However, this leads to higher wholesale prices for rival buyers.

[12]For a similar analysis but with a different upstream pricing regime, see Normann (2009).

[13]We also concentrate on the case of a single (or marginal) vertical merger. Recently, Nocke and Whinston (2010) considered the case in which multiple horizontal mergers might arise over time and showed under which conditions the optimal policy for an antitrust authority is myopic.

produce a homogenous good and compete in quantities on the downstream market. The inverse demand function is $P(Q)$, where $P(Q)$ is the market clearing price for the aggregate quantity $Q \equiv q_I + \sum_{j=1}^{N} q_j$ satisfying $P'(Q) < 0$, where q_i is firm i's quantity with $i = I, 1, ..., N$. To produce the final good firms require a fixed input, referred to as capacity. The technology has constant returns to scale, which implies that the short-run cost function of firm i can be written as

$$c(q_i, k_i) = k_i c\left(\frac{q_i}{k_i}, 1\right) \equiv k_i C\left(\frac{q_i}{k_i}\right),$$

where k_i is the firm's capacity, and $C'(q_i/k_i) \geq 0$ and $C''(q_i/k_i) > 0$.[14]

Capacity is supplied competitively with an inverse supply function of $R(K)$, with $R'(K) > 0$ and $K \equiv k_I + \sum_{j=1}^{N} k_j$, i.e., K is the aggregate purchase of capacity. Firm I is partially vertically integrated, that is, it owns $\underline{k} \geq 0$ units of capacity. We refer to $\underline{k}$ as its degree of vertical integration, which is taken as given.[15] In other words, our focus in this article is on a single aspect of upstream vertical integration: the commitment to the acquisition at cost of a minimum amount of an input prior to competition among rivals for the input. Consider, for example, the acquisition of bauxite mines by an incumbent aluminum manufacturer (e.g. Alcoa, prior to the famous antitrust case of 1945, *United States v. Alcoa*, 148 F.2d 416 (2d Cir. 1945)). The incumbent anticipates competition with new rivals for the purchases of bauxite in the upstream market. But prior to the entry of rivals the incumbent purchases a subset of the mines available. This gives it ownership, post-entry, to a fraction of available capacity.[16]

The timing of the game is as follows: In the first stage, the capacity stage, all firms i simultaneously choose their level of capacity k_i. The degree of vertical integration $\underline{k}$ is common knowledge. Firm I purchases $k_I - \underline{k}$ units of capacity at the market price $R(K)$. Thus, the profit function of firm I is given by

$$\Pi_I(q_I, k_I) = P(Q)q_I - k_I C\left(\frac{q_I}{k_I}\right) - (k_I - \underline{k})R(K), \tag{1}$$

and the one of a non-integrated firm j is $\Pi_j(q_j, k_j) = P(Q)q_j - k_j C(q_j/k_j) - k_j R(K)$. In the second stage, the quantity stage, all firms simultaneously choose their quantities after having observed all capacity levels $\mathbf{k} = (k_I, k_1, .., k_N)$. The aggregate quantity Q determines the market clearing price $P(Q)$, and payoffs are realized.

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[14]We focus on the case in which the integrated firm and the non-integrated firms have the same cost function. Riordan (1998) allows firm I to have a fixed cost advantage per unit of output. We analyze this extension in the working paper and show that our results carry over.

[15]Section 6 provides the analysis of an augmented model in which $\underline{k}$ is determined endogenously.

[16]Note that, in contrast to the bauxite-aluminum example, we allow for variable proportions in production.

Equation (1) implies that firm I has the opportunity to supply undesired capacity to an outside market, which occurs if $k_I < \underline{k}$, that is, $\underline{k}$ is not firm-specific. An increase in the (exogenous) degree of vertical integration reduces the number of infra-marginal units of capacity $k_I - \underline{k}$ on which the integrated firm bears the market clearing price $R(K)$ when purchasing additional units of capacity. Because the firm owns $\underline{k}$ units, it captures internally the effect of a higher capacity price on the first $\underline{k}$ units. As we will see shortly, this reduction in the number of infra-marginal units induces the integrated firm to purchase capacity more aggressively because the effective cost of capacity is lower.

We focus on symmetric subgame perfect equilibria, where symmetry means that the non-integrated firms play the same strategies. To ensure interior solutions and a unique equilibrium, we make some shape assumptions on the demand, supply and cost function. We suppose that $\lim_{Q \to \infty} P(Q) = 0$, that $P''(Q)$ is not too positive and that $P'''(Q)$, $C'''(q_i/k_i)$ and $R''(K)$ are not too negative. These assumptions are relatively mild and guarantee a unique equilibrium. A special case that satisfies these assumptions is the linear-quadratic model, in which $P(Q) = \alpha - \beta Q$ for $Q \in [0, \alpha/\beta]$, $R(K) = \delta K$, $C(q_i/k_i) = \frac{c}{2}\left(\frac{q_i}{k_i}\right)^2$, where α, β, c and δ are positive constants.

3 Equilibrium

We solve the game by backward induction.

The Quantity Stage (Stage 2)

At the quantity stage, $\mathbf{k}$ is already determined. As $\underline{k}$ has a direct effect only on k_I but not on q_I, the first-order condition for a profit maximum for each firm does not depend directly on $\underline{k}$. Consequently, the first-order condition of any firm i in the subgame of the quantity stage is given by[17]

$$P + P'q_i = C_i'. \tag{2}$$

It is easy to see that the second-order conditions are satisfied given that P'' is not too positive, which we assumed above. Our assumptions also imply that firm i's reaction function has a negative slope greater than -1. Therefore, every quantity-stage subgame has a unique equilibrium. We denote by $q_i^*(\mathbf{k})$ the equilibrium quantity of firm i, given any vector of capacities $\mathbf{k}$. From the first-order conditions we get the following intuitive lemma.

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[17]To simplify notation, in the following we abbreviate $P(Q)$ by P, $C(q_i/k_i)$ by C_i and $R(K)$ by R. We do so also for all derivatives.

Lemma 1

$$\frac{dq_i^*(\mathbf{k})}{dk_i} > 0 \quad and \quad \frac{dq_i^*(\mathbf{k})}{dk_j} < 0 \quad for\ all \quad i \neq j,\ i,j = I, 1, ..., N.$$

Therefore, all own effects are positive and all cross effects are negative. That is, a firm's optimal quantity increases in its own capacity and falls in the capacity of its rivals, independently of the type of the firm. Next we obtain the following result:

Lemma 2 $\frac{q_i^*(\mathbf{k})}{k_i}$ *decreases in* k_i *for all* $I, 1, ..., N$.

The same result is obtained by Riordan (1998). As observed above, a firm with a larger capacity produces a larger quantity, but because it produces more infra-marginal units, it suffers more from a fall in the final output price. As a consequence, it utilizes its capacity less intensively than firms with lower capacity. This means that q_i^*/k_i is smaller.

The Capacity Stage (Stage 1)

We now move on to the first stage of the game, the capacity choice game. Using the envelope theorem and dropping all arguments, the first-order condition of a non-integrated firm j in the capacity stage is given by

$$\frac{\partial \Pi_j}{\partial k_j} = P'\frac{dQ_{-j}^*}{dk_j}q_j^* - C_j + C_j'\frac{q_j^*}{k_j^r} - R - k_j^r R' = 0, \tag{3}$$

where Q_{-j}^* is the equilibrium quantity of all firms but firm j and the superscript r indicates that k_j^r is the best-response capacity of firm j to the capacity choices of its rivals $(k_I, k_1, .., k_{j-1}, k_{j+1}, .., k_N)$. The first-order condition of the integrated firm I is given by

$$\frac{\partial \Pi_I}{\partial k_I} = P'\frac{dQ_{-I}^*}{dk_I}q_I^* - C_I + C_I'\frac{q_I^*}{k_I^r} - R - (k_I^r - \underline{k})R' = 0, \tag{4}$$

where k_I^r is I's best-response capacity to $(k_1, .., k_N)$.

Showing that an equilibrium exists and, if it does, is unique is more involved in the capacity stage than in the quantity stage. The reason is that now a change in firm i's capacity has an effect on the equilibrium quantity of each firm in the second stage. Thus, the expression for the reaction function is more complicated than in a standard single stage game.[18] Nevertheless, the next lemma establishes that an equilibrium exists and is indeed unique.

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[18]Moreover, the game is not an aggregative game. The reaction of a non-integrated firm is different if firm I changes its capacity than if a non-integrated firm changes its capacity because this has different effects on the overall quantity produced in the second stage.

Lemma 3 *There exists a unique symmetric equilibrium in the capacity stage. In this equilibrium, k_I^* and k_j^*, $j = 1, ..., N$, are determined by (3) and (4).*

From the two first-order conditions we can now derive the following lemma:

Lemma 4 *Capacities are strategic substitutes, that is,*

$$\frac{dk_i^r}{dk_j} < 0 \quad for \ all \quad i \neq j, \ i, j = I, 1, ..., N.$$

Two economic forces are the key to the intuition for this result: First, in stage 2 quantity choices of the firms are strategic substitutes, and this translates to strategic substitutability of the capacity choices. As shown in Lemma 1, if a firm purchases more capacity in stage 2, it will increase its quantity, thereby inducing the rivals to lower their quantities. When producing a lower quantity, an additional unit of capacity becomes then less valuable for each rival firm, implying that each rival optimally lowers its capacity. Second, if a firm purchases more capacity, the capacity price R increases. Therefore, buying capacity becomes more expensive for the rivals, inducing each of them to lower its optimal capacity.

Drawing on the previous lemma, the next result then states how equilibrium capacities k_i^*, with $i = I, 1, .., N$, are affected by changes in $\underline{k}$:

Lemma 5

$$\frac{dk_I^*}{d\underline{k}} > 0 \quad and \quad \frac{dk_j^*}{d\underline{k}} < 0, \quad j = 1, ..., N.$$

That k_I^* increases and k_j^* decreases in $\underline{k}$, is intuitive. If $\underline{k}$ increases, firm I owns more capacity units. Thus, firm I's marginal opportunity costs of purchasing additional units of capacity are lower because the number of infra-marginal units for which it has to pay the capacity price R on the upstream market decreases. As a consequence, firm I finds it optimal to increase its overall amount of capacity. This effect reflects the reduction in oligopsony distortion. Although $\underline{k}$ does not directly influence the optimal capacity of the non-integrated firms, we know from Lemma 4 that capacities are strategic substitutes. Therefore, each non-integrated firm optimally acquires less capacity as $\underline{k}$ rises. Hence, non-integrated firms become (partially) foreclosed as $\underline{k}$ increases.

It follows immediately from Lemma 5 that $k_I^* > k_j^*$ for $\underline{k} > 0$, that is, $\underline{k} > 0$ commits firm I to a more aggressive reaction in stage 1. Thus, if firm I is vertically integrated to some extent, its equilibrium capacity is larger than the one of the non-integrated firms. From Lemma 2 we

know that this implies that its capacity utilization q_I^*/k_I^* is lower than for the non-integrated firms.

4   Competitive Effects of Vertical Integration

We now turn to the analysis of the effects of vertical integration on consumer surplus. As competition authorities both in the U.S. and in Europe base their decisions mainly on the effects on consumer surplus, this analysis is highly relevant for competition policy.

Competitive Threshold

We first analyze under which conditions vertical integration is pro - or anticompetitive, that is, whether a marginal change in $\underline{k}$ increases or decreases the aggregate equilibrium quantity supplied in the downstream market. From above, it follows that an increase in $\underline{k}$ has a direct positive effect on k_I and an indirect negative effect on all k_j.[19] This in turn leads to an increase in q_I and to a decrease in all q_j. Thus, vertical integration is procompetitive at the margin if and only if

$$\frac{dQ}{d\underline{k}} = \left(\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I} \right) \frac{dk_I}{d\underline{k}} + N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j} \right) \frac{dk_j}{d\underline{k}} > 0, \quad i \neq j,$$

or equivalently

$$\frac{\left(\frac{dk_j}{d\underline{k}} \right)}{\left(\frac{dk_I}{d\underline{k}} \right)} > -\frac{\frac{dq_I}{dk_I} + N\frac{dq_j}{dk_I}}{N\left(\frac{dq_I}{dk_j} + \frac{dq_j}{dk_j} + (N-1)\frac{dq_i}{dk_j} \right)}, \quad i \neq j. \tag{5}$$

The left-hand side of (5) expresses the relative change of a non-integrated firm's capacity with $\underline{k}$ to the change in the integrated firm's capacity at the equilibrium. Because capacity choices are strategic substitutes, this relative change is negative. The right-hand side gives a benchmark against which to compare this term. The inequality says that if the relative change is small enough in absolute terms, then vertical integration is procompetitive. Intuitively, if k_j does not fall by much after firm I becomes more integrated, the positive effect resulting from the increase in q_I dominates the negative effect that stems from the decrease in q_j of all non-integrated firms.

Inserting the respective derivatives (derived in the proof of Lemma 1) into the right-hand side of (5) and simplifying yields

$$\left(\frac{dk_j}{d\underline{k}} \right) / \left(\frac{dk_I}{d\underline{k}} \right) > -\frac{C_I'' \frac{q_I}{k_I}(C_j'' - k_j P')}{NC_j'' \frac{q_j}{k_j}(C_I'' - k_I P')}. \tag{6}$$

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[19]To simplify notation here and in what follows we omit the superscript $*$ on equilibrium quantities and capacities.

To gain some intuition for this formula suppose that $\underline{k}$ is zero. In this case all $N + 1$ firms are the same and we have $q_I = q_j$, $k_I = k_j$ and thus $C_I'' = C_j''$. As a consequence, the right-hand side of (6) simplifies to $-1/N$. Thus, to keep overall output constant, the aggregate capacity reduction of the non-integrated firms must be the same as the increase in the capacity of firm I. Because all N non-integrated firms are symmetric, each of them must lower its capacity by $1/N$ of the increase in the integrated firm's capacity.

Suppose now that $\underline{k} > 0$. From the above lemmas we know that in this case $k_I > k_j$, $q_I/k_I < q_j/k_j$ and thus $C_I'' < C_j''$. Then, the right-hand side of (6) is in absolute value smaller than $1/N$. The reason is that the integrated firm utilizes its capacity less intensively than a non-integrated firm. As a consequence, if all non-integrated firms reduced their capacity in sum by the same amount as the capacity increase of the integrated firm, overall output would fall because capacity is shifted to the less efficient firm. Thus, to keep output constant the reduction in capacity by non-integrated firms has to be smaller and overall capacity must rise.

To characterize how vertical integration changes overall output, we begin with the case where $\underline{k}$ is small.

Proposition 1 *For any finite N, there exists a competitive threshold $\underline{k}^* > 0$, such that for all $\underline{k} < \underline{k}^*$, vertical integration is procompetitive at the margin.*

Intuitively, if $\underline{k}$ is small, firm I utilizes its capacity only slightly less intensively than its non-integrated rivals. However, the aggregate reaction of the rivals to an increase in $\underline{k}$ is always smaller than the increase in k_I. Thus, the aggregate equilibrium capacity increases and overall output rises.

Next assume that $\underline{k}$ is so large that the equilibrium value of k_I is large enough to induce $k_j = 0$ for all $j \neq I$ and define $\bar{\underline{k}}$ as the degree of vertical integration at which $k_j = 0$. Observe that this implies $q_j = 0$. In words, at $\underline{k} = \bar{\underline{k}}$, only the integrated firm is active and its non-integrated rivals are fully foreclosed.[20] Accordingly, we refer to the case where $\underline{k}$ approaches $\bar{\underline{k}}$ as vertical integration to full foreclosure.

Proposition 2 *For any finite N, vertical integration to full foreclosure can be procompetitive at the margin.*

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[20]Such a $\bar{\underline{k}}$ necessarily exists because $dk_I/d\underline{k} > 0$ and $dk_j/d\underline{k} < 0$. In addition, variable production costs $c(q_j, k_j)$ are decreasing in k_j. Thus, costs are increasing in $\underline{k}$ for a non-integrated firm j, whereas revenue is decreasing. For $\underline{k}$ large enough, j's costs are thus too high relative to $P(Q)$ and its optimal production is zero.

Thus, even marginal vertical integration that leads to a complete foreclosure of rival firms is not necessarily detrimental to consumer surplus. In addition, as we will show below, vertical integration to full foreclosure may not only be locally procompetitive, that is, when $\underline{k}$ is close to $\bar{\underline{k}}$, but also globally, that is, for any $\underline{k} \in [0, \bar{\underline{k}})$. This implies that starting from any $\underline{k} \in [0, \bar{\underline{k}})$ vertical integration to $\bar{\underline{k}}$ may increase consumer surplus. This can occur because vertical integration reduces the monopsony distortion, thereby inducing the integrated firm to acquire more capacity, and to produce more output. If a firm acquires such a large amount of capacity that its competitors stop producing, the overall capacity used in production can become so large that the resulting monopoly quantity is larger than the oligopoly quantity without the capacity increase. Interestingly, in this case the efficiency effect of vertical integration outweighs the negative effects of monopolization on the downstream market.

We now turn to the analysis of intermediate values of $\underline{k}$, that is, values of $\underline{k} \in (\underline{k}^*, \bar{\underline{k}})$. It is of particular interest to explore if there is a unique threshold of $\underline{k}$ below which vertical integration is procompetitive and above which vertical integration is anticompetitive. Moreover, if no such threshold exists, is vertical integration procompetitive over the whole range from 0 to $\bar{\underline{k}}$? The expressions that are involved in the calculations are too complicated to allow us to answer this question in general. Nonetheless, we are able to show that the threshold, provided it exists, is indeed unique for the widely used linear-quadratic specification introduced above.[21]

Proposition 3 *For any finite N, in the linear-quadratic model, vertical integration is always procompetitive or there exists a unique $\underline{k}^* \in (0, \bar{\underline{k}})$, such that vertical integration is procompetitive at the margin for all $\underline{k} < \underline{k}^*$ and anticompetitive at the margin for all $\underline{k} > \underline{k}^*$.*

The proposition is important because it shows that the threshold is unique (given that it exists) in the general linear-quadratic specification used in many industrial organization models. This indicates that the threshold is unique also for specifications that are close to the linear-quadratic one and suggests that the threshold may be unique even more generally.[22]

Comparison to the Dominant Firm Model Our result that the efficiency gains of vertical integration are often larger than the foreclosure effects contrasts with the findings obtained in the Riordan (1998) model where a dominant firm faces a competitive fringe. In both models,

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[21]The proof of this proposition can be found in the working paper.

[22]An open question is whether there are any cases where the left-hand and right-hand side of (6) are the same for different values of $\underline{k}$. Despite having tried various alternative specifications, we are not aware of any such counter examples.

vertical integration increases rivals' costs and thereby leads to (partial) foreclosure. As fringe firms have no market power, their marginal cost is equal to the final consumer price. As a consequence of foreclosure, a positive mass of fringe firms exits the market, which has highly detrimental effects on the aggregate output because fringe firms utilize their capacity intensively. In contrast, oligopolistic non-integrated firms also exert market power, restricting their output to keep the final goods price high. Following an increase in vertical integration and the associated foreclosure effect, each oligopolistic rival uses its capacity more intensively (because it buys a smaller amount). Therefore, the detrimental effects of foreclosure through vertical integration are partly offset by rivals' more efficient production, whose marginal costs become closer to consumer prices as vertical integration increases. As a result, in the dominant firm model the output contraction of fringe firms after foreclosure is larger than the reaction of rival firms under oligopoly.

Relation to Horizontal Mergers It is also instructive to contrast the results of our model with those arising in models of horizontal mergers such as Perry and Porter (1985). For that purpose, consider a game that only consists of stage 2 of our model, where each of the $N + 1$ firms compete in quantities on a product market, with each firm being endowed with some units of capacity. Following Perry and Porter (1985), we can then analyze the effects of a merger between two firms, where the newly merged firm uses the aggregate capacity of both stand-alone firms.

There are three substantial differences between such horizontal mergers and our analysis of vertical integration. First, the explicit modeling of an upstream market for capacity makes firms' capacities endogenous. This allows us to focus on equilibrium values of capacities rather than given capacities, which is the working assumption in the analysis of horizontal mergers.[23] Second, in stage 2 of our model a horizontal merger is always anticompetitive because it decreases the number of firms and because larger firms utilize their capacity less intensively (Lemma 2). In contrast, vertical integration even to full foreclosure can be procompetitive. Therefore, despite the similarity of the economic forces at work, the models' predictions are almost reversed, and so are the models' implications for antitrust policy. Third, an important lesson from the analysis of Perry and Porter (1985) is that the profitability of horizontal mergers hinges critically on the (exogenous) endowment of capacities. For example, if all firms are

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[23]For similar arguments, see also Riordan (1998).

symmetric, it is easily shown that a horizontal merger is not profitable for more than three firms. In contrast, marginal increases in $\underline{k}$ at $\underline{k} = 0$ – that is, when all firms are ex ante symmetric – are always profitable in our model, as we will show in Section 6 below. Therefore, the models' predictions are, again, almost reversed.

Anticompetitive Integration with Many Rival Firms

We now consider the effect of a change in the number of firms on the competitive effects of vertical integration. Understanding this relationship is particularly relevant for antitrust policy. We start by looking at the case in which the number of firms becomes large. This case is also of theoretical interest as the limit corresponds to the model with a dominant firm facing a competitive fringe.

Proposition 4 *If $N \to \infty$, then vertical integration is anticompetitive for all $\underline{k} \in [0, \bar{\underline{k}}]$.*

Hence, as the number of firms grows large, vertical integration is always anticompetitive. Intuitively, the aggregate reaction of the non-integrated firms to an increase in $\underline{k}$ is larger, the more firms are in the market. Therefore, the aggregate capacity reduction and, hence, the quantity reduction of the non-integrated firms increases in their number. As N goes to infinity, this effect dominates any efficiency effect of the integrated firm. Thus in the limit, as the market power of the non-integrated firms vanishes, we obtain the result of Riordan (1998). As the integrated firm has no first-mover advantage in our model, but has one in Riordan's, Proposition 4 also shows that his strong result stems genuinely from the dominant firm's market power rather than from the first-mover advantage.[24]

We now turn to the case of N being finite, and analyze how $\underline{k}^*$ changes with N. We start with the case of a sufficiently inelastic supply function.[25] Here we obtain the following result:[26]

Proposition 5 *Suppose that $R(K)$ is sufficiently inelastic. If $C(\cdot)$ is quadratic, then $\underline{k}^*(N)$ is strictly decreasing in N.*

If $R(K)$ is sufficiently inelastic, there also exists a unique $\underline{k}^*$ as in the linear-quadratic specification. This is because the capacity reaction of a non-integrated firm to a change in

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[24]That Riordan's result is obtained as the limit of an imperfectly competitive model is not trivial and has been an open question hitherto.

[25]A steeply increasing supply curve exists in many high technological industries. For example, dedicated fiber optic cables or semiconductor devices like customized integrated circuits that are produced in specialized plants exhibit large production costs that are steeply increasing once a plant produces close to its capacity limit.

[26]The proof can also be found in the working paper.

$\underline{k}$ is independent of the value of $\underline{k}$. Therefore, $(dk_j/d\underline{k})/(dk_I/d\underline{k})$ stays constant as $\underline{k}$ varies. However, the right-hand side of (6) is strictly increasing because firm I utilizes its capacity less intensively with further integration. Thus, there is a unique intersection point between the left-hand and the right-hand-side of (6). Proposition 5 then shows that this $\underline{k}^*$ is strictly decreasing in N if the cost function is quadratic. Although the proposition is restricted by the assumption that $C''(\cdot)$ is a constant, the basic insight does not seem to be confined to this case. It is easy to demonstrate numerically that the result also holds for $C(q_i/k_i) = (q_i/k_i)^g$ with $g > 1$. We also note that the result is independent of the exact shape of the demand function.

Proposition 5 shows that, with quadratic costs and a sufficiently inelastic supply of capacity, the competitive threshold $\underline{k}^*(N)$ decreases in the number of non-integrated rivals the integrated firm faces. Whereas the result may come as a surprise at first glance, the intuition behind it is relatively simple. As the number of non-integrated firms increases, each of them becomes smaller and thus utilizes its capacity more intensively. Because the non-integrated firms are foreclosed through integration, overall capacity utilization in the industry falls. This effect is more likely to dominate the counteracting force that integration leads to an increase in the overall capacity, if the number of non-integrated rivals is larger.

For the linear-quadratic specification, numerical computations also demonstrate that the threshold $\underline{k}^*(N)$ decreases in N, even if $R(K)$ is elastic.[27] This is displayed in Figure 2 in Section 6, where $\underline{k}^*(N)$ is depicted by $\underline{k}^*$. As the figure shows, there is a flat segment at the beginning. This is because for $N = 1$ and $N = 2$ vertical integration is procompetitive for all $\underline{k}$, that is, with few non-integrated rivals vertical integration to full foreclosure is procompetitive. For these values the curve does not depict $\underline{k}^*$ but $\bar{k}$.

We note that this result, like all our previous results, also holds if all rival firms are vertically integrated to the same extent. That is, none of our results depend on only one firm being integrated.[28] Interestingly, when all rival firms are integrated, the competitive threshold increases in the degree of integration of the rivals because more integrated rivals are less responsive to increases in the input price, which reduces the foreclosure effect. This insight puts additional emphasis on the notion that antitrust authorities should be less wary of vertical integration the more market power the integrating firm's rivals have.

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[27]All computations were done in Python and are available upon request. The numerical computations are based on the parameterization $\alpha = \beta = c = \delta = 1$.

[28]The arguments underlying this statement are provided in the working paper.

Quantifying the Effects of Vertical Integration

So far we have been focusing on the direction or sign of output changes upon vertical integration. This leaves open the question how important these effects are quantitatively.

To shed light on this question, we use numerical computations for the linear-quadratic model. Figure 1a displays the percentage change in consumer surplus $CS(\underline{k})$ as a function of N when the integrated firm's (exogenous) degree of vertical integration increases marginally whereas its downstream market share is kept fixed at 50%.[29],[30] The results displayed in Figure 1a show that the marginal effect of vertical integration is positive and large when N is small and negative yet still sizeable in absolute terms when N is large. Of course, vertical integration is often not a continuous process but involves acquiring a non-negligible fraction of the intermediate good market. Thus, the computation shows that even in industries with a large number of rivals, the absolute effect of a discrete vertical merger is sizeable.

Another important feature of our model is that vertical integration up to full foreclosure of the non-integrated firms can enhance consumer surplus. Figure 1b illustrates the order of magnitude of these effects in the linear-quadratic model. It depicts the difference in consumer surplus between vertical integration to full foreclosure and no vertical integration, i.e., $CS(\underline{\bar{k}}) - CS(0)$, as percentage of $CS(0)$ as a function of N. If the only objective were to maximize consumer surplus and if the degree of vertical integration were 0, then vertical integration that would lead to full foreclosure should be permitted when the number of competitors is small (absent vertical integration to full foreclosure) but not when it is large.
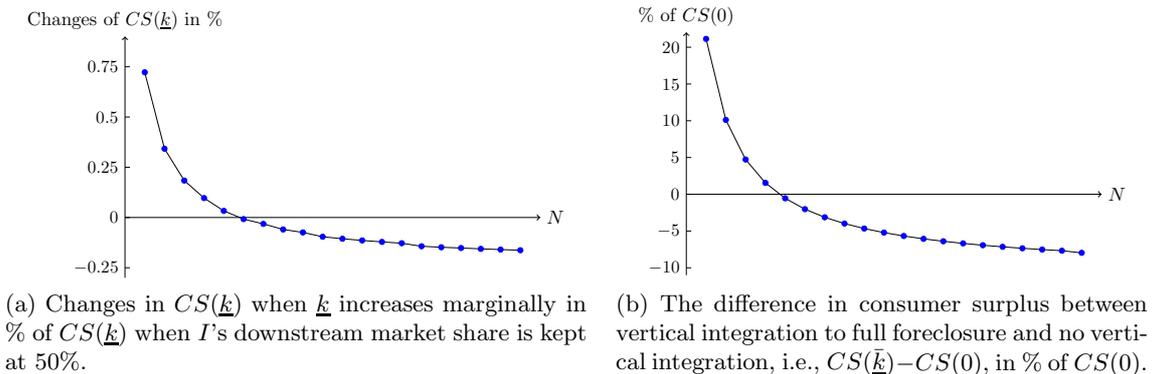

(a) Changes in $CS(\underline{k})$ when $\underline{k}$ increases marginally in % of $CS(\underline{k})$ when I's downstream market share is kept at 50%.

(b) The difference in consumer surplus between vertical integration to full foreclosure and no vertical integration, i.e., $CS(\underline{\bar{k}}) - CS(0)$, in % of $CS(0)$.

Figure 1: Quantifying the effects of vertical integration of consumer surplus.

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[29]Put formally, Figure 1a displays $100(CS(\underline{k} + 0.01) - CS(\underline{k}))/CS(\underline{k})$ at the point where $\underline{k}$ is such that $q_I^*/Q^* = 1/2$. Here, 0.01 is the smallest increment for changes in $\underline{k}$ that we used in our simulations.

[30]This exercise is also insightful as it captures the way in which many antitrust authorities may think about evaluating the competitive effects of vertical integration.

It might seem that vertical integration is most harmful to consumers when there are few non-integrated rivals, as opposed to many rivals. This notion would lead to the recommendation that vertical merger policy should follow horizontal merger policy in considering higher concentration among non-merging firms to be a negative factor in the decision to allow a merger. For example, the guidelines of the U.S. Department of Justice for non-horizontal mergers express the view that "(a)dverse competitive effects are likely only if overall concentration, or the largest firm's market share, is high" and state that the Department is unlikely to challenge a merger "unless overall concentration of the acquired firm's market is above 1800 HHI," with a lower concentration sufficing under certain conditions.[31]

The numerical results show that the opposite is true in our model. When the incentive to vertically integrated flows from the internalization of monopsony distortions, high concentration outside the merging parties should favor a merger.

5   Welfare Effects

Our focus thus far has been on the competitive effects of vertical integration. Yet, it is also important to analyze the implications of vertical integration on social welfare, which can be expressed as

$$
W = \int_0^Q P(x)dx - k_I C\left(\frac{q_I}{k_I}\right) - N k_j C\left(\frac{q_j}{k_j}\right) - \int_0^K R(y)dy.
$$

The first term is gross consumer surplus, the second term is the variable cost of the integrated firm, whereas the third term represents the variable cost of all non-integrated firms. The last term is the opportunity cost of capacity. Differentiating this expression with respect to $\underline{k}$ (and dropping arguments), we obtain that welfare is increasing in $\underline{k}$ if and only if

$$
P\frac{dQ}{d\underline{k}} - N C_j \frac{dk_j}{d\underline{k}} - N k_j C_j' \left(\frac{1}{k_j}\frac{dq_j}{d\underline{k}} - \frac{q_j}{k_j^2}\frac{dk_j}{d\underline{k}}\right) - C_I \frac{dk_I}{d\underline{k}} - k_I C_I' \left(\frac{1}{k_I}\frac{dq_I}{d\underline{k}} - \frac{q_I}{k_I^2}\frac{dk_I}{d\underline{k}}\right) - R\frac{dK}{d\underline{k}} > 0. \quad (7)
$$

We can now solve the first-order conditions of the quantity stage, given by (2), for C_j' and C_I', and insert them into (7). Similarly, inserting C_j' and C_I' from (2) into the first-order conditions from the capacity stage, (3) and (4), and solving them for C_j and C_I, we can replace

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[31]See http://www.justice.gov/atr/public/guidelines/2614.pdf.

C_j and C_I in (7). After rearranging we obtain

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} > -\frac{-P'\left(q_I\frac{dQ}{dk_I} + q_j\frac{dQ_{-I}}{dk_I}\right) + R'(k_I - \underline{k})}{N\left[-P'\left(q_j\frac{dQ}{dk_j} + (N-1)q_j\frac{dq_i}{dk_j} + q_I\frac{dq_I}{dk_j}\right) + R'k_j\right]}. \tag{8}$$

This inequality has a similar structure as (5). The left-hand side is again the equilibrium ratio of the response of k_j to a change in $\underline{k}$ over the response of k_I. The right-hand side is now different because when considering social welfare we have to take into account that the cost structure and therefore the absolute value of overall costs changes as $\underline{k}$ varies. Nevertheless, one can show that for any finite N there exists a $\underline{k}^*_W > 0$ such that for all $\underline{k} < \underline{k}^*_W$ vertical integration is welfare increasing at the margin. It is also possible that vertical integration to full foreclosure increases overall welfare.[32]

The intuition is similar to the one for Propositions 1 and 2. If the (exogenous) degree of vertical integration is low, further vertical integration mainly increases final output. Therefore, it is welfare increasing. On the other hand, if $\underline{k}$ is already large, the overall quantity may decrease and, in addition, the less efficient firm produces more, which raises production costs even for a given quantity.

A result that is akin to Proposition 3 can also be shown: If the model is linear-quadratic, then for any finite N there either exists a unique $\underline{k}^*_W \in (0, \bar{k})$ so that vertical integration is welfare enhancing at the margin for all $\underline{k} < \underline{k}^*_W$ and welfare reducing at the margin for all $\underline{k} > \underline{k}^*_W$, or vertical integration is always welfare enhancing.[33]

The analysis so far resembles the one of the previous section. However, the threshold value of $\underline{k}$ obtained in the welfare analysis is different from the one obtained for consumer surplus because, as mentioned, the variable costs of production and the opportunity costs of capacity change with an increase in $\underline{k}$. The next proposition shows that for the linear-quadratic specification, a comparison of these thresholds delivers a clear-cut result.[34]

Proposition 6 *In the linear-quadratic case, $\underline{k}^*_W < \underline{k}^*$.*

The proposition states that marginal vertical integration is less likely to enhance welfare than consumer surplus. The intuition for this is two-fold: First, an increase in the degree of vertical integration leads to an increase in aggregate capacity K. This is because the rise in

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[32]A formal statement and a sketch of the proof are in the working paper.

[33]The sketch of the proof is in the working paper.

[34]The proof of this proposition can be found in the working paper.

k_I following an increase in $\underline{k}$ is larger than the fall in the capacity of non-integrated rivals. Therefore, capacity costs increase. Second, firm I utilizes its capacity less intensively than a non-integrated firm. This implies that vertical integration increases overall production costs at $\underline{k}^*$ for constant aggregate quantity. Thus, even if aggregate quantity increases slightly, the effect of increased production costs dominates and welfare falls.[35] The result is interesting because it seems natural to conjecture that procompetitive vertical integration also improves welfare because firms' profits should rise as the industry becomes more integrated. However, what is missing in this reasoning is that vertical integration shifts production costs between firms. Proposition 6 shows that this effect can be so large that procompetitive but welfare reducing mergers are possible.[36]

Another important issue for practical application is to derive conclusions about the welfare effects of vertical integration that are based on observable market conditions.[37] For the linear-quadratic specification, one can numerically compute the critical input or output market shares of the integrated firm, given the thresholds $\underline{k}^*$ and $\underline{k}_W^*$, beyond which further vertical integration reduces consumer surplus or social welfare. In line with our previous results, these critical input and output market shares fall in the number of rivals. In addition, these critical market shares are almost identical in the input and the output market, suggesting that it may be sufficient for antitrust authorities to look at either of the two markets.

6   Incentives for Vertical Integration

Our model, like Riordan's (1998), assumes that one firm – firm I – owns some exogenous amount of capacity $\underline{k}$ at the outset, thereby taking vertical integration as given. Therefore, an important question is whether firm I has the incentive to acquire capacity $\underline{k}$ in the first place, and if so, what its optimal ownership stake of capacity would be. The answers to these questions are far from obvious because, as noted by Rey and Tirole (2007), a rise in $\underline{k}$ leads to a higher wholesale price, which increases firm I's expenses to buy the ownership stake. This higher wholesale price could dissuade firm I from buying the ownership stake in the first place.[38] To address these questions, we now extend our model by adding an ex ante stage, in which firm I can make an

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[35]The comparison between the two thresholds is illustrated in Figure 2 in the next section.

[36]If the integrated firm has an additional cost advantage that is sufficiently strong, the result can be reversed. In this case, anticompetitive but welfare enhancing mergers can occur. We show this result in the working paper.

[37]A particularly nice feature of Riordan's (1998) dominant firm model is that it establishes an indicator about the welfare effects of vertical integration that is based on the ratio of input to output market shares.

[38]Notice also that these have been open questions even for the analytically simpler model of a dominant firm facing a competitive fringe.
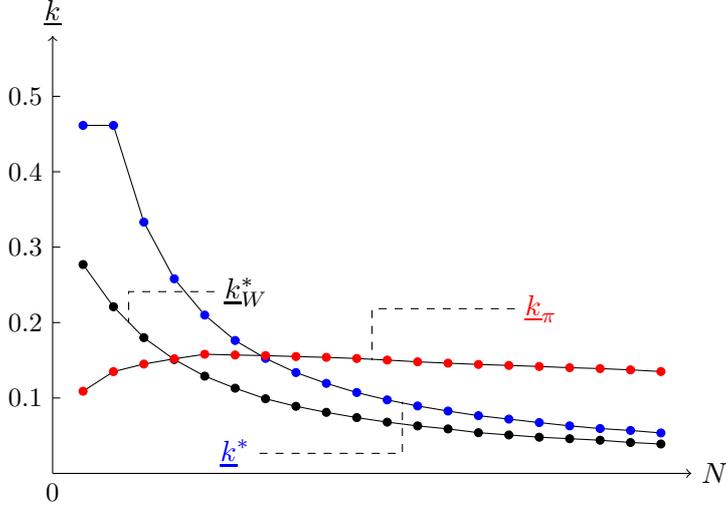
Figure 2: The competitive threshold $\underline{k}^*$, the welfare threshold $\underline{k}^*_W$, and the profitability threshold $\underline{k}_\pi$ in the linear-quadratic model.

offer to purchase $\underline{k}$ units of capacity from upstream suppliers. We show analytically that firm I always has an incentive to acquire at least a small stake of the competitively supplied capacity input, and we use numerical computations for the linear-quadratic model to determine firm I's profit maximizing level of integration.

Setup To fix ideas, we assume that the upstream market consists of a continuum of suppliers with heterogeneous costs of producing one unit of capacity, which yields the perfectly competitive inverse supply function $R(K)$ for aggregate capacity K, with $R'(K) > 0$.

We now consider a three-stage game, whose first stage – called stage 0 or ex ante stage – precedes the game analyzed thus far. In stage 0, firm I makes a public offer to buy $\underline{k}$ units of capacity at price r.[39],[40] Stages 1 and 2 are then exactly the same as stage 1 and 2 in the previous analysis, i.e., conditional on the acquired capacity of firm I, all $N + 1$ firms choose their capacities in stage 1 and then compete in quantities in stage 2.

Analysis We start with the optimal decision of an upstream supplier. Because each supplier has measure 0, firm I's level of vertical integration will be $\underline{k}$ if $\underline{k}$ other suppliers sell their unit of capacity at price r. Any supplier who does not sell his capacity at price r in the ex ante stage can therefore sell it at price $R(K(\underline{k}))$ on the capacity spot market in stage 1, where $K(\underline{k})$

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[39]With some abuse of notation we denote the maximum quantity firm I is willing to buy and the quantity it buys by $\underline{k}$. As shown shortly, these two quantities will be the same in equilibrium.

[40]The assumption that only firm I can make an offer to the upstream suppliers in the ex ante stage is in line with the assumption of a single integrated firm that underlies our main analysis.

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is the aggregate equilibrium capacity given that I is vertically integrated to the degree $\underline{k}$. It thus follows that firm I can acquire $\underline{k}$ units ex ante if and only if $r \geq R(K(\underline{k}))$. As there is no point for I to leave money on the table, it will set $r = R(K(\underline{k}))$. Given the offer to buy $\underline{k}$ at price $r = R(K(\underline{k}))$, $\underline{k}$ is the unique capacity sold to I in equilibrium. To see this, recall that K is increasing in $\underline{k}$ and R increasing in K. Therefore, if suppliers offered less (more) than $\underline{k}$, the price offer by I would beat (be worse than) the spot market price.[41] Therefore, if firm I wants to acquire $\underline{k}$ units of capacity in the ex ante stage, its overall cost of doing so is $\underline{k}R(K(\underline{k}))$.

For a given degree of vertical integration $\underline{k}$, the profit accruing to firm I in stages 1 and 2 is

$$\Pi_I(\underline{k}) = P(Q(\underline{k}))q_I(\underline{k}) - k_I(\underline{k})C\left(\frac{q_I(\underline{k})}{k_I(\underline{k})}\right) - (k_I(\underline{k}) - \underline{k})R(K(\underline{k})), \tag{9}$$

where $q_I(\underline{k})$, $k_I(\underline{k})$, and $Q(\underline{k})$ are, respectively, the equilibrium values of I's quantity, I's capacity and aggregate equilibrium quantity for a given amount of vertical integration $\underline{k}$. Differentiating this with respect to $\underline{k}$ and using the envelope theorem yields

$$\frac{\partial \Pi_I(\underline{k})}{\partial \underline{k}} = N\left[P'(Q(\underline{k}))q_I(\underline{k})\frac{\partial q_j(\underline{k})}{\partial \underline{k}} - (k_I(\underline{k}) - \underline{k})R'(K(\underline{k}))\frac{\partial k_j(\underline{k})}{\partial \underline{k}}\right] + R(K(\underline{k})) > 0,$$

where the inequality follows because, by Lemmas 1 and 5, $\partial q_j/\partial \underline{k} < 0$ and $\partial k_j/\partial \underline{k} < 0$.

We now evaluate the incentives to acquire a small (i.e., marginal) stake of upstream capacity for firm I at $\underline{k} = 0$. To do so, we compare the marginal change in profit in stages 1 and 2 evaluated at $\underline{k} = 0$, $(\partial \Pi_I(\underline{k})/\partial \underline{k})|_{\underline{k}=0}$, with the marginal change in the cost of procurement, which is the derivative of $\underline{k}R(K(\underline{k}))$ with respect to $\underline{k}$. Evaluated at $\underline{k} = 0$, this derivative is simply $R(K(0))$, yielding the following result

$$\frac{\partial \Pi_I(\underline{k})}{\partial \underline{k}}|_{\underline{k}=0} = N\left[P'(Q(0))q_I(0)\frac{\partial q_j(0)}{\partial \underline{k}} - k_I(0)R'(K(0))\frac{\partial k_j(0)}{\partial \underline{k}}\right] + R(K(0))$$
$$> R(K(0)) = \frac{\partial[\underline{k}R(K(\underline{k}))]}{\partial \underline{k}}|_{\underline{k}=0}.$$

Notice that this inequality holds for any positive N. This implies that some vertical integration is always profitable for firm I. Summarizing, we have established the following:

Proposition 7 *For any N, the acquisition of a marginal ownership stake at $\underline{k} = 0$ is always profitable for firm I.*

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[41]The fact $R(K(\underline{k}))$ is increasing in $\underline{k}$ reflects the well-known free riding phenomenon in the takeover literature (see e.g., Burkart, Gromb, and Panunzi (1998)): The larger the ownership stake a firm wants to acquire, the higher the price it has to pay for it; see also Joskow and Tirole (2000) and Gilbert, Neuhoff, and Newbery (2004).

Interestingly, a small degree of vertical integration is always profitable, independent of the shape of the demand, supply, and cost functions. This shows that even if firm I needs to pay for its acquisition, it finds it profitable to buy at least a small share. The result is of importance in relation to Proposition 1, according to which marginal vertical integration is procompetitive at $\underline{k} = 0$. Proposition 7 shows that, for the fairly natural capacity acquisition game analyzed here, such marginal vertical integration will indeed take place. Consequently, the market will implement these gains in consumer surplus.

To derive the profit maximizing level of vertical integration, we need to compare the benefits and costs from acquisition for an arbitrary level of $\underline{k}$. When acquiring $\underline{k}$, firm I's profit is given by (9), whereas the costs of procurement are $\underline{k}R(K(\underline{k}))$. Subtracting the latter from the former gives

$$\Pi_I(\underline{k}) = P(Q(\underline{k}))q_I(\underline{k}) - k_I(\underline{k})C\left(q_I(\underline{k})/k_I(\underline{k})\right) - k_I(\underline{k})R(K(\underline{k})). \tag{10}$$

The optimal level of integration is the maximizer of $\Pi_I(\underline{k})$ in (10) with respect to $\underline{k}$, which we denote by $\underline{k}_\pi$ and evaluate numerically for the linear-quadratic model. Figure 2 shows $\underline{k}_\pi$ together with $\underline{k}^*$ and $\underline{k}^W$.

It is evident that $\underline{k}_\pi$ is lower than $\underline{k}^*$ and $\underline{k}^W$ for small values of N but above $\underline{k}^*$ and $\underline{k}^W$ for high values of N. This implies that for small values of N, there is too little integration in equilibrium, whereas for larger values, the incentives to vertically integrate are excessive. As N becomes large, any degree of vertical integration is anticompetitive (and reduces welfare because $\underline{k}_W^* < \underline{k}^*$ by Proposition 6) but some vertical integration will occur in equilibrium. Therefore, what emerges from this analysis of equilibrium incentives to integrate is in line with the general theme that emerges from our article. More vigilance vis-à-vis vertical integration is called for in markets with a large number of non-integrated rivals. Figure 2 shows that as the number of rival firms increases, the incentives to vertically integrate are larger than the socially desirable degree of integration, independent of whether the objective is consumer surplus or welfare.

7   Conclusions

We have analyzed a model in which the effects of vertical integration on consumer and overall welfare depend on the underlying market structure. We have shown that, perhaps surprisingly, vertical integration is more likely to be procompetitive exactly when the market structure consists of a small number of non-integrated rivals. More generally, in our model vertical

integration is procompetitive under fairly wide circumstances because efficiency effects due to a reduction in the oligopsony distortion tend to dominate foreclosure effects. Because of this, even vertical integration that leads to full foreclosure of the rivals can be procompetitive. However, vertical integration can also increase consumer surplus and decrease total welfare because final output may be produced at higher costs after integration. With regards to the incentives to vertically integrate, we find that a small amount of integration is always profitable despite the free-riding problem the integrating firm faces. In addition, the private incentives to integrate tend to be too weak when the number of rivals is small and excessive when it is large. Our numerical results also indicate that —within the confines of our model— the effects of seemingly intuitive but ultimately misguided policy recommendations can be sizeable.

Our model considers the case in which upstream suppliers are perfectly competitive whereas downstream firms have full oligopsony power, leaving the open question for future research of what happens when input suppliers exert market power as well. We expect that the main effects will still be at work if input suppliers have some limited bargaining power. If a downstream firm integrates with an upstream supplier, the newly integrated firm owns more capacity units and therefore will purchase more capacity from other upstream suppliers on the input market. At the same time, it will produce more infra-marginal units of output and thus utilize its capacity less intensively than non-integrated downstream firms.

In contrast, the effects we identified will not be at work if downstream firms have no market power on the wholesale price as is the case, for example, in the well-known model of Salinger (1988), in which only upstream firms can influence the market price. Our model can therefore be seen as analyzing the other end of the spectrum, in which market power in the input market is purely oligopsonistic instead of oligopolistic, so that the two models are complementary to each other.

We also note that the policy implications of models in which only upstream firms have market power in the intermediate good market differ from ours. For example, in Inderst and Valletti (2011a), the foreclosure effect becomes more pronounced if the downstream market consists of a smaller number of firms, because the integrated firm then comes close to monopolization of the market. A similar effect is present in Salinger (1988). As a consequence, an important determinant for policy implications on vertical integration is the distribution of market power in the upstream market. Our main policy implication that, if our model is perceived as a plausible description of the relevant markets, antitrust authorities should be wary of vertical

integration in less concentrated industries is therefore most relevant if downstream firms have considerable market power in the intermediate good market.

Our main result can be tested empirically using data from industries with different levels of market concentration. The prediction of our model is that the likelihood that efficiency effects dominate is increasing in market concentration. Existing empirical studies on the competitive effects of vertical integration have focused on highly concentrated industries and found that efficiency effects dominate foreclosure effects in almost all cases (see Lafontaine and Slade, 2007). Our results indicate that this finding could be overturned when examining less concentrated markets.

Appendix

A Proofs

Proof of Lemma 1

Let $j \neq h, j \neq I$ and $h \neq I$. Totally differentiating (2) with respect to k_j yields[42]

$$P'\frac{dQ}{dk_j} + P'\frac{dq_j}{dk_j} + P''q_j\frac{dQ}{dk_j} = -C''_j\frac{q_j}{k_j^2} + C''_j\frac{1}{k_j}\frac{dq_j}{dk_j}. \tag{11}$$

We can write dQ/dk_j as $dQ/dk_j = dq_I/dk_j + \sum_{h \neq j} dq_h/dk_j + dq_j/dk_j$, which under the symmetry assumption that $k_h = k_j$ for all $h, j \in \{1, ..., N\}$, becomes

$$\frac{dQ}{dk_j} = \frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j} + \frac{dq_j}{dk_j}.$$

Therefore, (11) can be written as an equation that depends on the three variables dq_h/dk_j, dq_j/dk_j and dq_I/dk_j, which we wish to determine.

Totally differentiating the first-order condition of firm h, which is analogous to (2), with respect to k_j yields

$$P'\frac{dQ}{dk_j} + P'\frac{dq_h}{dk_j} + P''q_h\frac{dQ}{dk_j} = C''_h\frac{1}{k_h}\frac{dq_h}{dk_j}, \tag{12}$$

and differentiating (2) for $i = I$ with respect to k_j yields

$$P'\frac{dQ}{dk_j} + P'\frac{dq_I}{dk_j} + P''q_I\frac{dQ}{dk_j} = C''_I\frac{1}{k_I}\frac{dq_I}{dk_j}. \tag{13}$$

The system of the three equations (11), (12) and (13) is linear in the three unknowns dq_h/dk_j, dq_j/dk_j and dq_I/dk_j. Its unique solution, after imposing symmetry, i.e. $q_h = q_j$, $k_h = k_j$ and $C''_h = C''_j$, is

$$\frac{dq_I}{dk_j} = \frac{C''_j q_j k_I (P' + P''q_I)}{\eta k_j} < 0 \quad \text{for} \quad j \neq I, \tag{14}$$

$$\frac{dq_h}{dk_j} = \frac{C''_j q_j (C''_I - P'k_I)(P' + P''q_j)}{\eta(C''_j - P'k_j)} < 0 \quad \text{for} \quad j \neq h \tag{15}$$

and

$$\frac{dq_j}{dk_j} = \frac{C''_j q_j[(P')^2 k_j k_I(N+1) + P'(P''k_j k_I(q_I + (N-1)q_j) - 2C''_j k_I - C''_I k_j N)]}{\eta k_j(C''_j - P'k_j)} \tag{16}$$

$$+ \frac{C''_j q_j[C''_j C''_I - P''(C''_j k_I q_I + (N-1)C''_I k_j q_j)]}{\eta k_j(C''_j - P'k_j)} > 0,$$

---

[42]To simplify notation, we omit the superscript $*$ on equilibrium quantities and equilibrium capacities throughout this appendix.

where $\eta \equiv (P')^2(N+2)k_I k_j + P'[P''k_j k_I(q_I + Nq_j) - C'_I k_j(N+1) - 2k_I C''_j] + C''_I C''_j - P''(C''_j q_I k_I + C''_I q_j k_j N) > 0$. The inequality sign follows from P'' being negative or not too positive.

Totally differentiating the first-order conditions of firm I and j with respect to k_I yields

$$P'\frac{dQ}{dk_I} + P'\frac{dq_I}{dk_I} + P''q_I\frac{dQ}{dk_I} = -C''_I\frac{q_I}{k_I^2} + C''_I\frac{1}{k_I}\frac{dq_I}{dk_I} \qquad (17)$$

and

$$P'\frac{dQ}{dk_I} + P'\frac{dq_j}{dk_I} + P''q_j\frac{dQ}{dk_I} = C''_j\frac{1}{k_j}\frac{dq_j}{dk_I}, \qquad (18)$$

respectively, where under symmetry $dQ/dk_I = dq_I/dk_I + Ndq_j/dk_I$. Using the last equation to replace dQ/dk_I in (17) and (18) yields a system of two linear equations in the two unknowns dq_I/dk_I and dq_j/dk_I. The solution is

$$\frac{dq_j}{dk_I} = \frac{C''_I q_I k_j(P''q_j + P')}{k_I\eta} < 0 \quad \text{and} \quad \frac{dq_I}{dk_I} = -\frac{C''_I q_I[k_j(P'(N+1) + P''Nq_j) - C''_j]}{k_I\eta} > 0. \qquad (19)$$

Again, the inequality sign follows from P'' not being too positive. $\blacksquare$

Proof of Lemma 2

From Lemma 1 we know that $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i}) \Leftrightarrow \hat{k}_i > k_i$. Now suppose to the contrary of the claim in the lemma that $q_i(\hat{k}_i, \mathbf{k}_{-i})/\hat{k}_i \geq q_i(k_i, \mathbf{k}_{-i})/k_i$. Because $C''_i > 0$, this is equivalent to the right-hand side of (2) being weakly greater for $\hat{k}_i$ than for k_i.

Now we can turn to the left-hand side of (2). From (1) we can calculate dQ/dk_j and dQ/dk_I to get

$$\frac{dQ}{dk_j} = \frac{q_j C''_j(C''_I - k_I P')}{k_j\eta} > 0 \quad \text{and} \quad \frac{dQ}{dk_I} = \frac{q_I C''_I(C''_j - k_j P')}{k_I\eta} > 0.$$

Because $P' < 0$, the first term of the left-hand side of (2) is smaller for $\hat{k}_i$ than for k_i. Also, because $q_i(\hat{k}_i, \mathbf{k}_{-i}) > q_i(k_i, \mathbf{k}_{-i})$, $P' < 0$ and P'' is negative or not too positive, the second term on the left-hand side of (2) is either smaller for $\hat{k}_i$ than for k_i or only slightly bigger. Therefore, the left-hand side of (2) is strictly smaller for $\hat{k}_i$ than k_i, which is the desired contradiction. $\blacksquare$

Proof of Lemma 3

Differentiating (3) with respect to k_j and (4) with respect to k_I yields the second-order conditions

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = P'\frac{dq_j}{dk_j}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + P'q_j\left[\frac{d^2q_I}{dk_j^2} + (N-1)\frac{d^2q_h}{dk_j^2}\right] + \qquad (20)$$

$$+P''q_j \left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j} \right] \left[\frac{dq_j}{dk_j} + \frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j} \right] + C''_j \frac{q_j}{k_j^2} \left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j} \right) - 2R' - k_j R'' < 0$$

and

$$\frac{\partial^2 \Pi_I}{\partial k_I^2} = P' \frac{dq_I}{dk_I} N \frac{dq_j}{dk_I} + P' q_I N \frac{d^2 q_j}{dk_I^2} + \tag{21}$$

$$+P''q_I N \frac{dq_j}{dk_I} \left[\frac{dq_I}{dk_I} + N \frac{dq_j}{dk_I} \right] + C''_I \frac{q_I}{k_I^2} \left(\frac{dq_I}{dk_I} - \frac{q_I}{k_I} \right) - 2R' - (k_I - \underline{k})R'' < 0,$$

with $h \neq j$, $h, j = 1, ..., N$. In the following we show that (20) is indeed fulfilled when the first-order conditions are satisfied. The second-order condition for the integrated firm can then be shown to be fulfilled in exactly the same way.

In the proof of Lemma 1 we determined the equilibrium expressions for dq_i/dk_j, $i = I, 1, ..., N$, that appear in (20). To determine the sign of $\partial^2 \Pi_j / \partial k_j^2$ we still have to determine $d^2 q_I / dk_j^2$ and $d^2 q_h / dk_j^2$. To that end we now state the expressions for dq_I/dk_j and dq_h/dk_j without imposing symmetry, i.e., explicitly distinguishing between non-integrated firm h and j, that is between q_h and q_j, k_h and k_j and C''_h and C''_j. This gives us

$$\frac{dq_I}{dk_j} = \frac{C''_j q_j k_I (P' + q_I P'')(C''_h - P' k_h)}{k_j \nu} \quad \text{and} \quad \frac{C''_I q_j k_h (P' + q_h P'')(C''_I - P' k_I)}{k_j \nu}, \tag{22}$$

with

$$\nu = -k_I k_j k_h (N+2)(P')^3 + (3C''_h k_j k_I + k_I k_j (N+1)C''_j + k_h k_j (N+1)C''_I - P'' k_I k_h k_j ((N-1)q_h + q_I + q_j))(P')^2 +$$

$$((C''_j k_h k_I (q_I + (N-1)q_h) + C''_h k_h k_j (q_j + (N-1)q_h) + C''_h k_I k_j (q_j + q_I))P'' - N k_h C''_I C''_j - 2k_I C''_j C''_h - 2k_j C''_I C''_h)P'$$

$$-((N-1)C''_I C''_j q_h k_h + C''_h (q_j k_j C''_j + q_I k_I C''_j))P'' + C''_h C''_I C''_j.^{43}$$

Differentiating both equations of (22) with respect to k_j, using dq_h/dk_j, dq_j/dk_j and dq_I/dk_j from the proof of Lemma 1, and inserting the resulting expressions into the second-order condition yields

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} = -\frac{q_j^2 \left(\sum_{s=1}^9 (P')^s (\sum_{h=1}^3 \kappa_{sh}(P'')^h + \kappa_{s4}P''' + \kappa_{s5}C'''_j + \kappa_{s6}C'''_I + \kappa_{s7}) \right)}{k_j^2 (C''_j - k_j P')^3 \eta^3} - 2R' - k_j R'', \tag{23}$$

where we have used that in equilibrium $q_h = q_j$, $k_h = k_j$ and $C''_h = C''_j$. In equation (23) $\kappa_{sh} = \kappa_{sh}(q_j, k_j, q_I, k_I, C''_j, C''_I, P', P'', N)$, $s \in \{1, ..., 9\}$ and $h \in \{1, ..., 7\}$. We do not specify the exact expressions for κ_{sh} here because they stand for rather complex expressions consisting of several terms. Yet, in each case the sign of these expressions is easy to determine and this

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[43]One can easily check that if $q_h = q_j$, $k_h = k_j$ and, therefore, $C''_h = C''_j$ (which is the case in equilibrium), these formulas yield the expressions in (19).

is the only point of relevance for our purpose. These signs are the following: For $h = \{1, 2, 3\}$ $\kappa_{sh} \geq 0$, if both s and h are either even or odd and $\kappa_{sh} \leq 0$ if one is even and the other one is odd. κ_{s4}, κ_{s5}, $\kappa_{s6} \geq 0$ for s even and κ_{s4}, κ_{s5}, $\kappa_{s6} \leq 0$ for s odd. $\kappa_{s7} > 0$ for s even and $\kappa_{s7} < 0$ for s odd. Thus, the numerator in the fraction is positive because P'' is not too positive and P''' and C''' are not too negative. Because $\eta > 0$, the denominator is positive as well. Therefore, the first term in (23) is negative. Because R'' is not too negative as well, we get that $\partial^2 \Pi_j / \partial k_j^2 < 0$. In exactly the same way we can show that the second-order condition for firm I is satisfied. Thus, the profit function of each firm is quasiconcave in its own capacity and we have an interior equilibrium.

We now turn to the question of uniqueness. From Kolstad and Mathiesen (1987) and Vives (1999) we know that the equilibrium is unique if and only if the Jacobian determinant of minus the marginal profits is positive. In our case this determinant is given by

$$
|J| = \begin{vmatrix} -\frac{\partial^2 \Pi_j}{\partial k_j^2} & -\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} & \cdots & -\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \\ -\frac{\partial^2 \Pi_h}{\partial k_h \partial k_j} & -\frac{\partial^2 \Pi_h}{\partial k_h^2} & \cdots & -\frac{\partial^2 \Pi_h}{\partial k_h \partial k_I} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j} & -\frac{\partial^2 \Pi_I}{\partial k_I \partial k_h} & \cdots & -\frac{\partial^2 \Pi_I}{\partial k_I^2} \end{vmatrix}, \tag{24}
$$

with $h \neq j$, $h, j = 1, ..., N$. The terms that are relevant for this determinant are given by the second-order conditions, (20) and (21), and the terms $\partial^2 \Pi_j / (\partial k_j \partial k_I)$, $\partial^2 \Pi_j / (\partial k_j \partial k_h)$, $\partial^2 \Pi_h / (\partial k_h \partial k_j)$, $\partial^2 \Pi_I / (\partial k_I \partial k_j)$ and $\partial^2 \Pi_I / (\partial k_I \partial k_h)$. Because of symmetry we know that in equilibrium $\partial^2 \Pi_h / (\partial k_h \partial k_j) = \partial^2 \Pi_j / (\partial k_j \partial k_h)$ and $\partial^2 \Pi_I / (\partial k_I \partial k_h) = \partial^2 \Pi_I / (\partial k_I \partial k_j)$. The remaining terms can be written as

$$
\begin{aligned}
\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} =\ & P' \frac{dq_j}{dk_I} \left[\frac{dq_I}{dk_j} + (N-1) \frac{dq_h}{dk_j} \right] + P' q_j \left[\frac{d^2 q_I}{dk_j dk_I} + (N-1) \frac{d^2 q_h}{dk_j dk_I} \right] \\
& + P'' q_j \left[\frac{dq_I}{dk_j} + (N-1) \frac{dq_h}{dk_j} \right] \left[\frac{dq_I}{dk_I} + N \frac{dq_j}{dk_I} \right] + C_j'' \frac{q_j}{k_j^2} \frac{dq_j}{dk_I} - R' - k_j R'',
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} =\ & P' \frac{dq_j}{dk_h} \left[\frac{dq_I}{dk_j} + (N-1) \frac{dq_h}{dk_j} \right] + P' q_j \left[\frac{d^2 q_I}{dk_j dk_h} + (N-2) \frac{d^2 q_k}{dk_j dk_h} + \frac{d^2 q_h}{dk_j dk_h} \right] \\
& + P'' q_j \left[\frac{dq_I}{dk_j} + (N-1) \frac{dq_h}{dk_j} \right] \left[\frac{dq_h}{dk_h} + \frac{dq_I}{dk_h} + (N-1) \frac{dq_j}{dk_h} \right] + C_j'' \frac{q_j}{k_j^2} \frac{dq_j}{dk_h} - R' - k_j R'',
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j} =\ & P' \frac{dq_I}{dk_j} N \frac{dq_j}{dk_I} + P' q_I \left[\frac{d^2 q_j}{dk_j dk_I} + (N-1) \frac{d^2 q_h}{dk_j dk_I} \right] \\
& + P'' q_I N \frac{dq_I}{dk_j} \left[\frac{dq_I}{dk_I} + N \frac{dq_j}{dk_I} \right] + C_I'' \frac{q_I}{k_I^2} \frac{dq_I}{dk_I} - R' - (k_I - \underline{k}) R'',
\end{aligned} \tag{27}
$$

The second derivatives that appear in these expressions can be derived in the same way as above where we checked that the second-order conditions are satisfied.

Proceeding in a similar way as Kolstad and Mathiesen (1987), i.e., subtracting the first column in (24) from the other columns, and then dividing the i-th row by $\partial^2\Pi_i/\partial k_i\partial k_j - \partial^2\Pi_i/\partial k_i^2$, with $i = I, 1, ..., N$, yields

$$|J| = \begin{vmatrix} -\dfrac{\frac{\partial^2\Pi_j}{\partial k_j^2}}{\frac{\partial^2\Pi_j}{\partial k_j\partial k_h} - \frac{\partial^2\Pi_j}{\partial k_j^2}} & -1 & -1 & \cdots & -\dfrac{-\frac{\partial^2\Pi_j}{\partial k_j\partial k_I} + \frac{\partial^2\Pi_j}{\partial k_j^2}}{\frac{\partial^2\Pi_j}{\partial k_j\partial k_h} - \frac{\partial^2\Pi_j}{\partial k_j^2}} \\ -\dfrac{\frac{\partial^2\Pi_h}{\partial k_h\partial k_j}}{\frac{\partial^2\Pi_h}{\partial k_h\partial k_j} - \frac{\partial^2\Pi_h}{\partial k_h^2}} & 1 & 0 & 0 & \cdots -\dfrac{-\frac{\partial^2\Pi_h}{\partial k_h\partial k_I} + \frac{\partial^2\Pi_h}{\partial k_h\partial k_j}}{\frac{\partial^2\Pi_h}{\partial k_h\partial k_j} - \frac{\partial^2\Pi_h}{\partial k_h^2}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\dfrac{\frac{\partial^2\Pi_I}{\partial k_I\partial k_j}}{\frac{\partial^2\Pi_I}{\partial k_I\partial k_j} - \frac{\partial^2\Pi_I}{\partial k_I^2}} & 0 & 0 & \cdots & 1 \end{vmatrix}.$$

We can then calculate the determinant in a relatively straightforward way. Cumbersome but otherwise routine manipulations show that this determinant is unambiguously positive and, therefore, that the equilibrium of the capacity stage is unique. ∎

Proof of Lemma 4

As the second-order conditions are satisfied, we can use the Implicit Function Theorem to show that the sign of dk_j/dk_I equals the sign of $\partial^2\Pi_j/\partial k_j\partial k_I$, given by (25). Similarly, the sign of dk_j/dk_h equals the sign of $\partial^2\Pi_j/\partial k_j\partial k_h$ given by (26) and the sign of dk_I/dk_j equals the sign of $\partial^2\Pi_I/\partial k_I\partial k_j$ given by (27). In what follows, we will determine the sign of the representative term $\partial^2\Pi_j/\partial k_j\partial k_I$. Determining the sign of the other terms works in exactly the same way.

Proceeding along the same lines as in the proof of Lemma 3, where we determined the sign of the expression $\partial^2\Pi_j/\partial k_j^2$, we obtain

$$\frac{\partial^2\Pi_j}{\partial k_j\partial k_I} = \frac{q_j\left(\sum_{s=0}^{8}(P')^s(\sum_{h=1}^{3}\kappa_{sh}(P'')^h + \kappa_{s4}P''' + \kappa_{s5}C_j''' + \kappa_{s6}C_I''' + \kappa_{s7})\right)}{k_j^2 k_I(C_j'' - k_j P')^2\eta^3} - R' - k_j R''. \tag{28}$$

In this expression, $\kappa_{sh} = \kappa_{sh}(q_j, k_j, q_I, k_I, C_j'', C_I'', P', P'', N)$, $s \in \{0, ..., 8\}$ and $h \in \{1, ..., 7\}$. As above, we do not specify the explicit expressions for κ_{sh} here but only spell out their respective signs. These signs are the following: For $h = \{1, 2, 3\}$ $\kappa_{sh} \leq 0$, if both s and h are either even (including 0) or odd and $\kappa_{sh} \geq 0$ if one is even and the other one is odd. κ_{s4}, κ_{s5}, $\kappa_{s6} \leq 0$ for s even (including 0) and κ_{s4}, κ_{s5}, $\kappa_{s6} \geq 0$ for s odd. Finally, $\kappa_{s7} < 0$ for s even (including 0) and $\kappa_{s7} > 0$ for s odd, implying that the numerator in the fraction is negative because P'' is negative or not too positive and P''' and C''' are positive or not too negative.

As in Lemma 3, the denominator is positive because $\eta > 0$. Therefore, the first term in (28) is negative. Because $R' > 0$ and R'' is either positive or not too negative as well, we obtain $\partial^2 \Pi_j / \partial k_j \partial k_I < 0$. ∎

Proof of Lemma 5

Differentiating (3) and (4) with respect to $\underline{k}$ yields

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} \frac{dk_j}{d\underline{k}} + (N-1)\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} \frac{dk_h}{d\underline{k}} + \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{dk_I}{d\underline{k}} = 0$$

and

$$\frac{\partial^2 \Pi_I}{\partial k_I^2} \frac{dk_I}{d\underline{k}} + N\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j} \frac{dk_j}{d\underline{k}} + \frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}} = 0.$$

Using the fact that in equilibrium $dk_h/d\underline{k} = dk_j/d\underline{k}$ for $h, j \neq I$ we get

$$\frac{dk_j}{d\underline{k}} = \frac{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}}}{\frac{\partial^2 \Pi_j}{\partial k_j^2} \frac{\partial^2 \Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} \frac{\partial^2 \Pi_I}{\partial k_I^2} - N\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}} \tag{29}$$

and

$$\frac{dk_I}{d\underline{k}} = -\frac{\frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}} \left(\frac{\partial^2 \Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} \right)}{\frac{\partial^2 \Pi_j}{\partial k_j^2} \frac{\partial^2 \Pi_I}{\partial k_I^2} + (N-1)\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} \frac{\partial^2 \Pi_I}{\partial k_I^2} - N\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}}. \tag{30}$$

The terms that appear in these expressions are given by (20), (21), (25), (26), (27) and by

$$\frac{\partial^2 \Pi_I}{\partial k_I \partial \underline{k}} = R' > 0.$$

From the proofs of the previous lemmas we know that all terms in (25), (26) and (27) have a negative sign. Thus, the numerators of the fractions on the right-hand side of (29) and (30) are both negative. The denominator in these fractions is the same in both equations. It is easy to show that $|\partial^2 \Pi_j / \partial k_j^2| > |\partial^2 \Pi_j / (\partial k_j \partial k_h)|$ which implies that

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} \frac{\partial^2 \Pi_I}{\partial k_I^2} > \frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} \frac{\partial^2 \Pi_I}{\partial k_I^2}. \tag{31}$$

In addition one can also easily show that $|\partial^2 \Pi_I / \partial k_I^2| > |\partial^2 \Pi_I / (\partial k_I \partial k_j)|$. Tedious calculations then reveal that

$$\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h} \frac{\partial^2 \Pi_I}{\partial k_I^2} > \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}. \tag{32}$$

The inequalities in (31) and (32) then imply that the denominator is positive. As a consequence, we get that $dk_j/d\underline{k} < 0$ and $dk_I/d\underline{k} > 0$. ∎

Proof of Proposition 1

We start with the right-hand side of (6). As mentioned in the main text, if $\underline{k} = 0$, the right-hand side of (6) simplifies to $-1/N$.

We now turn to the left-hand side of (6). From equations (29) and (30) we obtain that it is given by

$$\frac{\left(\frac{dk_j^*}{d\underline{k}}\right)}{\left(\frac{dk_I^*}{d\underline{k}}\right)} = -\frac{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I}}{\frac{\partial^2 \Pi_j}{\partial k_j^2} + (N-1)\frac{\partial^2 \Pi_j}{\partial k_j \partial k_h}} < 0, \quad h \neq j, \, h, j = 1, ..., N. \tag{33}$$

At $\underline{k} = 0$, we know that there is no difference between firm I and any of the non-integrated firms. This implies that $\partial^2 \Pi_j/(\partial k_j \partial k_h) = \partial^2 \Pi_j/(\partial k_j \partial k_I)$. Now, because all the second derivatives appearing in (33) are known to be negative (from the proof of Lemmas 3 and 4), $(dk_j/d\underline{k})/(dk_I/d\underline{k}) > -1/N$ is equivalent to $\partial^2 \Pi_j/\partial k_j^2 - \partial^2 \Pi_j/\partial k_j \partial k_I < 0$. Because at $\underline{k} = 0$ all firms are symmetric, subtracting (25) from (20) yields

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} - \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} = P'N\frac{dq_h}{dk_j}\left[\frac{dq_j}{dk_j} - \frac{dq_h}{dk_j}\right]$$

$$+P'q_j\left[N\frac{d^2 q_h}{dk_j^2} - \frac{d^2 q_I}{dk_j dk_I} - (N-1)\frac{d^2 q_h}{dk_j dk_I}\right] + C_j''\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j} - \frac{dq_h}{dk_j}\right) - R'.$$

In the proof of Lemma 1 we determined dq_j/dk_j and dq_h/dk_j. Evaluating these expressions at $q_I = q_j$ and $k_I = k_j$ we can determine $dq_j/dk_j - dq_h/dk_j$ and $dq_j/dk_j - q_j/k_j - dq_h/dk_j$ to get

$$\frac{dq_j}{dk_j} - \frac{dq_h}{dk_j} = \frac{q_j C_j''}{k_j(C_j'' - P'k_j)} \quad \text{and} \quad \left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j} - \frac{dq_h}{dk_j}\right) = \frac{q_j P'}{C_j'' - P'k_j}. \tag{34}$$

Determining the second derivatives $(d^2 q_h)/(dk_j^2)$, $(d^2 q_I)/(dk_j dk_I)$ and $(d^2 q_h)/(dk_j dk_I)$ and using (34) we obtain, after simplifying,

$$\frac{\partial^2 \Pi_j}{\partial k_j^2} - \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} = -\frac{q_j^2 P'\xi}{k_j^2(C_j'' - (2+N)k_j P' - (1+N)k_j q_j P'')(C_j'' - P'k_j)^3} - R' < 0,$$

where

$$\xi = k_j^2\left(q_j C_j''' N + k_j C_j''(3N+2)\right)(P')^3+$$

$$\left(q_j k_j^2(k_j(1+3N)C_j'' + q_j C_j''' N)P'' - k_j C_j''(3k_j(N+2)C_j'' + q_j C_j''')\right)(P')^2+$$

$$\left(-q_j k_j C_j''(k_j C_j''(3N+4) + q_j C_j''')P'' + 5(C_j'')^3 k_j\right)P' - (C_j'')^3(C_j'' - 3q_j P''k_j) < 0.$$

That is, $\partial^2 \Pi_j/\partial k_j^2$ is larger in absolute terms than $(\partial^2 \Pi_j)/(\partial k_j \partial k_I)$. As a consequence, $(dk_j/d\underline{k})/(dk_I/d\underline{k}) > -1/N$, which implies that the left-hand side of (6) is larger than the right-hand side. Thus, at $\underline{k} = 0$ vertical integration is procompetitive at the margin. By continuity, it follows that vertical integration is procompetitive at the margin for all $\underline{k}$ below a certain, positive threshold denoted by $\underline{k}^\star$. ∎

Proof of Proposition 2

We show that for any finite N there either exists a $\underline{k}^{**} < \bar{\underline{k}}$, such that vertical integration is anticompetitive at the margin for all $\underline{k} > \underline{k}^{**}$, or it is procompetitive at the margin for all $\underline{k}$ close to $\bar{\underline{k}}$.

Let $\underline{k} = \bar{\underline{k}}$, so that $k_j = 0$ for all $j \neq I$. We first have to determine q_j/k_j in this case. Because C_j is strictly convex, C'_j is invertible and equation (2) can be written as $q_j = k_j C'^{-1}_j (P(Q) + P'(Q)q_j)$. It follows directly that if $k_j = 0$ we also have $q_j = 0$.

Observe that the inverse $C'^{-1}_j(.)$ is strictly increasing and that it is zero if and only if its argument is zero. By using the rule of L'Hôpital we get $q_j/k_j = C'^{-1}_j (P(q_I)) > 0$, if $q_j = 0$ and $k_j = 0$. To simplify notation in the following we denote $\rho \equiv C'^{-1}_j(P(q_I))$.

We now turn to (6). The right-hand side of (6) in the case of $\underline{k} = \bar{\underline{k}}$ can be written as

$$-\frac{C''_I \frac{q_I}{k_I}}{\rho(C''_I - k_I P')N}.$$

The left-hand side of (6) in case of $q_j = k_j = 0$ can be calculated from (29) and (30). To do so we first have to determine the second derivatives in (20), (21), (25), (26) and (27) at $q_j = k_j = 0$. From the right-hand side of (20) we know that $\partial^2\Pi_j/\partial k_j^2$ at $q_j = k_j = 0$ is given by

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = P'\frac{dq_j}{dk_j}\left[\frac{dq_I}{dk_j} + (N-1)\frac{dq_h}{dk_j}\right] + C''_j\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j}\right) - 2R'. \tag{35}$$

We can then calculate dq_I/dk_j, dq_h/dk_j and dq_j/dk_j at $q_j = k_j = 0$ from (15) and (16). Taking into account that $q_j/k_j = \rho$ we get, by using the rule of L'Hôpital, that

$$\frac{dq_I}{dk_j} = -\frac{k_I\rho(P' + q_I P'')}{2k_I P' + q_I k_I P'' - C''_I}, \quad \frac{dq_h}{dk_j} = 0 \quad \text{and} \quad \frac{dq_j}{dk_j} = \rho.$$

Calculating the second term of the right-hand side in (35) at $q_j = k_j = 0$ gives us, again by using L'Hôpital's rule, that

$$C''_j\frac{q_j}{k_j^2}\left(\frac{dq_j}{dk_j} - \frac{q_j}{k_j}\right) = \frac{\rho^2 P'(3k_I P' + q_I k_I P'' - 2C''_I)}{2k_I P' + q_I k_I P'' - C''_I}.$$

Inserting these terms into (35) and simplifying then yields

$$\frac{\partial^2\Pi_j}{\partial k_j^2} = 2\frac{\rho^2 P'(k_I P' - 2C''_I)}{2k_I P' + q_I k_I P'' - C''_I} - 2R'.$$

In the same way we can determine the expressions for $\partial^2\Pi_I/\partial k_I^2$, $\partial^2\Pi_j/(\partial k_j\partial k_I)$, $\partial^2\Pi_j/(\partial k_j\partial k_h)$ and $\partial^2\Pi_I/(\partial k_I\partial k_j)$ at $q_j = k_j = 0$. Inserting them in (29) and (30) and simplifying we obtain

that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} = -\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{(N+1)\left(\rho^2 P'(C_I'' - k_I P') + \sigma\right)},$$

with $\sigma \equiv R'k_I\left(2P'k_I + P''k_I q_I - C_I''\right) < 0$.

It follows that

$$\frac{\frac{dk_j}{d\underline{k}}}{\frac{dk_I}{d\underline{k}}} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_I P')N}$$

if and only if

$$-\left(\frac{N}{1+N}\right)\left(\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma}\right) < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_I P')}. \tag{36}$$

But the left-hand side of (36) can either be larger or smaller than the right-hand side. To see this suppose first that σ is small in absolute terms. In this case, the second term of the left-hand side is approximately the same as the right-hand side. But because $-N/(1+N) > -1$, the left-hand side is larger. By continuity, vertical integration can be procompetitive at the margin for all $\underline{k}$ close to $\bar{k}$. On the other hand, suppose that N is very large. In this case, $N/(1+N)$ is close to 1. We then have that vertical integration is anticompetitive at the margin if

$$-\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_I P')}. \tag{37}$$

Obviously the left-hand side equals the right-hand side if $\sigma = 0$. But because $\sigma < 0$ and $\rho^2 P'(C_I'' - k_I P') < P'C_I''\rho\frac{q_I}{k_I} < 0$, the inequality in (37) is fulfilled. ∎

Proof of Proposition 4

We first show that $q_j \to 0$ and $k_j \to 0$, $j \neq I$, as $N \to \infty$. Suppose to the contrary that $q_j > 0$. But because $Q = q_I + Nq_j$ and $P(Q) \leq 0$, as $N \to \infty$, the first-order condition given by (2) cannot be satisfied if $q_j > 0$, because the right-hand side is positive whereas the left-hand side would be negative. Therefore, $q_j \to 0$, as $N \to \infty$. Given this, suppose now that $k_j > 0$. But then in the first-order condition of the capacity stage, (3), the left-hand side would be negative whereas the right-hand side is zero. In order to fulfill this condition we must have $k_j \to 0$. Therefore, as $N \to \infty$, $q_j \to 0$ and $k_j \to 0$.

In the proof of Proposition 2 we already calculated the case of $q_j \to 0$ and $k_j \to 0$. In addition, for $N \to \infty$ we obtain from (36) that vertical integration is anticompetitive if

$$-\frac{C_I''P'\rho\frac{q_I}{k_I} + \sigma}{\rho^2 P'(C_I'' - k_I P') + \sigma} < -\frac{C_I''\frac{q_I}{k_I}}{\rho(C_I'' - k_I P')},$$

where ρ and σ are defined in the proof of Proposition 2. But we already showed in this proof that the inequality is fulfilled. Therefore, vertical integration is anticompetitive if $N \to \infty$. ∎

References

BOLTON, P., AND M. WHINSTON (1993): "Incomplete Contracts, Vertical Integration, and Supply Assurance," *Review of Economic Studies*, 60(1), 121–48.

BURKART, M., D. GROMB, AND F. PANUNZI (1998): "Why Higher Takeover Premia Protect Minority Shareholders.," *Journal of Political Economy*, 106(1), 172 – 204.

CHEN, Y. (2001): "On Vertical Mergers and their Competitive Effects," *RAND Journal of Economics*, 32, 667–685.

CHEN, Y., AND M. RIORDAN (2007): "Vertical integration, exclusive dealing and ex post cartelization," *RAND Journal of Economics*, 38, 1–21.

CHOI, J.-P., AND S.-S. YI (2000): "Vertical Foreclosure with the Choice of Input Specifications," *RAND Journal of Economics*, 31, 717–743.

CHURCH, J. (2008): "Vertical Mergers," in *Issues in Competition Law and Policy, Volume II*, ed. by W. Collins. American Bar Association, Chicago, 1455-1501.

DE FONTENAY, C., AND J. GANS (2005): "Vertical Integration in the Presence of Upstream Competition," *RAND Journal of Economics*, 36, 544–572.

ESÖ, P., V. NOCKE, AND L. WHITE (2010): "Competition for Scarce Resources," *The RAND Journal of Economics*, 41(3), 524–548.

GANS, J. (2007): "Concentration-Based Merger Tests and Vertical Market Structure," *Journal of Law and Economics*, 50, 661–680.

GILBERT, R., K. NEUHOFF, AND D. NEWBERY (2004): "Allocating Transmission to Mitigate Market Power in Electricity Networks.," *RAND Journal of Economics*, 35(4), 691 – 709.

HART, O., AND J. TIROLE (1990): "Vertical Integration and Market Foreclosure," *Brookings Papers on Economic Activity: Microeconomics*, pp. 205–276.

HENDRICKS, K., AND R. MCAFEE (2010): "A Theory of Bilateral Oligopoly," *Economic Inquiry*, 48(2), 391 414.

INDERST, R., AND T. M. VALLETTI (2011a): "Incentives for Input Foreclose," *European Economic Review*, 55(6), 820–831.

——— (2011b): "Buyer Power And The Waterbed Effect," *Journal of Industrial Economics*, 59(1), 1–20.

Joskow, P. L., and J. Tirole (2000): "Transmission Rights and Market Power on Electric Power Networks.," *RAND Journal of Economics*, 31(3), 450 – 487.

Kolstad, C. D., and L. Mathiesen (1987): "Necessary and Sufficient Conditions for a Cournot Equilibrium," *Review of Economic Studies*, LIV, 681–690.

Lafontaine, F., and M. Slade (2007): "Vertical Integration and Firm Boundaries: The Evidence," *Journal of Economic Literature*, 45, 631–687.

Nocke, V., and M. Whinston (2010): "Dynamic Merger Review," *Journal of Political Economy*, 118(6), 1200–1251.

Nocke, V., and L. White (2007): "Do Vertical Mergers Facilitate Upstream Collusion?," *American Economic Review*, 97(4), 1321–1339.

Normann, H.-T. (2009): "Vertical Integration, Raising Rival's Costs and Upstream Collusion," *European Economic Review*, 53, 461–480.

Ordover, J. A., G. Saloner, and S. C. Salop (1990): "Equilibrium Vertical Foreclosure," *American Economic Review*, 71, 127–42.

Perry, M. K. (1978): "Vertical Integration: The Monopsony Case," *American Economic Review*, 68(4), 561–70.

Perry, M. K., and R. H. Porter (1985): "Oligopoly and the Incentive for Horizontal Merger," *American Economic Review*, 75(1), 219–27.

Quirmbach, H. (1986): "The Path of Price Changes in Vertical Integration," *Journal of Political Economy*, 94(5), 1100–9.

Rey, P., and J. Tirole (2007): "A Primer on Foreclosure," in *Handbook of Industrial Organization, III*, ed. by M. Armstrong, and R. Porter. Elsevier, North-Holland, 2145-2220.

Riordan, M. H. (1998): "Anticompetitive Vertical Integration by a Dominant Firm," *American Economic Review*, 88, 1232–48.

——— (2008): "Competitive Effects of Vertical Integration," in *Handbook of Antitrust Economics*, ed. by P. Buccirossi. MIT Press, Cambridge, 145-182.

SALINGER, M. A. (1988): "Vertical Mergers and Market Foreclosure," *Quarterly Journal of Eonomics*, 103, 345–56.

VIVES, X. (1999): *Oligopoly Pricing*. MIT Press, Cambridge.